Computer Algebra
Independent Integration Tests

Summer 2023 edition

4-Trig-functions/4.2-Cosine/85-4.2.12-e-x-^m-a+b-cos-c+d-x^n-^p

Nasser M. Abbasi

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Contents

CHAPTER 1

INTRODUCTION

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [99]. This is test number [85].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.3 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (99)	0.00 (0)
Mathematica	100.00 (99)	0.00 (0)
Fricas	91.92 (91)	8.08 (8)
Maple	87.88 (87)	12.12 (12)
Maxima	69.70 (69)	30.30 (30)
Giac	52.53 (52)	47.47 (47)
Sympy	34.34 (34)	65.66 (65)
Mupad	30.30 (30)	69.70 (69)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

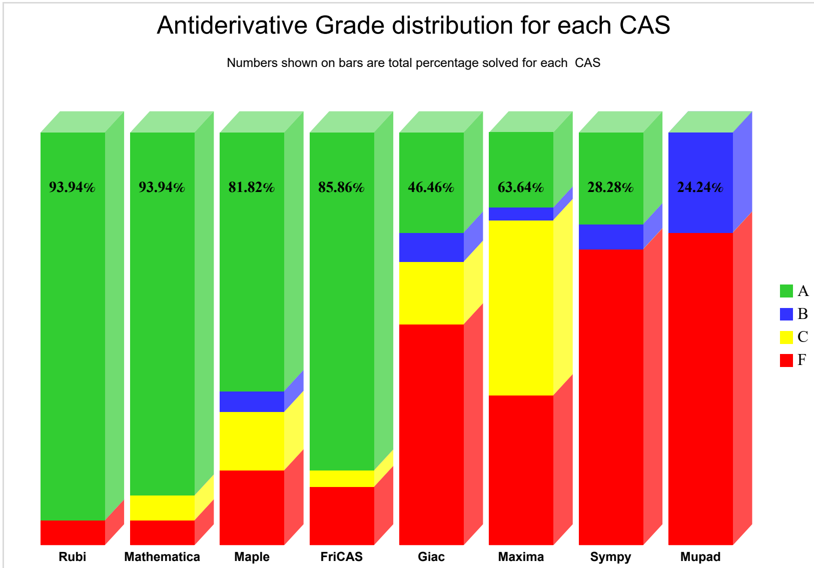
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

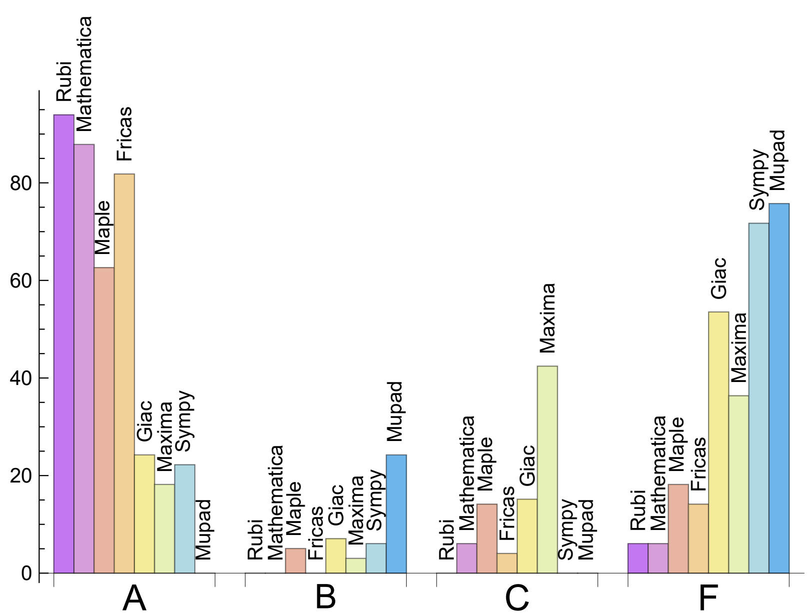
System	% A grade	% B grade	% C grade	% F grade
Rubi	93.939	0.000	0.000	6.061
Mathematica	87.879	0.000	6.061	6.061
Fricas	81.818	0.000	4.040	14.141
Maple	62.626	5.051	14.141	18.182
Giac	24.242	7.071	15.152	53.535
Sympy	22.222	6.061	0.000	71.717
Maxima	18.182	3.030	42.424	36.364
Mupad	0.000	24.242	0.000	75.758

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	0	0.00	0.00	0.00
Fricas	8	100.00	0.00	0.00
Maple	12	100.00	0.00	0.00
Maxima	30	60.00	0.00	40.00
Giac	47	100.00	0.00	0.00
Sympy	65	100.00	0.00	0.00
Mupad	69	0.00	100.00	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Rubi	0.16
Fricas	0.24
Maxima	0.43
Mathematica	0.50
Giac	0.57
Maple	1.00
Sympy	2.67
Mupad	10.90

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Mupad	32.23	0.88	26.50	0.83
Fricas	75.74	0.85	62.00	0.81
Sympy	87.74	1.40	45.00	1.13
Mathematica	90.46	0.93	81.00	0.96
Rubi	103.99	1.00	85.00	1.00
Maxima	109.64	1.22	73.00	1.00
Giac	114.50	1.29	65.50	1.16
Maple	145.47	1.23	64.00	0.93

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

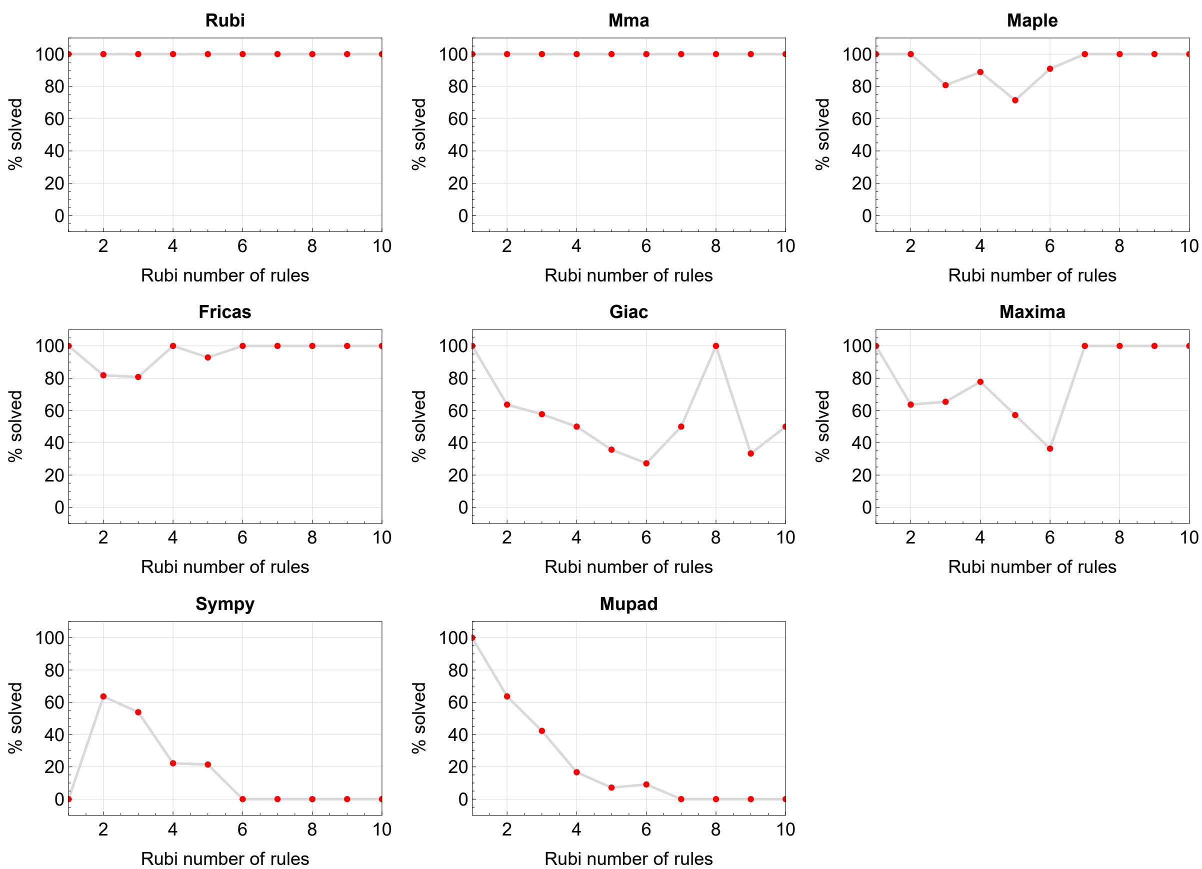


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

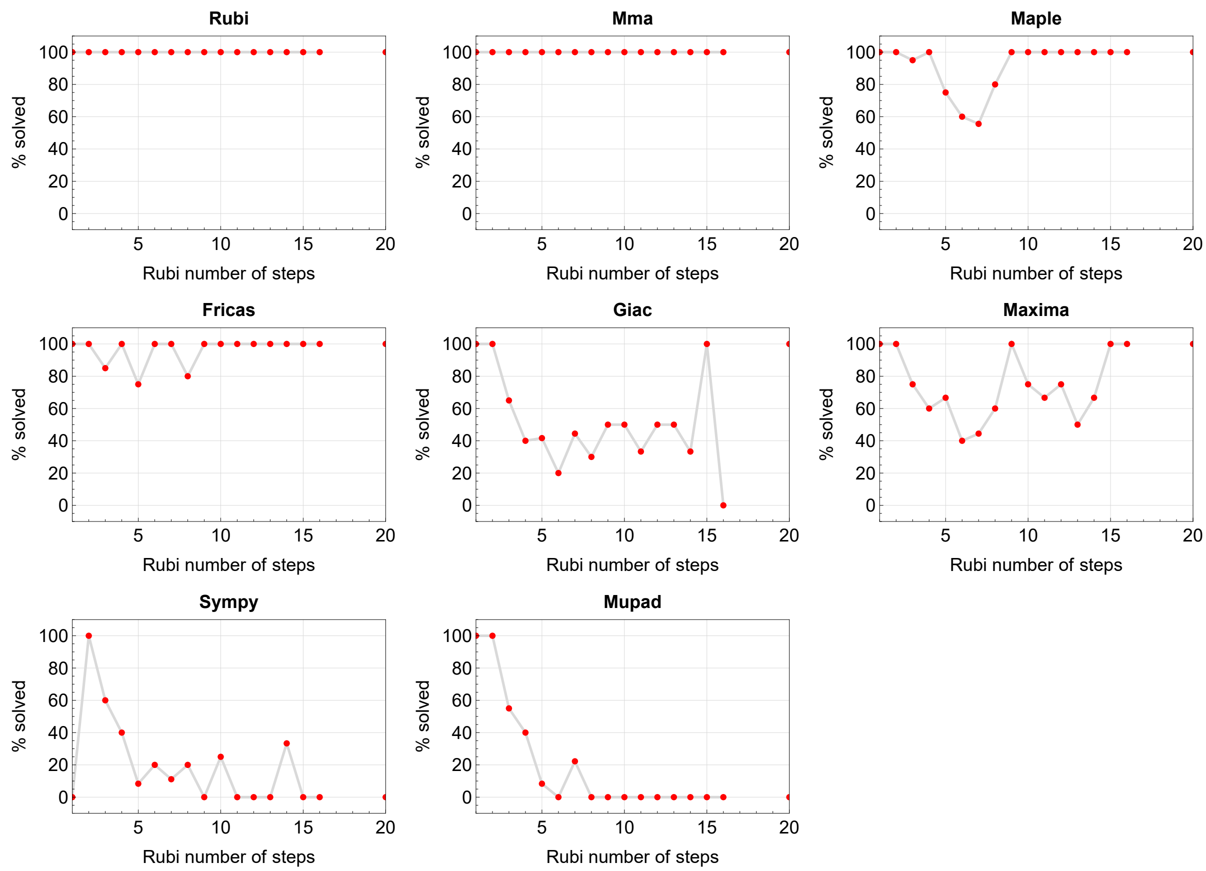


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

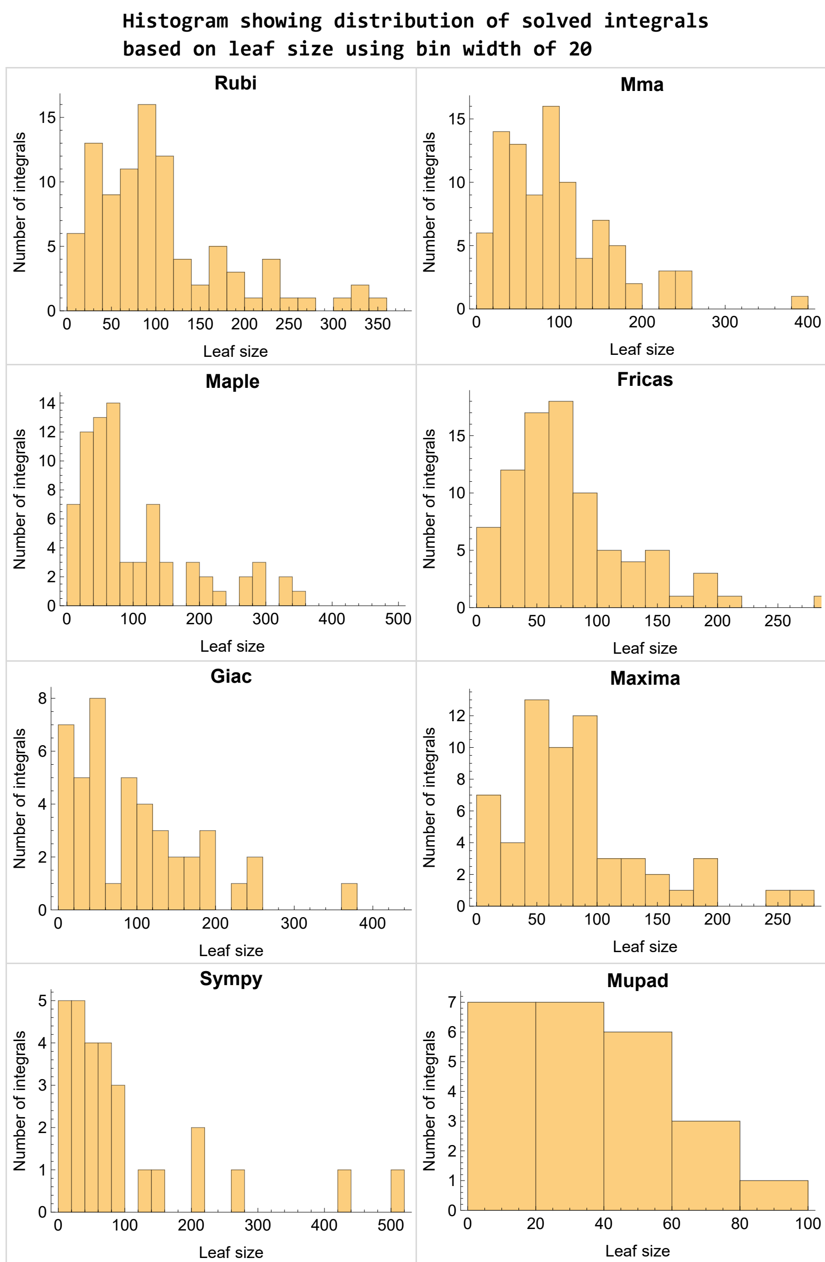


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

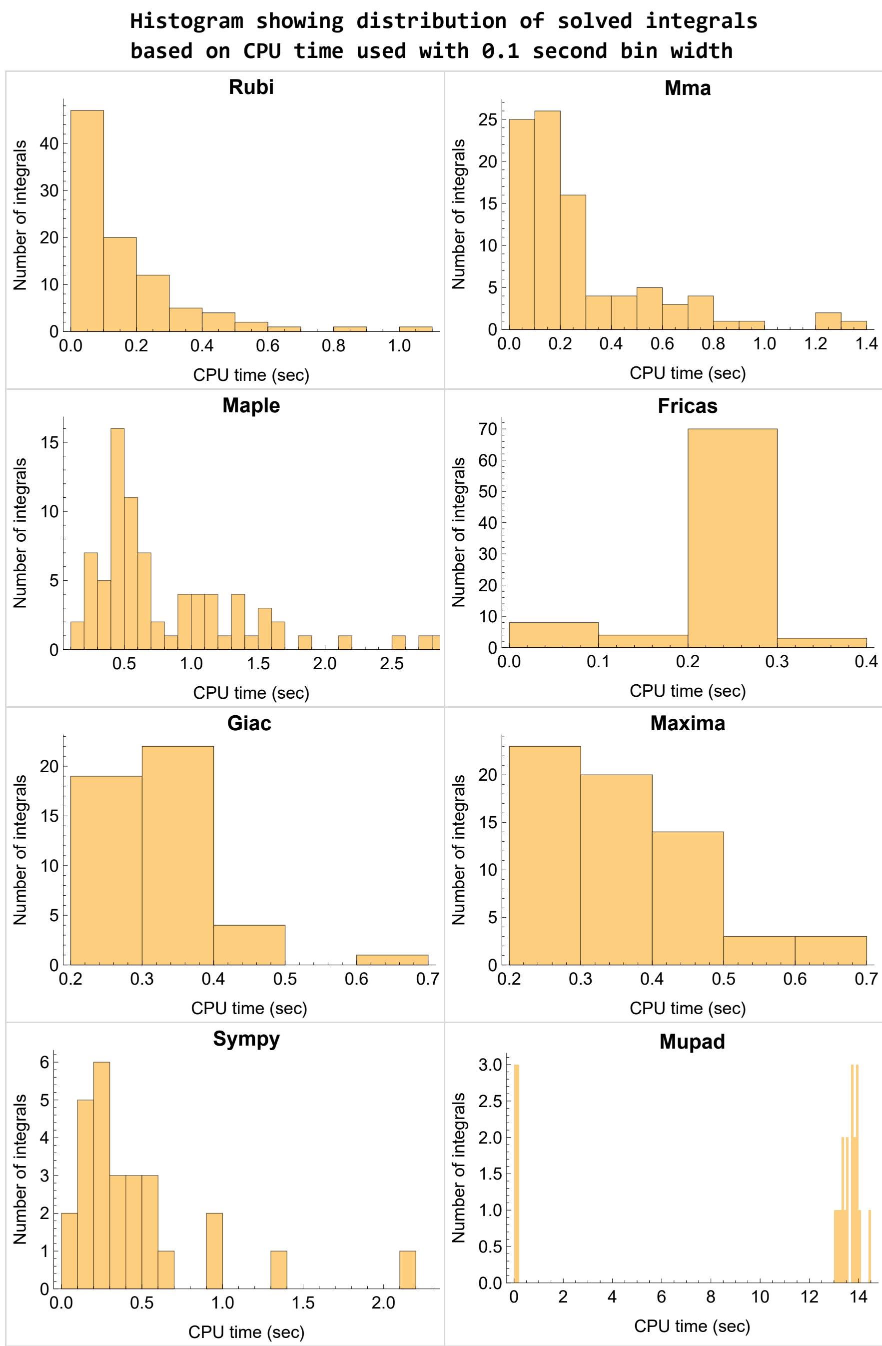


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

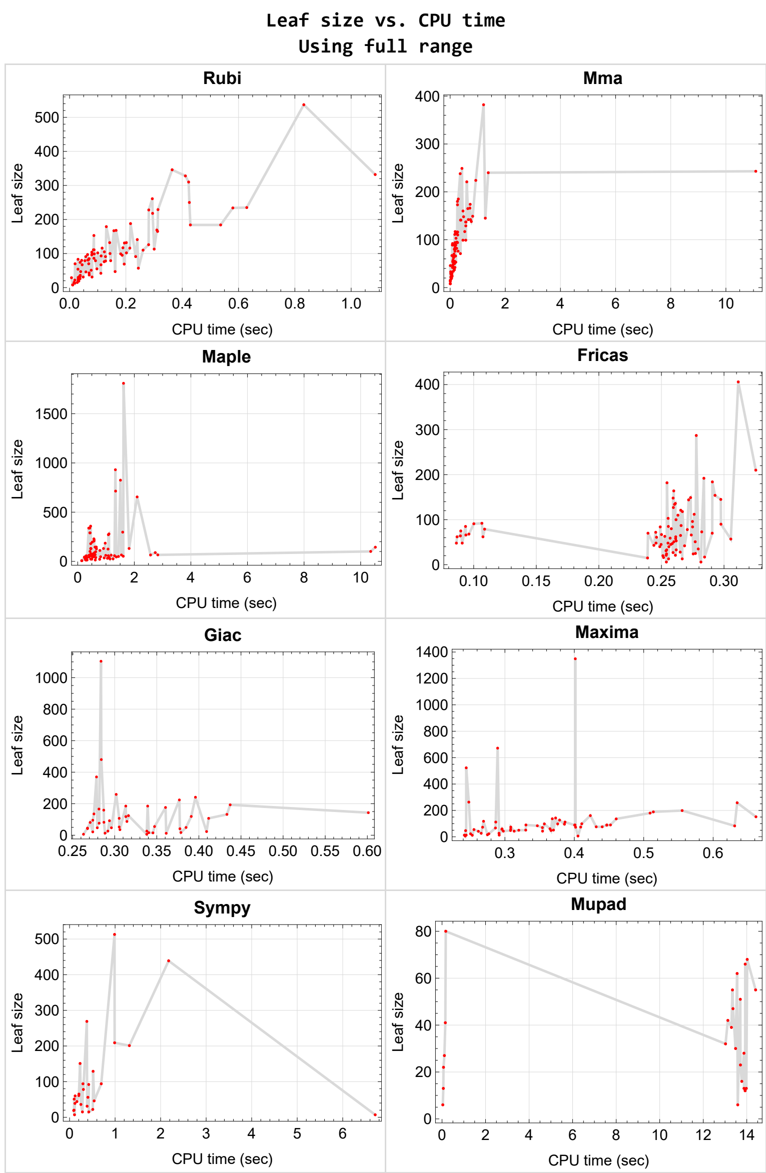


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

$\{63, 64, 66, 68, 88, 89\}$

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {}

Mathematica {}

Maple $\{12, 14, 19, 21\}$

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
```

```
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

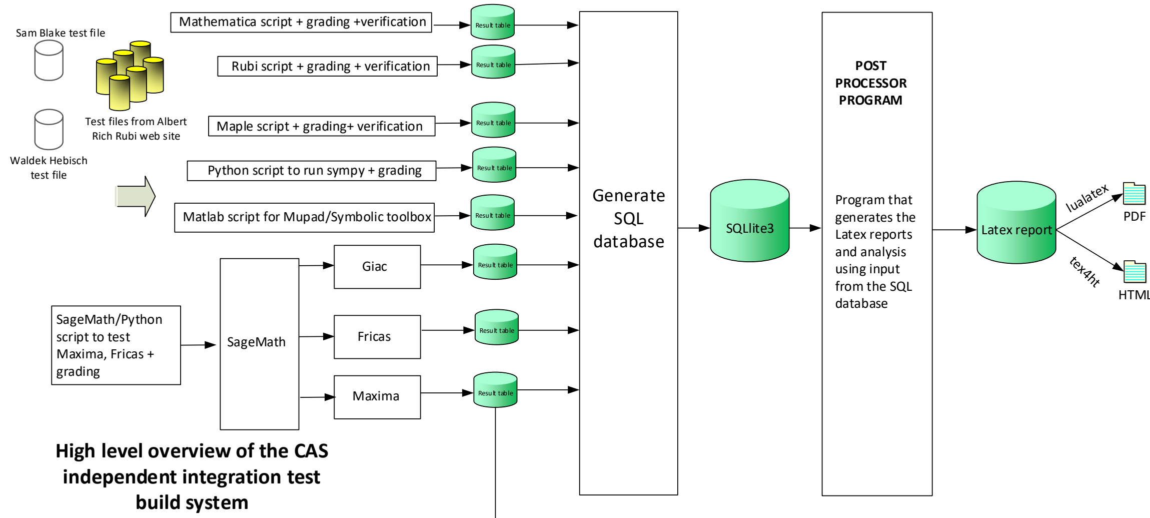
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer, the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"
13. String. Small string description of why the grade was given.
14. integer. 1 if result was verified or 0 if not verified. (For mma, rubi and maple only)

*The following fields are present only in **Rubi** Table file*

15. integer. Number of steps used.
16. integer. Number of rules used.
17. integer. Integrand leaf size.
18. real number. Ratio. Field 16 over field 17
19. String of form "{n,n,..}" which is list of the rules used by Rubi
20. String. The optimal antiderivative in Mathematica syntx

Nasser M. Abbasi
June 27, 2023
Design.vsdx

CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 65, 67, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 65, 67, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 91, 92, 96, 97 }

B grade { }

C grade { 90, 93, 94, 95, 98, 99 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 13, 15, 16, 17, 18, 20, 22, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 69, 70, 71, 72, 79, 80, 81, 82, 83, 84, 85, 86, 87, 91, 92, 97 }

B grade { 90, 93, 94, 95, 96 }

C grade { 12, 14, 19, 21, 23, 24, 25, 26, 27, 28, 73, 76, 98, 99 }

F normal fail { 29, 30, 31, 32, 33, 34, 65, 67, 74, 75, 77, 78 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 69, 70, 71, 72, 79, 80, 81, 82, 83, 84, 85, 86, 87, 90, 91, 92, 95, 96, 97 }

B grade { }

C grade { 93, 94, 98, 99 }

F normal fail { 65, 67, 73, 74, 75, 76, 77, 78 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maxima

A grade { 1, 3, 8, 10, 15, 17, 22, 37, 43, 45, 46, 47, 48, 61, 62, 91, 92, 97 }

B grade { 90, 95, 96 }

C grade { 2, 4, 5, 6, 7, 9, 11, 12, 13, 14, 16, 18, 19, 20, 21, 35, 36, 38, 39, 40, 41, 42, 44, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 69, 70, 71, 72, 85, 86, 87 }

F normal fail { 65, 67, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 93, 94, 98, 99 }

F(-1) timedout fail { }

F(-2) exception fail { 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34 }

Giac

A grade { 1, 3, 5, 8, 10, 12, 15, 17, 19, 22, 37, 38, 43, 45, 46, 47, 48, 61, 62, 90, 91, 92, 96, 97 }

B grade { 7, 14, 21, 35, 36, 39, 95 }

C grade { 2, 4, 9, 11, 16, 18, 49, 50, 51, 55, 56, 57, 85, 86, 87 }

F normal fail { 6, 13, 20, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 40, 41, 42, 44, 52, 53, 54, 58, 59, 60, 65, 67, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 93, 94, 98, 99 }

F(-1) timedout fail { }

F(-2) exception fail { }

Mupad

A grade { }

B grade { 1, 3, 4, 8, 10, 15, 17, 22, 37, 38, 39, 42, 43, 45, 46, 47, 48, 61, 62, 85, 86, 87, 92, 97 }

C grade { }

F normal fail { }

F(-1) timedout fail { 2, 5, 6, 7, 9, 11, 12, 13, 14, 16, 18, 19, 20, 21, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 40, 41, 44, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 65, 67, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 90, 91, 93, 94, 95, 96, 98, 99 }

F(-2) exception fail { }

Sympy

A grade { 1, 3, 4, 8, 11, 15, 17, 18, 22, 36, 37, 38, 39, 43, 46, 47, 48, 62, 90, 91, 92, 97 }

B grade { 2, 9, 10, 16, 45, 61 }

C grade { }

F normal fail { 5, 6, 7, 12, 13, 14, 19, 20, 21, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 40, 41, 42, 44, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 65, 67, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 93, 94, 95, 96, 98, 99 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name **`Mathematica`** was abbreviated to **`MMA`**.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	29	31	27	27	36	45	27
N.S.	1	1.00	0.85	0.91	0.79	0.79	1.06	1.32	0.79
time (sec)	N/A	0.036	0.064	1.050	0.292	0.259	0.249	0.268	0.109

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	B	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	82	58	67	72	209	135	0
N.S.	1	1.00	0.90	0.64	0.74	0.79	2.30	1.48	0.00
time (sec)	N/A	0.057	0.167	1.082	0.286	0.245	0.989	0.275	0.000

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	19	13	13
N.S.	1	1.00	1.00	0.93	0.87	0.87	1.27	0.87	0.87
time (sec)	N/A	0.016	0.005	0.621	0.242	0.267	0.091	0.289	0.060

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	A	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	57	44	48	61	61	95	51
N.S.	1	1.00	0.81	0.63	0.69	0.87	0.87	1.36	0.73
time (sec)	N/A	0.020	0.102	0.236	0.308	0.246	0.202	0.274	13.703

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	24	22	43	21	0	21	0
N.S.	1	1.00	0.96	0.88	1.72	0.84	0.00	0.84	0.00
time (sec)	N/A	0.030	0.060	0.289	0.354	0.261	0.000	0.274	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	81	57	73	70	0	0	0
N.S.	1	1.00	1.01	0.71	0.91	0.88	0.00	0.00	0.00
time (sec)	N/A	0.043	0.196	0.301	0.402	0.239	0.000	0.000	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	42	39	48	40	0	87	0
N.S.	1	1.00	1.00	0.93	1.14	0.95	0.00	2.07	0.00
time (sec)	N/A	0.111	0.083	0.333	0.366	0.249	0.000	0.315	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	40	42	42	45	78	76	41
N.S.	1	1.00	0.78	0.82	0.82	0.88	1.53	1.49	0.80
time (sec)	N/A	0.077	0.139	0.524	0.262	0.257	0.304	0.282	0.153

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	B	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	87	63	90	84	201	118	0
N.S.	1	1.00	0.96	0.69	0.99	0.92	2.21	1.30	0.00
time (sec)	N/A	0.127	0.181	0.452	0.330	0.249	1.313	0.313	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	27	23	23	28	60	26	22
N.S.	1	1.00	0.87	0.74	0.74	0.90	1.94	0.84	0.71
time (sec)	N/A	0.048	0.044	0.459	0.277	0.265	0.122	0.338	0.068

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	A	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	67	45	70	59	56	82	0
N.S.	1	1.00	0.96	0.64	1.00	0.84	0.80	1.17	0.00
time (sec)	N/A	0.068	0.063	0.404	0.354	0.257	0.403	0.287	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	C	A	F	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	37	37	34	68	51	31	0	35	0
N.S.	1	1.00	0.92	1.84	1.38	0.84	0.00	0.95	0.00
time (sec)	N/A	0.070	0.113	0.616	0.370	0.254	0.000	0.306	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	76	62	83	66	0	0	0
N.S.	1	1.00	1.00	0.82	1.09	0.87	0.00	0.00	0.00
time (sec)	N/A	0.123	0.175	0.398	0.631	0.274	0.000	0.000	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	C	A	F	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	57	57	50	98	61	48	0	107	0
N.S.	1	1.00	0.88	1.72	1.07	0.84	0.00	1.88	0.00
time (sec)	N/A	0.244	0.134	0.635	0.365	0.246	0.000	0.305	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	55	66	58	58	92	92	66
N.S.	1	1.00	0.70	0.84	0.73	0.73	1.16	1.16	0.84
time (sec)	N/A	0.092	0.163	0.527	0.308	0.261	0.420	0.294	13.928

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	B	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	188	160	130	143	148	439	259	0
N.S.	1	1.00	0.85	0.69	0.76	0.79	2.34	1.38	0.00
time (sec)	N/A	0.216	0.467	0.457	0.373	0.259	2.176	0.302	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	26	27	25	44	26	28
N.S.	1	1.00	1.00	0.79	0.82	0.76	1.33	0.79	0.85
time (sec)	N/A	0.037	0.019	1.250	0.266	0.277	0.162	0.292	13.870

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	A	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	116	101	112	121	129	185	0
N.S.	1	1.00	0.76	0.66	0.73	0.79	0.84	1.21	0.00
time (sec)	N/A	0.086	0.254	0.434	0.387	0.265	0.517	0.339	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	C	A	F	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	55	55	50	125	89	47	0	47	0
N.S.	1	1.00	0.91	2.27	1.62	0.85	0.00	0.85	0.00
time (sec)	N/A	0.097	0.156	1.072	0.400	0.259	0.000	0.279	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	166	128	152	136	0	0	0
N.S.	1	1.00	0.99	0.76	0.90	0.81	0.00	0.00	0.00
time (sec)	N/A	0.165	0.716	0.661	0.662	0.261	0.000	0.000	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	C	A	F	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	91	91	90	185	98	80	0	185	0
N.S.	1	1.00	0.99	2.03	1.08	0.88	0.00	2.03	0.00
time (sec)	N/A	0.235	0.190	0.972	0.376	0.258	0.000	0.313	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	54	50	55	51	94	52	55
N.S.	1	1.00	0.81	0.75	0.82	0.76	1.40	0.78	0.82
time (sec)	N/A	0.045	0.093	1.441	0.255	0.277	0.696	0.305	14.405

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	113	229	0	64	0	0	0
N.S.	1	1.00	1.02	2.06	0.00	0.58	0.00	0.00	0.00
time (sec)	N/A	0.087	0.210	0.572	0.000	0.089	0.000	0.000	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	111	290	0	62	0	0	0
N.S.	1	1.00	1.00	2.61	0.00	0.56	0.00	0.00	0.00
time (sec)	N/A	0.085	0.168	0.439	0.000	0.087	0.000	0.000	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	89	290	0	48	0	0	0
N.S.	1	1.00	1.10	3.58	0.00	0.59	0.00	0.00	0.00
time (sec)	N/A	0.067	0.106	0.429	0.000	0.086	0.000	0.000	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	89	338	0	48	0	0	0
N.S.	1	1.00	1.10	4.17	0.00	0.59	0.00	0.00	0.00
time (sec)	N/A	0.065	0.079	0.390	0.000	0.091	0.000	0.000	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	114	338	0	66	0	0	0
N.S.	1	1.00	1.16	3.45	0.00	0.67	0.00	0.00	0.00
time (sec)	N/A	0.080	0.205	0.426	0.000	0.094	0.000	0.000	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	117	358	0	75	0	0	0
N.S.	1	1.00	1.12	3.44	0.00	0.72	0.00	0.00	0.00
time (sec)	N/A	0.079	0.180	0.456	0.000	0.090	0.000	0.000	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	132	132	142	0	0	92	0	0	0
N.S.	1	1.00	1.08	0.00	0.00	0.70	0.00	0.00	0.00
time (sec)	N/A	0.202	0.738	0.000	0.000	0.107	0.000	0.000	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	132	132	142	0	0	91	0	0	0
N.S.	1	1.00	1.08	0.00	0.00	0.69	0.00	0.00	0.00
time (sec)	N/A	0.142	0.659	0.000	0.000	0.100	0.000	0.000	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	100	100	99	0	0	68	0	0	0
N.S.	1	1.00	0.99	0.00	0.00	0.68	0.00	0.00	0.00
time (sec)	N/A	0.144	0.566	0.000	0.000	0.096	0.000	0.000	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	96	96	99	0	0	62	0	0	0
N.S.	1	1.00	1.03	0.00	0.00	0.65	0.00	0.00	0.00
time (sec)	N/A	0.079	0.464	0.000	0.000	0.107	0.000	0.000	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	117	117	137	0	0	79	0	0	0
N.S.	1	1.00	1.17	0.00	0.00	0.68	0.00	0.00	0.00
time (sec)	N/A	0.189	0.563	0.000	0.000	0.109	0.000	0.000	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	116	116	137	0	0	85	0	0	0
N.S.	1	1.00	1.18	0.00	0.00	0.73	0.00	0.00	0.00
time (sec)	N/A	0.116	0.573	0.000	0.000	0.093	0.000	0.000	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	39	57	33	0	132	0
N.S.	1	1.00	1.00	1.26	1.84	1.06	0.00	4.26	0.00
time (sec)	N/A	0.080	0.038	0.640	0.296	0.254	0.000	0.433	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	21	43	20	15	41	0
N.S.	1	1.00	1.00	1.05	2.15	1.00	0.75	2.05	0.00
time (sec)	N/A	0.030	0.054	0.776	0.298	0.251	0.422	0.377	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	14	13	15	15	15	13
N.S.	1	1.00	1.00	1.08	1.00	1.15	1.15	1.15	1.00
time (sec)	N/A	0.018	0.006	0.293	0.292	0.239	0.282	0.345	13.872

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	29	35	51	33	31	49	30
N.S.	1	1.00	0.97	1.17	1.70	1.10	1.03	1.63	1.00
time (sec)	N/A	0.031	0.056	0.935	0.330	0.262	0.386	0.385	13.486

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	46	47	50	43	46	107	47
N.S.	1	1.00	1.00	1.02	1.09	0.93	1.00	2.33	1.02
time (sec)	N/A	0.058	0.007	0.810	0.320	0.244	0.540	0.412	13.366

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	80	57	127	73	0	0	0
N.S.	1	1.00	1.01	0.72	1.61	0.92	0.00	0.00	0.00
time (sec)	N/A	0.054	0.151	0.477	0.379	0.282	0.000	0.000	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	24	22	43	21	0	0	0
N.S.	1	1.00	0.96	0.88	1.72	0.84	0.00	0.00	0.00
time (sec)	N/A	0.033	0.049	0.496	0.313	0.254	0.000	0.000	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	62	48	98	65	0	0	55
N.S.	1	1.00	0.84	0.65	1.32	0.88	0.00	0.00	0.74
time (sec)	N/A	0.037	0.121	0.571	0.357	0.265	0.000	0.000	13.343

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	17	22	17	13
N.S.	1	1.00	1.00	0.93	0.87	1.13	1.47	1.13	0.87
time (sec)	N/A	0.027	0.005	0.332	0.244	0.284	0.510	0.379	13.978

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	88	64	74	84	0	0	0
N.S.	1	1.00	0.91	0.66	0.76	0.87	0.00	0.00	0.00
time (sec)	N/A	0.064	0.180	0.532	0.308	0.275	0.000	0.000	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	18	14	12	13	39	12	12
N.S.	1	1.00	0.95	0.74	0.63	0.68	2.05	0.63	0.63
time (sec)	N/A	0.033	0.036	0.295	0.253	0.256	0.117	0.361	13.923

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	7	6	6
N.S.	1	1.00	1.00	0.88	0.75	0.75	0.88	0.75	0.75
time (sec)	N/A	0.011	0.004	0.148	0.243	0.282	0.108	0.338	0.036

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	17	16	16	20	16	16
N.S.	1	1.00	1.00	0.77	0.73	0.73	0.91	0.73	0.73
time (sec)	N/A	0.019	0.025	0.271	0.275	0.252	0.098	0.340	13.769

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	31	34	23	24	51	23	23
N.S.	1	1.00	0.86	0.94	0.64	0.67	1.42	0.64	0.64
time (sec)	N/A	0.038	0.035	0.273	0.251	0.275	0.101	0.409	13.709

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	235	235	165	196	136	145	0	241	0
N.S.	1	1.00	0.70	0.83	0.58	0.62	0.00	1.03	0.00
time (sec)	N/A	0.629	0.640	0.516	0.460	0.297	0.000	0.396	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	141	131	112	118	0	193	0
N.S.	1	1.00	0.83	0.78	0.66	0.70	0.00	1.14	0.00
time (sec)	N/A	0.310	0.388	0.466	0.287	0.267	0.000	0.437	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	94	64	73	78	0	143	0
N.S.	1	1.00	0.95	0.65	0.74	0.79	0.00	1.44	0.00
time (sec)	N/A	0.182	0.177	0.428	0.268	0.271	0.000	0.601	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	110	78	74	96	0	0	0
N.S.	1	1.00	1.00	0.71	0.67	0.87	0.00	0.00	0.00
time (sec)	N/A	0.261	0.274	0.465	0.409	0.275	0.000	0.000	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	180	129	76	134	0	0	0
N.S.	1	1.00	0.98	0.70	0.41	0.73	0.00	0.00	0.00
time (sec)	N/A	0.429	0.263	0.468	0.440	0.260	0.000	0.000	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	250	250	238	180	76	164	0	0	0
N.S.	1	1.00	0.95	0.72	0.30	0.66	0.00	0.00	0.00
time (sec)	N/A	0.425	0.364	0.459	0.432	0.260	0.000	0.000	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	310	310	174	219	161	184	0	224	0
N.S.	1	1.00	0.56	0.71	0.52	0.59	0.00	0.72	0.00
time (sec)	N/A	0.423	0.727	0.579	0.423	0.291	0.000	0.377	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	218	218	148	145	137	144	0	176	0
N.S.	1	1.00	0.68	0.67	0.63	0.66	0.00	0.81	0.00
time (sec)	N/A	0.295	0.487	0.612	0.368	0.272	0.000	0.360	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	103	67	96	90	0	124	0
N.S.	1	1.00	1.01	0.66	0.94	0.88	0.00	1.22	0.00
time (sec)	N/A	0.202	0.212	0.525	0.386	0.298	0.000	0.316	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	116	87	88	100	0	0	0
N.S.	1	1.00	1.00	0.75	0.76	0.86	0.00	0.00	0.00
time (sec)	N/A	0.214	0.272	0.587	0.447	0.262	0.000	0.000	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	228	228	185	146	90	154	0	0	0
N.S.	1	1.00	0.81	0.64	0.39	0.68	0.00	0.00	0.00
time (sec)	N/A	0.281	0.296	0.656	0.452	0.293	0.000	0.000	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	328	328	249	207	90	192	0	0	0
N.S.	1	1.00	0.76	0.63	0.27	0.59	0.00	0.00	0.00
time (sec)	N/A	0.411	0.428	0.577	0.447	0.284	0.000	0.000	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	66	58	47	48	513	47	62
N.S.	1	1.00	0.77	0.67	0.55	0.56	5.97	0.55	0.72
time (sec)	N/A	0.086	0.105	1.300	0.243	0.255	0.988	0.296	13.557

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	7	6	6
N.S.	1	1.00	1.00	0.88	0.75	0.75	0.88	0.75	0.75
time (sec)	N/A	0.011	0.005	0.140	0.405	0.254	6.713	0.263	13.585

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	20	20	17	20	20
N.S.	1	1.00	1.11	1.00	1.11	1.11	0.94	1.11	1.11
time (sec)	N/A	0.024	1.532	0.328	1.220	0.274	8.542	0.944	13.233

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	22	22	19	22	22
N.S.	1	1.00	1.10	1.00	1.10	1.10	0.95	1.10	1.10
time (sec)	N/A	0.026	1.667	0.240	1.610	0.270	27.056	5.770	13.098

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	93	93	89	0	0	0	0	0	0
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.112	0.204	0.000	0.000	0.000	0.000	0.000	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	24	24	20	24	24
N.S.	1	1.00	1.09	1.00	1.09	1.09	0.91	1.09	1.09
time (sec)	N/A	0.060	1.750	0.403	1.421	0.274	8.259	0.940	13.325

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	131	131	149	0	0	0	0	0	0
N.S.	1	1.00	1.14	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.195	0.817	0.000	0.000	0.000	0.000	0.000	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	24	26	26	22	26	26
N.S.	1	1.00	1.08	1.00	1.08	1.08	0.92	1.08	1.08
time (sec)	N/A	0.059	1.853	0.371	1.683	0.263	25.848	5.988	14.403

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	24	25	90	24	0	0	0
N.S.	1	1.00	0.92	0.96	3.46	0.92	0.00	0.00	0.00
time (sec)	N/A	0.037	0.074	1.141	0.401	0.254	0.000	0.000	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	37	40	99	35	0	0	0
N.S.	1	1.00	0.86	0.93	2.30	0.81	0.00	0.00	0.00
time (sec)	N/A	0.069	0.153	1.111	0.411	0.279	0.000	0.000	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	53	52	180	51	0	0	0
N.S.	1	1.00	0.79	0.78	2.69	0.76	0.00	0.00	0.00
time (sec)	N/A	0.111	0.179	1.610	0.509	0.260	0.000	0.000	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	66	66	189	62	0	0	0
N.S.	1	1.00	0.84	0.84	2.39	0.78	0.00	0.00	0.00
time (sec)	N/A	0.146	0.196	2.831	0.514	0.261	0.000	0.000	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	92	75	0	0	0	0	0
N.S.	1	1.00	1.11	0.90	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.030	0.087	0.517	0.000	0.000	0.000	0.000	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	102	102	94	0	0	0	0	0	0
N.S.	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.100	0.267	0.000	0.000	0.000	0.000	0.000	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	179	179	173	0	0	0	0	0	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.131	0.266	0.000	0.000	0.000	0.000	0.000	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	115	111	0	0	0	0	0
N.S.	1	1.00	1.10	1.06	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.124	0.219	0.784	0.000	0.000	0.000	0.000	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	141	141	129	0	0	0	0	0	0
N.S.	1	1.00	0.91	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.240	0.567	0.000	0.000	0.000	0.000	0.000	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	229	229	221	0	0	0	0	0	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.314	0.603	0.000	0.000	0.000	0.000	0.000	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	45	45	0	45	0	0	0
N.S.	1	1.00	0.96	0.96	0.00	0.96	0.00	0.00	0.00
time (sec)	N/A	0.162	0.099	1.365	0.000	0.250	0.000	0.000	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	53	65	0	53	0	0	0
N.S.	1	1.00	0.77	0.94	0.00	0.77	0.00	0.00	0.00
time (sec)	N/A	0.194	0.205	2.574	0.000	0.262	0.000	0.000	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	95	101	0	85	0	0	0
N.S.	1	1.00	0.84	0.89	0.00	0.75	0.00	0.00	0.00
time (sec)	N/A	0.300	0.245	10.355	0.000	0.256	0.000	0.000	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	70	65	0	70	0	0	0
N.S.	1	1.00	0.90	0.83	0.00	0.90	0.00	0.00	0.00
time (sec)	N/A	0.128	0.139	1.545	0.000	0.291	0.000	0.000	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	82	89	0	87	0	0	0
N.S.	1	1.00	0.86	0.94	0.00	0.92	0.00	0.00	0.00
time (sec)	N/A	0.187	0.236	2.741	0.000	0.266	0.000	0.000	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	141	144	0	127	0	0	0
N.S.	1	1.00	0.85	0.87	0.00	0.77	0.00	0.00	0.00
time (sec)	N/A	0.313	0.377	10.520	0.000	0.260	0.000	0.000	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	76	131	258	112	0	159	80
N.S.	1	1.00	0.77	1.32	2.61	1.13	0.00	1.61	0.81
time (sec)	N/A	0.088	0.288	1.816	0.635	0.276	0.000	0.287	0.172

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	42	63	199	63	0	119	39
N.S.	1	1.00	0.89	1.34	4.23	1.34	0.00	2.53	0.83
time (sec)	N/A	0.039	0.175	1.182	0.555	0.253	0.000	0.391	13.295

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	36	84	40	0	55	32
N.S.	1	1.00	1.00	1.24	2.90	1.38	0.00	1.90	1.10
time (sec)	N/A	0.007	0.023	0.246	0.347	0.256	0.000	0.347	13.023

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	12	14	12	14	23	20	14	14
N.S.	1	1.00	1.17	1.00	1.17	1.92	1.67	1.17	1.17
time (sec)	N/A	0.010	2.301	0.153	0.418	0.246	0.993	0.308	13.149

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	12	14	12	14	23	22	14	14
N.S.	1	1.00	1.17	1.00	1.17	1.92	1.83	1.17	1.17
time (sec)	N/A	0.010	3.929	0.158	0.416	0.253	0.949	0.318	13.085

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	B	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	346	346	224	825	672	103	269	480	0
N.S.	1	1.00	0.65	2.38	1.94	0.30	0.78	1.39	0.00
time (sec)	N/A	0.364	0.930	1.511	0.290	0.255	0.379	0.284	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	71	297	263	67	151	166	0
N.S.	1	1.00	0.43	1.78	1.57	0.40	0.90	0.99	0.00
time (sec)	N/A	0.157	0.372	1.590	0.248	0.250	0.231	0.281	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	48	61	60	42	65	42	42
N.S.	1	1.00	0.89	1.13	1.11	0.78	1.20	0.78	0.78
time (sec)	N/A	0.030	0.103	0.990	0.296	0.268	0.205	0.268	13.131

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	145	271	0	149	0	0	0
N.S.	1	1.00	1.15	2.15	0.00	1.18	0.00	0.00	0.00
time (sec)	N/A	0.281	1.275	1.083	0.000	0.273	0.000	0.000	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	240	714	0	210	0	0	0
N.S.	1	1.00	1.30	3.88	0.00	1.14	0.00	0.00	0.00
time (sec)	N/A	0.536	1.382	1.339	0.000	0.325	0.000	0.000	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	B	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	537	537	382	1809	1349	182	0	1104	0
N.S.	1	1.00	0.71	3.37	2.51	0.34	0.00	2.06	0.00
time (sec)	N/A	0.832	1.212	1.619	0.401	0.254	0.000	0.284	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	261	261	117	655	523	110	0	370	0
N.S.	1	1.00	0.45	2.51	2.00	0.42	0.00	1.42	0.00
time (sec)	N/A	0.294	0.529	2.100	0.244	0.263	0.000	0.278	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	65	131	118	57	94	81	68
N.S.	1	1.00	0.76	1.54	1.39	0.67	1.11	0.95	0.80
time (sec)	N/A	0.069	0.157	0.913	0.269	0.305	0.293	0.271	14.020

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	234	234	243	279	0	287	0	0	0
N.S.	1	1.00	1.04	1.19	0.00	1.23	0.00	0.00	0.00
time (sec)	N/A	0.580	11.083	1.105	0.000	0.278	0.000	0.000	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	332	332	138	931	0	406	0	0	0
N.S.	1	1.00	0.42	2.80	0.00	1.22	0.00	0.00	0.00
time (sec)	N/A	1.085	0.768	1.332	0.000	0.311	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [35] had the largest ratio of [.6250000000000000000]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	3	3	1.00	12	0.250
2	A	4	4	1.00	12	0.333
3	A	2	2	1.00	10	0.200
4	A	3	3	1.00	8	0.375
5	A	3	3	1.00	12	0.250
6	A	4	4	1.00	12	0.333
7	A	5	5	1.00	12	0.417
8	A	3	3	1.00	14	0.214
9	A	6	5	1.00	14	0.357
10	A	3	3	1.00	12	0.250
11	A	5	4	1.00	10	0.400
12	A	5	4	1.00	14	0.286
13	A	6	6	1.00	14	0.429
14	A	7	6	1.00	14	0.429
15	A	4	4	1.00	14	0.286
16	A	10	5	1.00	14	0.357
17	A	3	2	1.00	12	0.167
18	A	8	4	1.00	10	0.400
19	A	8	4	1.00	14	0.286
20	A	9	5	1.00	14	0.357
21	A	12	6	1.00	14	0.429
22	A	3	2	1.00	12	0.167
23	A	4	3	1.00	14	0.214
24	A	4	3	1.00	14	0.214

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
25	A	3	2	1.00	14	0.143
26	A	3	2	1.00	14	0.143
27	A	4	3	1.00	14	0.214
28	A	4	3	1.00	14	0.214
29	A	7	5	1.00	16	0.312
30	A	7	5	1.00	16	0.312
31	A	6	4	1.00	16	0.250
32	A	6	4	1.00	16	0.250
33	A	7	5	1.00	16	0.312
34	A	7	6	1.00	16	0.375
35	A	5	5	1.00	8	0.625
36	A	3	3	1.00	12	0.250
37	A	2	2	1.00	12	0.167
38	A	3	3	1.00	12	0.250
39	A	4	3	1.00	12	0.250
40	A	5	5	1.00	8	0.625
41	A	3	3	1.00	12	0.250
42	A	4	4	1.00	12	0.333
43	A	2	2	1.00	12	0.167
44	A	5	5	1.00	12	0.417
45	A	3	3	1.00	14	0.214
46	A	2	2	1.00	12	0.167
47	A	3	3	1.00	6	0.500
48	A	3	3	1.00	8	0.375
49	A	13	7	1.00	16	0.438
50	A	10	7	1.00	16	0.438
51	A	7	7	1.00	16	0.438
52	A	8	7	1.00	16	0.438
53	A	11	7	1.00	16	0.438
54	A	14	7	1.00	16	0.438
55	A	15	10	1.00	18	0.556
56	A	12	9	1.00	18	0.500
57	A	9	8	1.00	18	0.444
58	A	10	9	1.00	18	0.500
59	A	12	10	1.00	18	0.556

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
60	A	16	9	1.00	18	0.500
61	A	7	5	1.00	8	0.625
62	A	2	2	1.00	12	0.167
63	N/A	0	0	1.00	18	0.000
64	N/A	0	0	1.00	20	0.000
65	A	3	3	1.00	20	0.150
66	N/A	0	0	1.00	22	0.000
67	A	5	5	1.00	22	0.227
68	N/A	0	0	1.00	24	0.000
69	A	3	3	1.00	12	0.250
70	A	5	4	1.00	14	0.286
71	A	8	4	1.00	14	0.286
72	A	8	4	1.00	14	0.286
73	A	3	2	1.00	8	0.250
74	A	5	3	1.00	10	0.300
75	A	8	3	1.00	10	0.300
76	A	3	2	1.00	12	0.167
77	A	5	3	1.00	14	0.214
78	A	8	3	1.00	14	0.214
79	A	5	5	1.00	16	0.312
80	A	7	6	1.00	18	0.333
81	A	12	6	1.00	18	0.333
82	A	6	5	1.00	16	0.312
83	A	8	6	1.00	18	0.333
84	A	14	6	1.00	18	0.333
85	A	7	6	1.00	12	0.500
86	A	5	4	1.00	10	0.400
87	A	1	1	1.00	8	0.125
88	N/A	0	0	1.00	12	0.000
89	N/A	0	0	1.00	12	0.000
90	A	14	3	1.00	18	0.167
91	A	8	3	1.00	16	0.188
92	A	3	3	1.00	14	0.214
93	A	8	4	1.00	18	0.222

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
94	A	10	6	1.00	18	0.333
95	A	20	4	1.00	18	0.222
96	A	11	4	1.00	16	0.250
97	A	4	3	1.00	14	0.214
98	A	11	4	1.00	18	0.222
99	A	13	6	1.00	18	0.333

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int x^3 \cos(a+bx^2)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 34

$$\int x^3 \cos(a+bx^2)\,dx = \frac{\cos(a+bx^2)}{2b^2} + \frac{x^2\sin(a+bx^2)}{2b}$$

```
[Out] 1/2*cos(b*x^2+a)/b^2+1/2*x^2*sin(b*x^2+a)/b
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3461, 3377, 2718}

$$\int x^3 \cos(a+bx^2)\,dx = \frac{\cos(a+bx^2)}{2b^2} + \frac{x^2\sin(a+bx^2)}{2b}$$

```
[In] Int[x^3*Cos[a + b*x^2],x]
```

```
[Out] Cos[a + b*x^2]/(2*b^2) + (x^2*Sin[a + b*x^2])/(2*b)
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}\text{Subst}\left(\int x\cos(a+bx)\,dx, x, x^2\right)\\ &= \frac{x^2\sin\left(a+bx^2\right)}{2b} - \frac{\text{Subst}\left(\int \sin(a+bx)\,dx, x, x^2\right)}{2b}\\ &= \frac{\cos\left(a+bx^2\right)}{2b^2} + \frac{x^2\sin\left(a+bx^2\right)}{2b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.85

$$\int x^3\cos\left(a+bx^2\right)\,dx = \frac{\cos\left(a+bx^2\right)+bx^2\sin\left(a+bx^2\right)}{2b^2}$$

```
[In] Integrate[x^3*Cos[a + b*x^2],x]
[Out] (Cos[a + b*x^2] + b*x^2*Sin[a + b*x^2])/(2*b^2)
```

Maple [A] (verified)

Time = 1.05 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

method	result
default	$\frac{\cos\left(b\,x^{2}+a\right)}{2b^{2}}+\frac{x^{2}\sin\left(b\,x^{2}+a\right)}{2b}$
risch	$\frac{\cos\left(b\,x^{2}+a\right)}{2b^{2}}+\frac{x^{2}\sin\left(b\,x^{2}+a\right)}{2b}$
parallelrisch	$\frac{1+\tan\left(\frac{a}{2}+\frac{b\,x^{2}}{2}\right)x^{2}b}{b^{2}\left(1+\tan^{2}\left(\frac{a}{2}+\frac{b\,x^{2}}{2}\right)\right)}$
norman	$\frac{\frac{1}{b^{2}}+\frac{x^{2}\tan\left(\frac{a}{2}+\frac{b\,x^{2}}{2}\right)}{b}}{1+\tan^{2}\left(\frac{a}{2}+\frac{b\,x^{2}}{2}\right)}$
meijerg	$\frac{\cos(a)\sqrt{\pi}\left(-\frac{1}{2\sqrt{\pi}}+\frac{\cos\left(b\,x^{2}\right)}{2\sqrt{\pi}}+\frac{x^{2}b\sin\left(b\,x^{2}\right)}{2\sqrt{\pi}}\right)}{b^{2}}-\frac{\sin(a)\sqrt{\pi}\left(-\frac{x^{2}b\cos\left(b\,x^{2}\right)}{2\sqrt{\pi}}+\frac{\sin\left(b\,x^{2}\right)}{2\sqrt{\pi}}\right)}{b^{2}}$
parts	$\frac{\sqrt{2}\,\sqrt{\pi}\,x^{3}\cos(a)\,\mathrm{C}\left(\frac{x\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}}\right)}{2\sqrt{b}}-\frac{\sqrt{2}\,\sqrt{\pi}\,x^{3}\sin(a)\,\mathrm{S}\left(\frac{x\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}}\right)}{2\sqrt{b}}-\frac{3\pi^{2}\left(\cos(a)\left(\frac{2\,\mathrm{C}\left(\frac{x\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}}\right)x^{3}b^{\frac{3}{2}}\sqrt{2}}{3\pi^{\frac{3}{2}}}-\frac{2x^{2}b\sin\left(b\,x^{2}\right)}{3\pi^{2}}-\frac{2\cos(b}{3\pi}\right.\right.}{}$

[In] int(x^3*cos(b*x^2+a),x,method=_RETURNVERBOSE)

[Out] 1/2*cos(b*x^2+a)/b^2+1/2*x^2*sin(b*x^2+a)/b

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.79

$$\int x^{3}\cos\left(a+bx^{2}\right)\,dx=\frac{bx^{2}\sin\left(bx^{2}+a\right)+\cos\left(bx^{2}+a\right)}{2\,b^{2}}$$

[In] integrate(x^3*cos(b*x^2+a),x, algorithm="fricas")

[Out] 1/2*(b*x^2*sin(b*x^2 + a) + cos(b*x^2 + a))/b^2

Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int x^{3}\cos\left(a+bx^{2}\right)\,dx=\begin{cases}\frac{x^{2}\sin\left(a+bx^{2}\right)}{2b}+\frac{\cos\left(a+bx^{2}\right)}{2b^{2}} & \text{for } b\neq 0\\ \frac{x^{4}\cos\left(a\right)}{4} & \text{otherwise}\end{cases}$$

[In] integrate(x**3*cos(b*x**2+a),x)

```
[Out] Piecewise((x**2*sin(a + b*x**2)/(2*b) + cos(a + b*x**2)/(2*b**2), Ne(b, 0))
, (x**4*cos(a)/4, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.79

$$\int x^3 \cos \left(a + bx^2\right) \, dx = \frac{bx^2 \sin \left(bx^2 + a\right) + \cos \left(bx^2 + a\right)}{2 \, b^2}$$

```
[In] integrate(x^3*cos(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/2*(b*x^2*sin(b*x^2 + a) + cos(b*x^2 + a))/b^2
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.32

$$\int x^3 \cos \left(a + bx^2\right) \, dx = -\frac{a \sin \left(bx^2 + a\right)}{2 \, b^2} + \frac{\left(bx^2 + a\right) \sin \left(bx^2 + a\right) + \cos \left(bx^2 + a\right)}{2 \, b^2}$$

```
[In] integrate(x^3*cos(b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/2*a*sin(b*x^2 + a)/b^2 + 1/2*((b*x^2 + a)*sin(b*x^2 + a) + cos(b*x^2 + a
))/b^2
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.79

$$\int x^3 \cos \left(a + bx^2\right) \, dx = \frac{\cos \left(b\,x^2 + a\right) + b\,x^2\,\sin \left(b\,x^2 + a\right)}{2\,b^2}$$

```
[In] int(x^3*cos(a + b*x^2),x)
```

```
[Out] (cos(a + b*x^2) + b*x^2*sin(a + b*x^2))/(2*b^2)
```

3.2 $\int x^2 \cos(a+bx^2)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 91

$$\int x^2 \cos(a+bx^2)\,dx = -\frac{\sqrt{\frac{\pi}{2}}\cos(a)\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)}{2b^{3/2}} - \frac{\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)\sin(a)}{2b^{3/2}} + \frac{x\sin(a+bx^2)}{2b}$$

```
[Out] 1/2*x*sin(b*x^2+a)/b-1/4*cos(a)*FresnelS(x*b^(1/2)*2^(1/2)/Pi^(1/2))*2^(1/2
)*Pi^(1/2)/b^(3/2)-1/4*FresnelC(x*b^(1/2)*2^(1/2)/Pi^(1/2))*sin(a)*2^(1/2)*
Pi^(1/2)/b^(3/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3467, 3434, 3433, 3432}

$$\int x^2 \cos(a+bx^2)\,dx = -\frac{\sqrt{\frac{\pi}{2}}\sin(a)\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)}{2b^{3/2}} - \frac{\sqrt{\frac{\pi}{2}}\cos(a)\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)}{2b^{3/2}} + \frac{x\sin(a+bx^2)}{2b}$$

```
[In] Int[x^2*Cos[a + b*x^2],x]
```

```
[Out] -1/2*(Sqrt[Pi/2]*Cos[a]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*x])/b^(3/2) - (Sqrt[Pi/
2]*FresnelC[Sqrt[b]*Sqrt[2/Pi]*x]*Sin[a])/(2*b^(3/2)) + (x*Sin[a + b*x^2])/
(2*b)
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3434

```
Int[Sin[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Sin[c], Int
[Cos[d*(e + f*x)^2], x], x] + Dist[Cos[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3467

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[e^(n
 - 1)*(e*x)^(m - n + 1)*(Sin[c + d*x^n]/(d*n)), x] - Dist[e^n*((m - n + 1)/
(d*n)), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{x\sin(a+bx^2)}{2b} - \frac{\int \sin(a+bx^2)\,dx}{2b}\\ &= \frac{x\sin(a+bx^2)}{2b} - \frac{\cos(a)\int \sin(bx^2)\,dx}{2b} - \frac{\sin(a)\int \cos(bx^2)\,dx}{2b}\\ &= -\frac{\sqrt{\frac{\pi}{2}}\cos(a)\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)}{2b^{3/2}} - \frac{\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)\sin(a)}{2b^{3/2}} + \frac{x\sin(a+bx^2)}{2b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.90

$$\int x^2\cos(a+bx^2)\,dx = \frac{-\sqrt{2\pi}\cos(a)\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right) - \sqrt{2\pi}\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)\sin(a) + 2\sqrt{b}x\sin(a+bx^2)}{4b^{3/2}}$$

```
[In] Integrate[x^2*Cos[a + b*x^2],x]
[Out] (-(Sqrt[2*Pi]*Cos[a]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*x]) - Sqrt[2*Pi]*FresnelC[
Sqrt[b]*Sqrt[2/Pi]*x]*Sin[a] + 2*Sqrt[b]*x*Sin[a + b*x^2])/(4*b^(3/2))
```

Maple [A] (verified)

Time = 1.08 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.64

method	result
default	$\frac{x\sin\left(b\,x^{2}+a\right)}{2b}-\frac{\sqrt{2}\,\sqrt{\pi}\,\left(\cos\left(a\right)\mathrm{S}\left(\frac{x\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}}\right)+\sin\left(a\right)\mathrm{C}\left(\frac{x\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}}\right)\right)}{4b^{\frac{3}{2}}}$
risch	$-\frac{i\,\mathrm{e}^{-ia}\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{ib}\,x\right)}{8b\sqrt{ib}}+\frac{i\,\mathrm{e}^{ia}\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{-ib}\,x\right)}{8b\sqrt{-ib}}+\frac{x\sin\left(b\,x^{2}+a\right)}{2b}$
meijerg	$\frac{\cos\left(a\right)\sqrt{\pi}\,\sqrt{2}\,\left(\frac{x\sqrt{2}\,\left(b^{2}\right)^{\frac{3}{4}}\sin\left(b\,x^{2}\right)}{2\sqrt{\pi}\,b}-\frac{\left(b^{2}\right)^{\frac{3}{4}}\mathrm{S}\left(\frac{x\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}}\right)}{2b^{\frac{3}{2}}}\right)}{2\left(b^{2}\right)^{\frac{3}{4}}}-\frac{\sin\left(a\right)\sqrt{\pi}\,\sqrt{2}\,\left(-\frac{x\sqrt{2}\,\sqrt{b}\,\cos\left(b\,x^{2}\right)}{2\sqrt{\pi}}+\frac{\mathrm{C}\left(\frac{x\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}}\right)}{2}\right)}{2b^{\frac{3}{2}}}$
parts	$\frac{\sqrt{2}\,\sqrt{\pi}\,x^{2}\cos\left(a\right)\mathrm{C}\left(\frac{x\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}}\right)}{2\sqrt{b}}-\frac{\sqrt{2}\,\sqrt{\pi}\,x^{2}\sin\left(a\right)\mathrm{S}\left(\frac{x\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}}\right)}{2\sqrt{b}}-\frac{\sqrt{2}\,\pi^{\frac{3}{2}}\,(\cos\left(a\right)(\frac{\mathrm{C}\left(\frac{x\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}}\right)x^{2}b}{\pi}-\frac{x\sqrt{b}\,\sqrt{2}\,\sin\left(b\,x^{2}\right)}{2\pi^{\frac{3}{2}}}+\frac{\mathrm{S}\left(\frac{x\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}}\right)}{2\pi}}{2b^{\frac{3}{2}}}$

[In] int(x^2*cos(b*x^2+a),x,method=_RETURNVERBOSE)

[Out] 1/2*x*sin(b*x^2+a)/b-1/4/b^(3/2)*2^(1/2)*Pi^(1/2)*(cos(a)*FresnelS(x*b^(1/2)*2^(1/2)/Pi^(1/2))+sin(a)*FresnelC(x*b^(1/2)*2^(1/2)/Pi^(1/2)))

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.79

$$\int x^2\cos\left(a+bx^2\right)\,dx = -\frac{\sqrt{2}\pi\sqrt{\frac{b}{\pi}}\cos\left(a\right)\mathrm{S}\left(\sqrt{2}x\sqrt{\frac{b}{\pi}}\right)+\sqrt{2}\pi\sqrt{\frac{b}{\pi}}\,\mathrm{C}\left(\sqrt{2}x\sqrt{\frac{b}{\pi}}\right)\sin\left(a\right)-2\,bx\sin\left(bx^2+a\right)}{4\,b^2}$$

[In] integrate(x^2*cos(b*x^2+a),x, algorithm="fricas")

[Out] -1/4*(sqrt(2)*pi*sqrt(b/pi)*cos(a)*fresnel_sin(sqrt(2)*x*sqrt(b/pi)) + sqrt(2)*pi*sqrt(b/pi)*fresnel_cos(sqrt(2)*x*sqrt(b/pi))*sin(a) - 2*b*x*sin(b*x^2 + a))/b^2

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 209 vs. 2(90) = 180.

Time = 0.99 (sec) , antiderivative size = 209, normalized size of antiderivative = 2.30

$$\int x^2\cos\left(a+bx^2\right)\,dx=\frac{b^{\frac{3}{2}}x^5\sqrt{\frac{1}{b}}\sin\left(a\right)\Gamma\left(\frac{3}{4}\right)\Gamma\left(\frac{5}{4}\right){}_2F_3\left(\begin{matrix}\frac{3}{4},\frac{5}{4}\\\frac{3}{2},\frac{7}{4},\frac{9}{4}\end{matrix}\middle|-\frac{b^2x^4}{4}\right)}{8\Gamma\left(\frac{7}{4}\right)\Gamma\left(\frac{9}{4}\right)}-\frac{\sqrt{b}x^3\sqrt{\frac{1}{b}}\cos\left(a\right)\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right){}_2F_3\left(\begin{matrix}\frac{1}{4},\frac{3}{4}\\\frac{1}{2},\frac{5}{4},\frac{7}{4}\end{matrix}\middle|-\frac{b^2x^4}{4}\right)}{8\Gamma\left(\frac{5}{4}\right)\Gamma\left(\frac{7}{4}\right)}-\frac{\sqrt{2}\sqrt{\pi}x^2\sqrt{\frac{1}{b}}\sin\left(a\right)S\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{\pi}}\right)}{2}+\frac{\sqrt{2}\sqrt{\pi}x^2\sqrt{\frac{1}{b}}\cos\left(a\right)C\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{\pi}}\right)}{2}$$

```
[In] integrate(x**2*cos(b*x**2+a),x)
```

```
[Out] b**(3/2)*x**5*sqrt(1/b)*sin(a)*gamma(3/4)*gamma(5/4)*hyper((3/4, 5/4), (3/2
, 7/4, 9/4), -b**2*x**4/4)/(8*gamma(7/4)*gamma(9/4)) - sqrt(b)*x**3*sqrt(1/
b)*cos(a)*gamma(1/4)*gamma(3/4)*hyper((1/4, 3/4), (1/2, 5/4, 7/4), -b**2*x*
*4/4)/(8*gamma(5/4)*gamma(7/4)) - sqrt(2)*sqrt(pi)*x**2*sqrt(1/b)*sin(a)*fr
esnels(sqrt(2)*sqrt(b)*x/sqrt(pi))/2 + sqrt(2)*sqrt(pi)*x**2*sqrt(1/b)*cos(
a)*fresnelc(sqrt(2)*sqrt(b)*x/sqrt(pi))/2
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.74

$$\int x^2\cos\left(a+bx^2\right)\,dx$$
$$=\frac{8\,b^2x\sin\left(bx^2+a\right)+\sqrt{2}\sqrt{\pi}\Big(\left(-(i+1)\cos\left(a\right)+(i-1)\sin\left(a\right)\right)\mathrm{erf}\left(\sqrt{i\,b}x\right)+\left((i-1)\cos\left(a\right)-(i+1)\sin\right.}{16\,b^3}$$

```
[In] integrate(x^2*cos(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/16*(8*b^2*x*sin(b*x^2 + a) + sqrt(2)*sqrt(pi)*((-(I + 1)*cos(a) + (I - 1)
*sin(a))*erf(sqrt(I*b)*x) + ((I - 1)*cos(a) - (I + 1)*sin(a))*erf(sqrt(-I*b
)*x))*b^(3/2))/b^3
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.48

$$\int x^2 \cos\left(a+bx^2\right)\,dx = -\frac{i\,xe^{\left(i\,bx^2+i\,a\right)}}{4\,b} + \frac{i\,xe^{\left(-i\,bx^2-i\,a\right)}}{4\,b} - \frac{\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}i\,\sqrt{2}x\left(\frac{i\,b}{|b|}+1\right)\sqrt{|b|}\right)e^{\left(i\,a\right)}}{8\,b\left(\frac{i\,b}{|b|}+1\right)\sqrt{|b|}} - \frac{\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(\frac{1}{2}i\,\sqrt{2}x\left(-\frac{i\,b}{|b|}+1\right)\sqrt{|b|}\right)e^{\left(-i\,a\right)}}{8\,b\left(-\frac{i\,b}{|b|}+1\right)\sqrt{|b|}}$$

```
[In] integrate(x^2*cos(b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/4*I*x*e^(I*b*x^2 + I*a)/b + 1/4*I*x*e^(-I*b*x^2 - I*a)/b - 1/8*sqrt(2)*s
qrt(pi)*erf(-1/2*I*sqrt(2)*x*(I*b/abs(b) + 1)*sqrt(abs(b)))*e^(I*a)/(b*(I*b
/abs(b) + 1)*sqrt(abs(b))) - 1/8*sqrt(2)*sqrt(pi)*erf(1/2*I*sqrt(2)*x*(-I*b
/abs(b) + 1)*sqrt(abs(b)))*e^(-I*a)/(b*(-I*b/abs(b) + 1)*sqrt(abs(b)))
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \cos\left(a+bx^2\right)\,dx = \int x^2\,\cos\left(b\,x^2+a\right)\,dx$$

```
[In] int(x^2*cos(a + b*x^2),x)
```

```
[Out] int(x^2*cos(a + b*x^2), x)
```

3.3 $\int x \cos(a+bx^2)\, dx$

Optimal result

Integrand size = 10, antiderivative size = 15

$$\int x \cos(a+bx^2)\, dx = \frac{\sin(a+bx^2)}{2b}$$

```
[Out] 1/2*sin(b*x^2+a)/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3461, 2717}

$$\int x \cos(a+bx^2)\, dx = \frac{\sin(a+bx^2)}{2b}$$

```
[In] Int[x*Cos[a + b*x^2],x]
```

```
[Out] Sin[a + b*x^2]/(2*b)
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
```

```
m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \cos(a+bx)\,dx, x, x^2\right)\\ &= \frac{\sin(a+bx^2)}{2b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int x\cos\left(a+bx^2\right)\,dx = \frac{\sin(a+bx^2)}{2b}$$

```
[In] Integrate[x*Cos[a + b*x^2],x]
[Out] Sin[a + b*x^2]/(2*b)
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result
derivativedivides	$\frac{\sin\left(b\,x^2+a\right)}{2b}$
default	$\frac{\sin\left(b\,x^2+a\right)}{2b}$
risch	$\frac{\sin\left(b\,x^2+a\right)}{2b}$
parallelrisch	$\frac{\sin\left(b\,x^2+a\right)}{2b}$
norman	$\frac{\tan\left(\frac{a}{2}+\frac{b\,x^2}{2}\right)}{b\left(1+\tan^2\left(\frac{a}{2}+\frac{b\,x^2}{2}\right)\right)}$
meijerg	$\frac{\cos(a)\sin\left(b\,x^2\right)}{2b}-\frac{\sin(a)\sqrt{\pi}\left(\frac{1}{\sqrt{\pi}}-\frac{\cos\left(b\,x^2\right)}{\sqrt{\pi}}\right)}{2b}$
parts	$-\frac{\sqrt{2}\,\sqrt{\pi}\,x\sin(a)\,\text{S}\left(\frac{x\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}}\right)}{2\sqrt{b}}+\frac{\sqrt{2}\,\sqrt{\pi}\,x\cos(a)\,\text{C}\left(\frac{x\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}}\right)}{2\sqrt{b}}-\frac{\sqrt{2}\,\sqrt{\pi}\left(\frac{\cos(a)\sqrt{2}\,\sqrt{\pi}\left(\frac{\text{C}\left(\frac{x\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}}\right)x\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}}-\frac{\sin\left(b\,x^2\right)}{\pi}\right.}{2\sqrt{b}}\right.}{2\sqrt{}}$

```
[In] int(x*cos(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*sin(b*x^2+a)/b
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int x \cos \left(a + bx^2\right) \, dx = \frac{\sin \left(bx^2 + a\right)}{2 \, b}$$

```
[In] integrate(x*cos(b*x^2+a),x, algorithm="fricas")
[Out] 1/2*sin(b*x^2 + a)/b
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.27

$$\int x \cos \left(a + bx^2\right) \, dx = \begin{cases} \frac{\sin \left(a+bx^2\right)}{2b} & \text{for}\ b \neq 0 \\ \frac{x^2 \cos \left(a\right)}{2} & \text{otherwise} \end{cases}$$

```
[In] integrate(x*cos(b*x**2+a),x)
[Out] Piecewise((sin(a + b*x**2)/(2*b), Ne(b, 0)), (x**2*cos(a)/2, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int x \cos \left(a + bx^2\right) \, dx = \frac{\sin \left(bx^2 + a\right)}{2 \, b}$$

```
[In] integrate(x*cos(b*x^2+a),x, algorithm="maxima")
[Out] 1/2*sin(b*x^2 + a)/b
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int x\cos\left(a+bx^2\right)\,dx=\frac{\sin\left(bx^2+a\right)}{2\,b}$$

```
[In] integrate(x*cos(b*x^2+a),x, algorithm="giac")
[Out] 1/2*sin(b*x^2 + a)/b
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int x\cos\left(a+bx^2\right)\,dx=\frac{\sin\left(b\,x^2+a\right)}{2\,b}$$

```
[In] int(x*cos(a + b*x^2),x)
[Out] sin(a + b*x^2)/(2*b)
```

3.4 $\int \cos(a+bx^2)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 70

$$\int \cos\left(a+bx^2\right)\,dx=\frac{\sqrt{\frac{\pi}{2}}\cos(a)\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)}{\sqrt{b}}-\frac{\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)\sin(a)}{\sqrt{b}}$$

```
[Out] 1/2*cos(a)*FresnelC(x*b^(1/2)*2^(1/2)/Pi^(1/2))*2^(1/2)*Pi^(1/2)/b^(1/2)-1/
2*FresnelS(x*b^(1/2)*2^(1/2)/Pi^(1/2))*sin(a)*2^(1/2)*Pi^(1/2)/b^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3435, 3433, 3432}

$$\int \cos\left(a+bx^2\right)\,dx=\frac{\sqrt{\frac{\pi}{2}}\cos(a)\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)}{\sqrt{b}}-\frac{\sqrt{\frac{\pi}{2}}\sin(a)\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)}{\sqrt{b}}$$

```
[In] Int[Cos[a + b*x^2],x]
```

```
[Out] (Sqrt[Pi/2]*Cos[a]*FresnelC[Sqrt[b]*Sqrt[2/Pi]*x])/Sqrt[b] - (Sqrt[Pi/2]*Fr
esnelS[Sqrt[b]*Sqrt[2/Pi]*x]*Sin[a])/Sqrt[b]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3435

```
Int[Cos[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Cos[c], Int
[Cos[d*(e + f*x)^2], x], x] - Dist[Sin[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \cos(a)\int \cos\left(bx^2\right)\,dx - \sin(a)\int \sin\left(bx^2\right)\,dx\\ &= \frac{\sqrt{\frac{\pi}{2}}\cos(a)\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)}{\sqrt{b}} - \frac{\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)\sin(a)}{\sqrt{b}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.81

$$\int \cos\left(a+bx^2\right)\,dx = \frac{\sqrt{\frac{\pi}{2}}\left(\cos(a)\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right) - \text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)\sin(a)\right)}{\sqrt{b}}$$

```
[In] Integrate[Cos[a + b*x^2],x]
[Out] (Sqrt[Pi/2]*(Cos[a]*FresnelC[Sqrt[b]*Sqrt[2/Pi]*x] - FresnelS[Sqrt[b]*Sqrt[
2/Pi]*x]*Sin[a]))/Sqrt[b]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.63

method	result	size
default	$\frac{\sqrt{2}\,\sqrt{\pi}\,\left(\cos(a)\,\mathrm{C}\left(\frac{x\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}}\right)-\sin(a)\,\mathrm{S}\left(\frac{x\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}}\right)\right)}{2\sqrt{b}}$	44
meijerg	$\frac{\cos(a)\,\mathrm{C}\left(\frac{x\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}}\right)\sqrt{2}\,\sqrt{\pi}}{2\sqrt{b}}-\frac{\mathrm{S}\left(\frac{x\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}}\right)\sin(a)\sqrt{2}\,\sqrt{\pi}}{2\sqrt{b}}$	52
risch	$\frac{\mathrm{e}^{-ia}\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{ib}\,x\right)}{4\sqrt{ib}}+\frac{\mathrm{e}^{ia}\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{-ib}\,x\right)}{4\sqrt{-ib}}$	52

```
[In] int(cos(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*2^(1/2)*Pi^(1/2)/b^(1/2)*(cos(a)*FresnelC(x*b^(1/2)*2^(1/2)/Pi^(1/2))-s
in(a)*FresnelS(x*b^(1/2)*2^(1/2)/Pi^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.87

$$\int \cos\left(a+bx^2\right)\,dx=\frac{\sqrt{2}\pi\sqrt{\frac{b}{\pi}}\cos(a)\,\mathrm{C}\left(\sqrt{2}x\sqrt{\frac{b}{\pi}}\right)-\sqrt{2}\pi\sqrt{\frac{b}{\pi}}\,\mathrm{S}\left(\sqrt{2}x\sqrt{\frac{b}{\pi}}\right)\sin(a)}{2\,b}$$

```
[In] integrate(cos(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(2)*pi*sqrt(b/pi)*cos(a)*fresnel_cos(sqrt(2)*x*sqrt(b/pi)) - sqrt(
2)*pi*sqrt(b/pi)*fresnel_sin(sqrt(2)*x*sqrt(b/pi))*sin(a))/b
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.87

$$\int \cos\left(a+bx^2\right)\,dx=\frac{\sqrt{2}\sqrt{\pi}\left(-\sin(a)S\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{\pi}}\right)+\cos(a)C\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{\pi}}\right)\right)\sqrt{\frac{1}{b}}}{2}$$

```
[In] integrate(cos(b*x**2+a),x)
```

```
[Out] sqrt(2)*sqrt(pi)*(-sin(a)*fresnels(sqrt(2)*sqrt(b)*x/sqrt(pi)) + cos(a)*fre
snelc(sqrt(2)*sqrt(b)*x/sqrt(pi)))*sqrt(1/b)/2
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.69

$$\int \cos\left(a+bx^2\right)\,dx=-\frac{\sqrt{2}\sqrt{\pi}\left(\left((i-1)\cos(a)+(i+1)\sin(a)\right)\operatorname{erf}\left(\sqrt{i\,b}x\right)+\left(-(i+1)\cos(a)-(i-1)\sin(a)\right)\operatorname{erf}\left(\sqrt{-i\,b}x\right)\right)}{8\sqrt{b}}$$

```
[In] integrate(cos(b*x^2+a),x, algorithm="maxima")
```

```
[Out] -1/8*sqrt(2)*sqrt(pi)*(((I - 1)*cos(a) + (I + 1)*sin(a))*erf(sqrt(I*b)*x) +
 (-(I + 1)*cos(a) - (I - 1)*sin(a))*erf(sqrt(-I*b)*x))/sqrt(b)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.36

$$\int \cos \left(a+bx^2\right)\, dx = \frac{i\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}i\sqrt{2}x\left(\frac{i\,b}{|b|}+1\right)\sqrt{|b|}\right)e^{(i\,a)}}{4\left(\frac{i\,b}{|b|}+1\right)\sqrt{|b|}} - \frac{i\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(\frac{1}{2}i\sqrt{2}x\left(-\frac{i\,b}{|b|}+1\right)\sqrt{|b|}\right)e^{(-i\,a)}}{4\left(-\frac{i\,b}{|b|}+1\right)\sqrt{|b|}}$$

```
[In] integrate(cos(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/4*I*sqrt(2)*sqrt(pi)*erf(-1/2*I*sqrt(2)*x*(I*b/abs(b) + 1)*sqrt(abs(b)))*
e^(I*a)/((I*b/abs(b) + 1)*sqrt(abs(b))) - 1/4*I*sqrt(2)*sqrt(pi)*erf(1/2*I*
sqrt(2)*x*(-I*b/abs(b) + 1)*sqrt(abs(b)))*e^(-I*a)/((-I*b/abs(b) + 1)*sqrt(
abs(b)))
```

Mupad [B] (verification not implemented)

Time = 13.70 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.73

$$\int \cos \left(a+bx^2\right)\, dx = \frac{\sqrt{2}\,\sqrt{\pi}\,\mathrm{C}\left(\frac{\sqrt{2}\,\sqrt{b}\,x}{\sqrt{\pi}}\right)\,\cos \left(a\right)}{2\,\sqrt{b}} - \frac{\sqrt{2}\,\sqrt{\pi}\,\mathrm{S}\left(\frac{\sqrt{2}\,\sqrt{b}\,x}{\sqrt{\pi}}\right)\,\sin \left(a\right)}{2\,\sqrt{b}}$$

```
[In] int(cos(a + b*x^2),x)
```

```
[Out] (2^(1/2)*pi^(1/2)*fresnelc((2^(1/2)*b^(1/2)*x)/pi^(1/2))*cos(a))/(2*b^(1/2)
) - (2^(1/2)*pi^(1/2)*fresnels((2^(1/2)*b^(1/2)*x)/pi^(1/2))*sin(a))/(2*b^(
1/2))
```

3.5 $\int \frac{\cos(a+bx^2)}{x} dx$

Optimal result

Integrand size = 12, antiderivative size = 25

$$\int \frac{\cos(a+bx^2)}{x} dx = \frac{1}{2}\cos(a)\,\text{CosIntegral}\left(bx^2\right) - \frac{1}{2}\sin(a)\text{Si}\left(bx^2\right)$$

```
[Out] 1/2*Ci(b*x^2)*cos(a)-1/2*Si(b*x^2)*sin(a)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3459, 3457, 3456}

$$\int \frac{\cos(a+bx^2)}{x} dx = \frac{1}{2}\cos(a)\,\text{CosIntegral}\left(bx^2\right) - \frac{1}{2}\sin(a)\text{Si}\left(bx^2\right)$$

```
[In] Int[Cos[a + b*x^2]/x,x]
```

```
[Out] (Cos[a]*CosIntegral[b*x^2])/2 - (Sin[a]*SinIntegral[b*x^2])/2
```

Rule 3456

```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3457

```
Int[Cos[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[CosIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3459

```
Int[Cos[(c_) + (d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Dist[Cos[c], Int[Cos[d*x
^n]/x, x], x] - Dist[Sin[c], Int[Sin[d*x^n]/x, x], x] /; FreeQ[{c, d, n}, x
]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \cos(a)\int \frac{\cos(bx^2)}{x}\,dx - \sin(a)\int \frac{\sin(bx^2)}{x}\,dx\\ &= \frac{1}{2}\cos(a)\,\text{CosIntegral}\left(bx^2\right) - \frac{1}{2}\sin(a)\text{Si}\left(bx^2\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{\cos(a+bx^2)}{x}\,dx = \frac{1}{2}\left(\cos(a)\,\text{CosIntegral}\left(bx^2\right) - \sin(a)\text{Si}\left(bx^2\right)\right)$$

```
[In] Integrate[Cos[a + b*x^2]/x,x]
[Out] (Cos[a]*CosIntegral[b*x^2] - Sin[a]*SinIntegral[b*x^2])/2
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

method	result	size
default	$\frac{\text{Ci}(b\,x^2)\cos(a)}{2}-\frac{\text{Si}(b\,x^2)\sin(a)}{2}$	22
risch	$\frac{i\pi\ \text{csgn}(b\,x^2)\text{e}^{-ia}}{4}-\frac{i\,\text{Si}(b\,x^2)\text{e}^{-ia}}{2}-\frac{\text{e}^{-ia}\,\text{Ei}_1(-ib\,x^2)}{4}-\frac{\text{e}^{ia}\,\text{Ei}_1(-ib\,x^2)}{4}$	63
meijerg	$\frac{\cos(a)\sqrt{\pi}\left(\frac{2\gamma+4\ln(x)+\ln(b^2)}{\sqrt{\pi}}-\frac{2\gamma}{\sqrt{\pi}}-\frac{2\ln(2)}{\sqrt{\pi}}-\frac{2\ln\left(\frac{b\,x^2}{2}\right)}{\sqrt{\pi}}+\frac{2\ \text{Ci}(b\,x^2)}{\sqrt{\pi}}\right)}{4}-\frac{\text{Si}(b\,x^2)\sin(a)}{2}$	72

```
[In] int(cos(b*x^2+a)/x,x,method=_RETURNVERBOSE)
[Out] 1/2*Ci(b*x^2)*cos(a)-1/2*Si(b*x^2)*sin(a)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{\cos(a+bx^2)}{x}\,dx = \frac{1}{2}\cos(a)\operatorname{Ci}(bx^2) - \frac{1}{2}\sin(a)\operatorname{Si}(bx^2)$$

```
[In] integrate(cos(b*x^2+a)/x,x, algorithm="fricas")
[Out] 1/2*cos(a)*cos_integral(b*x^2) - 1/2*sin(a)*sin_integral(b*x^2)
```

Sympy [F]

$$\int \frac{\cos(a+bx^2)}{x}\,dx = \int \frac{\cos(a+bx^2)}{x}\,dx$$

```
[In] integrate(cos(b*x**2+a)/x,x)
[Out] Integral(cos(a + b*x**2)/x, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.72

$$\int \frac{\cos(a+bx^2)}{x}\,dx = \frac{1}{4}\left(\operatorname{Ei}(i\,bx^2) + \operatorname{Ei}(-i\,bx^2)\right)\cos(a) + \frac{1}{4}\left(i\operatorname{Ei}(i\,bx^2) - i\operatorname{Ei}(-i\,bx^2)\right)\sin(a)$$

```
[In] integrate(cos(b*x^2+a)/x,x, algorithm="maxima")
[Out] 1/4*(Ei(I*b*x^2) + Ei(-I*b*x^2))*cos(a) + 1/4*(I*Ei(I*b*x^2) - I*Ei(-I*b*x^
2))*sin(a)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{\cos(a+bx^2)}{x}\,dx = \frac{1}{2}\cos(a)\operatorname{Ci}\left(bx^2\right) - \frac{1}{2}\sin(a)\operatorname{Si}\left(bx^2\right)$$

```
[In] integrate(cos(b*x^2+a)/x,x, algorithm="giac")
[Out] 1/2*cos(a)*cos_integral(b*x^2) - 1/2*sin(a)*sin_integral(b*x^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(a+bx^2)}{x}\,dx = \frac{\cos(a)\ \operatorname{cosint}(b\,x^2)}{2} - \frac{\sin(a)\ \operatorname{sinint}(b\,x^2)}{2}$$

```
[In] int(cos(a + b*x^2)/x,x)
[Out] (cos(a)*cosint(b*x^2))/2 - (sin(a)*sinint(b*x^2))/2
```

3.6 $\int \frac{\cos(a+bx^2)}{x^2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 80

$$\int \frac{\cos(a+bx^2)}{x^2}\,dx = -\frac{\cos(a+bx^2)}{x} - \sqrt{b}\sqrt{2\pi}\cos(a)\operatorname{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right) - \sqrt{b}\sqrt{2\pi}\operatorname{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)\sin(a)$$

```
[Out] -cos(b*x^2+a)/x-cos(a)*FresnelS(x*b^(1/2)*2^(1/2)/Pi^(1/2))*b^(1/2)*2^(1/2)
*Pi^(1/2)-FresnelC(x*b^(1/2)*2^(1/2)/Pi^(1/2))*sin(a)*b^(1/2)*2^(1/2)*Pi^(1
/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3469, 3434, 3433, 3432}

$$\int \frac{\cos(a+bx^2)}{x^2}\,dx = -\sqrt{2\pi}\sqrt{b}\sin(a)\operatorname{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right) - \sqrt{2\pi}\sqrt{b}\cos(a)\operatorname{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right) - \frac{\cos(a+bx^2)}{x}$$

```
[In] Int[Cos[a + b*x^2]/x^2,x]
```

```
[Out] -(Cos[a + b*x^2]/x) - Sqrt[b]*Sqrt[2*Pi]*Cos[a]*FresnelS[Sqrt[b]*Sqrt[2/Pi]
*x] - Sqrt[b]*Sqrt[2*Pi]*FresnelC[Sqrt[b]*Sqrt[2/Pi]*x]*Sin[a]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3434

```
Int[Sin[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Sin[c], Int
[Cos[d*(e + f*x)^2], x], x] + Dist[Cos[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3469

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_), x_Symbol] :> Simp[(e*x)
^(m + 1)*(Cos[c + d*x^n]/(e*(m + 1))), x] + Dist[d*(n/(e^n*(m + 1))), Int[(
e*x)^(m + n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\cos(a+bx^2)}{x} - (2b)\int \sin\left(a+bx^2\right)\,dx \\
&= -\frac{\cos(a+bx^2)}{x} - (2b\cos(a))\int \sin\left(bx^2\right)\,dx - (2b\sin(a))\int \cos\left(bx^2\right)\,dx \\
&= -\frac{\cos(a+bx^2)}{x} - \sqrt{b}\sqrt{2\pi}\cos(a)\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right) - \sqrt{b}\sqrt{2\pi}\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)\sin(a)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.01

$$\int \frac{\cos(a+bx^2)}{x^2}\,dx = -\frac{\cos(a)\cos(bx^2)}{x} - \sqrt{b}\sqrt{2\pi}\left(\cos(a)\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right) + \text{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)\sin(a)\right) + \frac{\sin(a)\sin(bx^2)}{x}$$

```
[In] Integrate[Cos[a + b*x^2]/x^2,x]
```

```
[Out] -((Cos[a]*Cos[b*x^2])/x) - Sqrt[b]*Sqrt[2*Pi]*(Cos[a]*FresnelS[Sqrt[b]*Sqrt
[2/Pi]*x] + FresnelC[Sqrt[b]*Sqrt[2/Pi]*x]*Sin[a]) + (Sin[a]*Sin[b*x^2])/x
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.71

method	result	size
default	$-\frac{\cos\left(b\,x^{2}+a\right)}{x}-\sqrt{b}\,\sqrt{2}\,\sqrt{\pi}\,\left(\cos\left(a\right)\mathrm{S}\left(\frac{x\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}}\right)+\sin\left(a\right)\mathrm{C}\left(\frac{x\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}}\right)\right)$	57
risch	$-\frac{i\mathrm{e}^{-i a} b\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{i b}\,x\right)}{2\sqrt{i b}}+\frac{i\mathrm{e}^{i a} b\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{-i b}\,x\right)}{2\sqrt{-i b}}-\frac{\cos\left(b\,x^{2}+a\right)}{x}$	69
meijerg	$\frac{\cos(a)\sqrt{\pi}\,\left(b^{2}\right)^{\frac{1}{4}}\sqrt{2}\,\left(-\frac{4\sqrt{2}\,\cos\left(b\,x^{2}\right)}{\sqrt{\pi}\,x\left(b^{2}\right)^{\frac{1}{4}}}-\frac{8\sqrt{b}\,\mathrm{S}\left(\frac{x\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}}\right)}{\left(b^{2}\right)^{\frac{1}{4}}}\right)}{8}-\frac{\sin(a)\sqrt{\pi}\,\sqrt{b}\,\sqrt{2}\,\left(-\frac{4\sqrt{2}\,\sin\left(b\,x^{2}\right)}{x\sqrt{b}\,\sqrt{\pi}}+8\,\mathrm{C}\left(\frac{x\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}}\right)\right)}{8}$	110

```
[In] int(cos(b*x^2+a)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -cos(b*x^2+a)/x-b^(1/2)*2^(1/2)*Pi^(1/2)*(cos(a)*FresnelS(x*b^(1/2)*2^(1/2)
/Pi^(1/2))+sin(a)*FresnelC(x*b^(1/2)*2^(1/2)/Pi^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.88

$$\int \frac{\cos\left(a+bx^2\right)}{x^2}\,dx = -\frac{\sqrt{2}\pi x\sqrt{\frac{b}{\pi}}\cos\left(a\right)\mathrm{S}\left(\sqrt{2}x\sqrt{\frac{b}{\pi}}\right)+\sqrt{2}\pi x\sqrt{\frac{b}{\pi}}\,\mathrm{C}\left(\sqrt{2}x\sqrt{\frac{b}{\pi}}\right)\sin\left(a\right)+\cos\left(bx^2+a\right)}{x}$$

```
[In] integrate(cos(b*x^2+a)/x^2,x, algorithm="fricas")
```

```
[Out] -(sqrt(2)*pi*x*sqrt(b/pi)*cos(a)*fresnel_sin(sqrt(2)*x*sqrt(b/pi)) + sqrt(2
)*pi*x*sqrt(b/pi)*fresnel_cos(sqrt(2)*x*sqrt(b/pi))*sin(a) + cos(b*x^2 + a)
)/x
```

Sympy [F]

$$\int \frac{\cos\left(a+bx^2\right)}{x^2}\,dx = \int \frac{\cos\left(a+bx^2\right)}{x^2}\,dx$$

```
[In] integrate(cos(b*x**2+a)/x**2,x)
```

```
[Out] Integral(cos(a + b*x**2)/x**2, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.40 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.91

$$\int \frac{\cos(a+bx^2)}{x^2}\,dx$$
$$= \frac{\sqrt{bx^2}\left(\left(-(i+1)\,\sqrt{2}\Gamma\left(-\frac{1}{2}, i\,bx^2\right) + (i-1)\,\sqrt{2}\Gamma\left(-\frac{1}{2}, -i\,bx^2\right)\right)\cos(a) + \left((i-1)\,\sqrt{2}\Gamma\left(-\frac{1}{2}, i\,bx^2\right) - (i+1\right.}{8\,x}$$

```
[In] integrate(cos(b*x^2+a)/x^2,x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(b*x^2)*((-(I + 1)*sqrt(2)*gamma(-1/2, I*b*x^2) + (I - 1)*sqrt(2)*g
amma(-1/2, -I*b*x^2))*cos(a) + ((I - 1)*sqrt(2)*gamma(-1/2, I*b*x^2) - (I +
 1)*sqrt(2)*gamma(-1/2, -I*b*x^2))*sin(a))/x
```

Giac [F]

$$\int \frac{\cos(a+bx^2)}{x^2}\,dx = \int \frac{\cos(bx^2+a)}{x^2}\,dx$$

```
[In] integrate(cos(b*x^2+a)/x^2,x, algorithm="giac")
```

```
[Out] integrate(cos(b*x^2 + a)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(a+bx^2)}{x^2}\,dx = \int \frac{\cos(b\,x^2+a)}{x^2}\,dx$$

```
[In] int(cos(a + b*x^2)/x^2,x)
```

```
[Out] int(cos(a + b*x^2)/x^2, x)
```

3.7 $\int \frac{\cos(a+bx^2)}{x^3}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 42

$$\int \frac{\cos(a+bx^2)}{x^3}\,dx = -\frac{\cos(a+bx^2)}{2x^2} - \frac{1}{2}b\,\text{CosIntegral}(bx^2)\sin(a) - \frac{1}{2}b\cos(a)\text{Si}(bx^2)$$

```
[Out] -1/2*cos(b*x^2+a)/x^2-1/2*b*cos(a)*Si(b*x^2)-1/2*b*Ci(b*x^2)*sin(a)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {3461, 3378, 3384, 3380, 3383}

$$\int \frac{\cos(a+bx^2)}{x^3}\,dx = -\frac{1}{2}b\sin(a)\,\text{CosIntegral}(bx^2) - \frac{1}{2}b\cos(a)\text{Si}(bx^2) - \frac{\cos(a+bx^2)}{2x^2}$$

```
[In] Int[Cos[a + b*x^2]/x^3,x]
```

```
[Out] -1/2*Cos[a + b*x^2]/x^2 - (b*CosIntegral[b*x^2]*Sin[a])/2 - (b*Cos[a]*SinIn
tegral[b*x^2])/2
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{\cos(a+bx)}{x^2}\,dx, x, x^2\right)\\
&= -\frac{\cos\left(a+bx^2\right)}{2x^2} - \frac{1}{2}b\text{Subst}\left(\int \frac{\sin(a+bx)}{x}\,dx, x, x^2\right)\\
&= -\frac{\cos\left(a+bx^2\right)}{2x^2} - \frac{1}{2}(b\cos(a))\text{Subst}\left(\int \frac{\sin(bx)}{x}\,dx, x, x^2\right)\\
&\quad - \frac{1}{2}(b\sin(a))\text{Subst}\left(\int \frac{\cos(bx)}{x}\,dx, x, x^2\right)\\
&= -\frac{\cos\left(a+bx^2\right)}{2x^2} - \frac{1}{2}b\,\text{CosIntegral}\left(bx^2\right)\sin(a) - \frac{1}{2}b\cos(a)\text{Si}\left(bx^2\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int \frac{\cos(a+bx^2)}{x^3}\,dx = -\frac{\cos(a+bx^2)+bx^2\,\text{CosIntegral}(bx^2)\sin(a)+bx^2\cos(a)\text{Si}(bx^2)}{2x^2}$$

```
[In] Integrate[Cos[a + b*x^2]/x^3,x]
[Out] -1/2*(Cos[a + b*x^2] + b*x^2*CosIntegral[b*x^2]*Sin[a] + b*x^2*Cos[a]*SinIn
tegral[b*x^2])/x^2
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.93

method	result
default	$-\frac{\cos(b\,x^2+a)}{2x^2}-b\left(\frac{\cos(a)\,\text{Si}(b\,x^2)}{2}+\frac{\sin(a)\,\text{Ci}(b\,x^2)}{2}\right)$
risch	$\frac{\mathrm{e}^{-ia}\pi\ \text{csgn}(b\,x^2)b}{4}-\frac{\mathrm{e}^{-ia}\,\text{Si}(b\,x^2)b}{2}+\frac{i\,\text{Ei}_1(-ib\,x^2)\mathrm{e}^{-ia}b}{4}-\frac{i\mathrm{e}^{ia}b\ \text{Ei}_1(-ib\,x^2)}{4}-\frac{\cos(b\,x^2+a)}{2x^2}$
meijerg	$\frac{\cos(a)\sqrt{\pi}\,\sqrt{b^2}\left(-\frac{4b^2\cos\left(x^2\sqrt{b^2}\right)}{x^2\left(b^2\right)^{\frac{3}{2}}\sqrt{\pi}}-\frac{4\ \text{Si}\left(x^2\sqrt{b^2}\right)}{\sqrt{\pi}}\right)}{8}-\frac{\sin(a)\sqrt{\pi}\,b\left(\frac{4\gamma-4+8\ln(x)+4\ln(b)}{\sqrt{\pi}}+\frac{4}{\sqrt{\pi}}-\frac{4\gamma}{\sqrt{\pi}}-\frac{4\ln(2)}{\sqrt{\pi}}-\frac{4\ln\left(\frac{b\,x^2}{2}\right)}{\sqrt{\pi}}-\frac{4\sin\left(b\,x^2\right)}{\sqrt{\pi}\,x^2b}+\right.}{8}$

```
[In] int(cos(b*x^2+a)/x^3,x,method=_RETURNVERBOSE)
[Out] -1/2*cos(b*x^2+a)/x^2-b*(1/2*cos(a)*Si(b*x^2)+1/2*sin(a)*Ci(b*x^2))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.95

$$\int \frac{\cos(a+bx^2)}{x^3}\,dx = -\frac{bx^2\,\text{Ci}(bx^2)\sin(a)+bx^2\cos(a)\,\text{Si}(bx^2)+\cos(bx^2+a)}{2\,x^2}$$

```
[In] integrate(cos(b*x^2+a)/x^3,x, algorithm="fricas")
[Out] -1/2*(b*x^2*cos_integral(b*x^2)*sin(a) + b*x^2*cos(a)*sin_integral(b*x^2) +
 cos(b*x^2 + a))/x^2
```

Sympy [F]

$$\int \frac{\cos(a+bx^2)}{x^3}\,dx = \int \frac{\cos(a+bx^2)}{x^3}\,dx$$

```
[In] integrate(cos(b*x**2+a)/x**3,x)
[Out] Integral(cos(a + b*x**2)/x**3, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.37 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.14

$$\int \frac{\cos(a+bx^2)}{x^3}\,dx = -\frac{1}{4}\left((i\,\Gamma(-1,i\,bx^2) - i\,\Gamma(-1,-i\,bx^2))\cos(a) + (\Gamma(-1,i\,bx^2) + \Gamma(-1,-i\,bx^2))\sin(a)\right)b$$

```
[In] integrate(cos(b*x^2+a)/x^3,x, algorithm="maxima")
[Out] -1/4*((I*gamma(-1, I*b*x^2) - I*gamma(-1, -I*b*x^2))*cos(a) + (gamma(-1, I*
b*x^2) + gamma(-1, -I*b*x^2))*sin(a))*b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. 2(36) = 72.

Time = 0.31 (sec) , antiderivative size = 87, normalized size of antiderivative = 2.07

$$\int \frac{\cos(a+bx^2)}{x^3}\,dx = -\frac{(bx^2+a)b^2\operatorname{Ci}(bx^2)\sin(a) - ab^2\operatorname{Ci}(bx^2)\sin(a) + (bx^2+a)b^2\cos(a)\operatorname{Si}(bx^2) - ab^2\cos(a)\operatorname{Si}(bx^2) + b^2}{2\,b^2x^2}$$

```
[In] integrate(cos(b*x^2+a)/x^3,x, algorithm="giac")
[Out] -1/2*((b*x^2 + a)*b^2*cos_integral(b*x^2)*sin(a) - a*b^2*cos_integral(b*x^2
)*sin(a) + (b*x^2 + a)*b^2*cos(a)*sin_integral(b*x^2) - a*b^2*cos(a)*sin_in
tegral(b*x^2) + b^2*cos(b*x^2 + a))/(b^2*x^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(a+bx^2)}{x^3}\,dx = \int \frac{\cos(b\,x^2+a)}{x^3}\,dx$$

```
[In] int(cos(a + b*x^2)/x^3,x)
[Out] int(cos(a + b*x^2)/x^3, x)
```

3.8 $\int x^3 \cos^2(a+bx^2)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 51

$$\int x^3 \cos^2\left(a+bx^2\right)\,dx = \frac{x^4}{8}+\frac{\cos^2(a+bx^2)}{8b^2}+\frac{x^2\cos(a+bx^2)\sin(a+bx^2)}{4b}$$

```
[Out] 1/8*x^4+1/8*cos(b*x^2+a)^2/b^2+1/4*x^2*cos(b*x^2+a)*sin(b*x^2+a)/b
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3461, 3391, 30}

$$\int x^3 \cos^2\left(a+bx^2\right)\,dx = \frac{\cos^2(a+bx^2)}{8b^2}+\frac{x^2\sin(a+bx^2)\cos(a+bx^2)}{4b}+\frac{x^4}{8}$$

```
[In] Int[x^3*Cos[a + b*x^2]^2,x]
```

```
[Out] x^4/8 + Cos[a + b*x^2]^2/(8*b^2) + (x^2*Cos[a + b*x^2]*Sin[a + b*x^2])/(4*b
)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
```

```
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}\text{Subst}\left(\int x\cos^2(a+bx)\,dx, x, x^2\right)\\ &= \frac{\cos^2(a+bx^2)}{8b^2} + \frac{x^2\cos(a+bx^2)\sin(a+bx^2)}{4b} + \frac{1}{4}\text{Subst}\left(\int x\,dx, x, x^2\right)\\ &= \frac{x^4}{8} + \frac{\cos^2(a+bx^2)}{8b^2} + \frac{x^2\cos(a+bx^2)\sin(a+bx^2)}{4b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.78

$$\int x^3\cos^2\left(a+bx^2\right)\,dx = \frac{\cos(2(a+bx^2)) + 2bx^2(bx^2+\sin(2(a+bx^2)))}{16b^2}$$

```
[In] Integrate[x^3*Cos[a + b*x^2]^2,x]
[Out] (Cos[2*(a + b*x^2)] + 2*b*x^2*(b*x^2 + Sin[2*(a + b*x^2)]))/(16*b^2)
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.82

method	result	size
default	$\frac{x^4}{8}+\frac{x^2\sin(2b\,x^2+2a)}{8b}+\frac{\cos(2b\,x^2+2a)}{16b^2}$	42
risch	$\frac{x^4}{8}+\frac{x^2\sin(2b\,x^2+2a)}{8b}+\frac{\cos(2b\,x^2+2a)}{16b^2}$	42
parallelrisch	$\frac{2x^4b^2+2x^2\sin(2b\,x^2+2a)b+\cos(2b\,x^2+2a)-1}{16b^2}$	44
norman	$\frac{\frac{x^4}{8}+\frac{x^4\left(\tan^2\left(\frac{a}{2}+\frac{b\,x^2}{2}\right)\right)}{4}+\frac{x^4\left(\tan^4\left(\frac{a}{2}+\frac{b\,x^2}{2}\right)\right)}{8}+\frac{x^2\tan\left(\frac{a}{2}+\frac{b\,x^2}{2}\right)}{2b}-\frac{x^2\left(\tan^3\left(\frac{a}{2}+\frac{b\,x^2}{2}\right)\right)}{2b}-\frac{\tan^2\left(\frac{a}{2}+\frac{b\,x^2}{2}\right)}{2b^2}}{\left(1+\tan^2\left(\frac{a}{2}+\frac{b\,x^2}{2}\right)\right)^2}$	119

```
[In] int(x^3*cos(b*x^2+a)^2,x,method=_RETURNVERBOSE)
[Out] 1/8*x^4+1/8/b*x^2*sin(2*b*x^2+2*a)+1/16/b^2*cos(2*b*x^2+2*a)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.88

$$\int x^3\cos^2\left(a+bx^2\right)\,dx=\frac{b^2x^4+2\,bx^2\cos\left(bx^2+a\right)\sin\left(bx^2+a\right)+\cos\left(bx^2+a\right)^2}{8\,b^2}$$

```
[In] integrate(x^3*cos(b*x^2+a)^2,x, algorithm="fricas")
[Out] 1/8*(b^2*x^4 + 2*b*x^2*cos(b*x^2 + a)*sin(b*x^2 + a) + cos(b*x^2 + a)^2)/b^
2
```

Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.53

$$\int x^3\cos^2\left(a+bx^2\right)\,dx$$
$$=\begin{cases}\frac{x^4\sin^2\left(a+bx^2\right)}{8}+\frac{x^4\cos^2\left(a+bx^2\right)}{8}+\frac{x^2\sin\left(a+bx^2\right)\cos\left(a+bx^2\right)}{4b}+\frac{\cos^2\left(a+bx^2\right)}{8b^2} & \text{for } b\neq 0\\ \frac{x^4\cos^2\left(a\right)}{4} & \text{otherwise}\end{cases}$$

```
[In] integrate(x**3*cos(b*x**2+a)**2,x)
[Out] Piecewise((x**4*sin(a + b*x**2)**2/8 + x**4*cos(a + b*x**2)**2/8 + x**2*sin
(a + b*x**2)*cos(a + b*x**2)/(4*b) + cos(a + b*x**2)**2/(8*b**2), Ne(b, 0))
, (x**4*cos(a)**2/4, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.82

$$\int x^3 \cos^2\left(a + bx^2\right) \, dx = \frac{2\,b^2x^4 + 2\,bx^2 \sin\left(2\,bx^2 + 2\,a\right) + \cos\left(2\,bx^2 + 2\,a\right)}{16\,b^2}$$

```
[In] integrate(x^3*cos(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/16*(2*b^2*x^4 + 2*b*x^2*sin(2*b*x^2 + 2*a) + cos(2*b*x^2 + 2*a))/b^2
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.49

$$\int x^3 \cos^2\left(a + bx^2\right) \, dx = -\frac{(2\,bx^2 + 2\,a + \sin\left(2\,bx^2 + 2\,a\right))a}{8\,b^2} + \frac{2\,\left(bx^2 + a\right)^2 + 2\,\left(bx^2 + a\right)\sin\left(2\,bx^2 + 2\,a\right) + \cos\left(2\,bx^2 + 2\,a\right)}{16\,b^2}$$

```
[In] integrate(x^3*cos(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/8*(2*b*x^2 + 2*a + sin(2*b*x^2 + 2*a))*a/b^2 + 1/16*(2*(b*x^2 + a)^2 + 2
*(b*x^2 + a)*sin(2*b*x^2 + 2*a) + cos(2*b*x^2 + 2*a))/b^2
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.80

$$\int x^3 \cos^2\left(a + bx^2\right) \, dx = \frac{\cos\left(2\,b\,x^2 + 2\,a\right)}{16\,b^2} + \frac{x^4}{8} + \frac{x^2\,\sin\left(2\,b\,x^2 + 2\,a\right)}{8\,b}$$

```
[In] int(x^3*cos(a + b*x^2)^2,x)
```

```
[Out] cos(2*a + 2*b*x^2)/(16*b^2) + x^4/8 + (x^2*sin(2*a + 2*b*x^2))/(8*b)
```

3.9 $\int x^2 \cos^2(a+bx^2)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 91

$$\int x^2 \cos^2(a+bx^2)\,dx = \frac{x^3}{6} - \frac{\sqrt{\pi}\cos(2a)\,\text{FresnelS}\left(\frac{2\sqrt{b}x}{\sqrt{\pi}}\right)}{16b^{3/2}} - \frac{\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{b}x}{\sqrt{\pi}}\right)\sin(2a)}{16b^{3/2}} + \frac{x\sin(2a+2bx^2)}{8b}$$

```
[Out] 1/6*x^3+1/8*x*sin(2*b*x^2+2*a)/b-1/16*cos(2*a)*FresnelS(2*x*b^(1/2)/Pi^(1/2
))*Pi^(1/2)/b^(3/2)-1/16*FresnelC(2*x*b^(1/2)/Pi^(1/2))*sin(2*a)*Pi^(1/2)/b
^(3/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3485, 3467, 3434, 3433, 3432}

$$\int x^2 \cos^2(a+bx^2)\,dx = -\frac{\sqrt{\pi}\sin(2a)\,\text{FresnelC}\left(\frac{2\sqrt{b}x}{\sqrt{\pi}}\right)}{16b^{3/2}} - \frac{\sqrt{\pi}\cos(2a)\,\text{FresnelS}\left(\frac{2\sqrt{b}x}{\sqrt{\pi}}\right)}{16b^{3/2}} + \frac{x\sin(2a+2bx^2)}{8b} + \frac{x^3}{6}$$

```
[In] Int[x^2*Cos[a + b*x^2]^2,x]
```

```
[Out] x^3/6 - (Sqrt[Pi]*Cos[2*a]*FresnelS[(2*Sqrt[b]*x)/Sqrt[Pi]])/(16*b^(3/2)) -
 (Sqrt[Pi]*FresnelC[(2*Sqrt[b]*x)/Sqrt[Pi]]*Sin[2*a])/(16*b^(3/2)) + (x*Sin
[2*a + 2*b*x^2])/(8*b)
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3434

```
Int[Sin[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Sin[c], Int
[Cos[d*(e + f*x)^2], x], x] + Dist[Cos[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3467

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[e^(n
 - 1)*(e*x)^(m - n + 1)*(Sin[c + d*x^n]/(d*n)), x] - Dist[e^n*((m - n + 1)/
(d*n)), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3485

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_)*((e_.)*(x_))^(m_.), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Cos[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{x^2}{2}+\frac{1}{2}x^2\cos\left(2a+2bx^2\right)\right)dx \\
&= \frac{x^3}{6}+\frac{1}{2}\int x^2\cos\left(2a+2bx^2\right)\,dx \\
&= \frac{x^3}{6}+\frac{x\sin\left(2a+2bx^2\right)}{8b}-\frac{\int\sin\left(2a+2bx^2\right)\,dx}{8b} \\
&= \frac{x^3}{6}+\frac{x\sin\left(2a+2bx^2\right)}{8b}-\frac{\cos(2a)\int\sin\left(2bx^2\right)\,dx}{8b}-\frac{\sin(2a)\int\cos\left(2bx^2\right)\,dx}{8b} \\
&= \frac{x^3}{6}-\frac{\sqrt{\pi}\cos(2a)\,\text{FresnelS}\left(\frac{2\sqrt{b}x}{\sqrt{\pi}}\right)}{16b^{3/2}}-\frac{\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{b}x}{\sqrt{\pi}}\right)\sin(2a)}{16b^{3/2}}+\frac{x\sin\left(2a+2bx^2\right)}{8b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.96

$$\int x^2 \cos^2\left(a+bx^2\right)\,dx$$
$$=\frac{-3\sqrt{\pi}\cos(2a)\,\text{FresnelS}\left(\frac{2\sqrt{b}x}{\sqrt{\pi}}\right)-3\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{b}x}{\sqrt{\pi}}\right)\sin(2a)+2\sqrt{b}x(4bx^2+3\sin\left(2(a+bx^2)\right))}{48b^{3/2}}$$

```
[In] Integrate[x^2*Cos[a + b*x^2]^2,x]
[Out] (-3*Sqrt[Pi]*Cos[2*a]*FresnelS[(2*Sqrt[b]*x)/Sqrt[Pi]] - 3*Sqrt[Pi]*Fresnel
C[(2*Sqrt[b]*x)/Sqrt[Pi]]*Sin[2*a] + 2*Sqrt[b]*x*(4*b*x^2 + 3*Sin[2*(a + b*
x^2)]))/(48*b^(3/2))
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.69

method	result	size
default	$\frac{x^3}{6}+\frac{x\sin(2b\,x^2+2a)}{8b}-\frac{\sqrt{\pi}\left(\cos(2a)\,\mathrm{S}\left(\frac{2x\sqrt{b}}{\sqrt{\pi}}\right)+\sin(2a)\,\mathrm{C}\left(\frac{2x\sqrt{b}}{\sqrt{\pi}}\right)\right)}{16b^{\frac{3}{2}}}$	63
risch	$\frac{x^3}{6}-\frac{i\mathrm{e}^{-2ia}\sqrt{\pi}\sqrt{2}\,\mathrm{erf}\left(\sqrt{2}\sqrt{ib}\,x\right)}{64b\sqrt{ib}}+\frac{i\mathrm{e}^{2ia}\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{-2ib}\,x\right)}{32b\sqrt{-2ib}}+\frac{x\sin(2b\,x^2+2a)}{8b}$	88

```
[In] int(x^2*cos(b*x^2+a)^2,x,method=_RETURNVERBOSE)
[Out] 1/6*x^3+1/8*x*sin(2*b*x^2+2*a)/b-1/16/b^(3/2)*Pi^(1/2)*(cos(2*a)*FresnelS(2
*x*b^(1/2)/Pi^(1/2))+sin(2*a)*FresnelC(2*x*b^(1/2)/Pi^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.92

$$\int x^2 \cos^2\left(a+bx^2\right)\,dx$$
$$=\frac{8\,b^2x^3+12\,bx\cos\left(bx^2+a\right)\sin\left(bx^2+a\right)-3\,\pi\sqrt{\frac{b}{\pi}}\cos\left(2\,a\right)\mathrm{S}\left(2\,x\sqrt{\frac{b}{\pi}}\right)-3\,\pi\sqrt{\frac{b}{\pi}}\,\mathrm{C}\left(2\,x\sqrt{\frac{b}{\pi}}\right)\sin\left(2\,a\right)}{48\,b^2}$$

```
[In] integrate(x^2*cos(b*x^2+a)^2,x, algorithm="fricas")
[Out] 1/48*(8*b^2*x^3 + 12*b*x*cos(b*x^2 + a)*sin(b*x^2 + a) - 3*pi*sqrt(b/pi)*co
s(2*a)*fresnel_sin(2*x*sqrt(b/pi)) - 3*pi*sqrt(b/pi)*fresnel_cos(2*x*sqrt(b
/pi))*sin(2*a))/b^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 201 vs. $2(85) = 170$.

Time = 1.31 (sec) , antiderivative size = 201, normalized size of antiderivative = 2.21

$$\int x^2\cos^2\left(a+bx^2\right)\,dx=\frac{b^{\frac{3}{2}}x^5\sqrt{\frac{1}{b}}\sin\left(2a\right)\Gamma\left(\frac{3}{4}\right)\Gamma\left(\frac{5}{4}\right){}_2F_3\left(\begin{matrix}\frac{3}{4},\frac{5}{4}\\ \frac{3}{2},\frac{7}{4},\frac{9}{4}\end{matrix}\middle|-b^2x^4\right)}{8\Gamma\left(\frac{7}{4}\right)\Gamma\left(\frac{9}{4}\right)}-\frac{\sqrt{b}x^3\sqrt{\frac{1}{b}}\cos\left(2a\right)\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right){}_2F_3\left(\begin{matrix}\frac{1}{4},\frac{3}{4}\\ \frac{1}{2},\frac{5}{4},\frac{7}{4}\end{matrix}\middle|-b^2x^4\right)}{16\Gamma\left(\frac{5}{4}\right)\Gamma\left(\frac{7}{4}\right)}+\frac{x^3}{6}-\frac{\sqrt{\pi}x^2\sqrt{\frac{1}{b}}\sin\left(2a\right)S\left(\frac{2\sqrt{b}x}{\sqrt{\pi}}\right)}{4}+\frac{\sqrt{\pi}x^2\sqrt{\frac{1}{b}}\cos\left(2a\right)C\left(\frac{2\sqrt{b}x}{\sqrt{\pi}}\right)}{4}$$

```
[In] integrate(x**2*cos(b*x**2+a)**2,x)
```

```
[Out] b**(3/2)*x**5*sqrt(1/b)*sin(2*a)*gamma(3/4)*gamma(5/4)*hyper((3/4, 5/4), (3
/2, 7/4, 9/4), -b**2*x**4)/(8*gamma(7/4)*gamma(9/4)) - sqrt(b)*x**3*sqrt(1/
b)*cos(2*a)*gamma(1/4)*gamma(3/4)*hyper((1/4, 3/4), (1/2, 5/4, 7/4), -b**2*
x**4)/(16*gamma(5/4)*gamma(7/4)) + x**3/6 - sqrt(pi)*x**2*sqrt(1/b)*sin(2*a
)*fresnels(2*sqrt(b)*x/sqrt(pi))/4 + sqrt(pi)*x**2*sqrt(1/b)*cos(2*a)*fresn
elc(2*sqrt(b)*x/sqrt(pi))/4
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.33 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.99

$$\int x^2\cos^2\left(a+bx^2\right)\,dx=\frac{64\,b^3x^3+48\,b^2x\sin\left(2\,bx^2+2\,a\right)-3\cdot 4^{\frac{1}{4}}\sqrt{2}\sqrt{\pi}\left(\left(\left(i+1\right)\cos\left(2\,a\right)-\left(i-1\right)\sin\left(2\,a\right)\right)\operatorname{erf}\left(\sqrt{2i\,b}x\right)+\left(-(i-\right.\right.}{384\,b^3}$$

```
[In] integrate(x^2*cos(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/384*(64*b^3*x^3 + 48*b^2*x*sin(2*b*x^2 + 2*a) - 3*4^(1/4)*sqrt(2)*sqrt(pi
)*(((I + 1)*cos(2*a) - (I - 1)*sin(2*a))*erf(sqrt(2*I*b)*x) + (-(I - 1)*cos
(2*a) + (I + 1)*sin(2*a))*erf(sqrt(-2*I*b)*x))*b^(3/2))/b^3
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.30

$$\int x^2\cos^2\left(a+bx^2\right)\,dx=\frac{1}{6}\,x^3-\frac{i\,xe^{\left(2i\,bx^2+2i\,a\right)}}{16\,b}+\frac{i\,xe^{\left(-2i\,bx^2-2i\,a\right)}}{16\,b}-\frac{\sqrt{\pi}\,\mathrm{erf}\left(-i\,\sqrt{b}x\left(\frac{i\,b}{|b|}+1\right)\right)e^{(2i\,a)}}{32\,b^{\frac{3}{2}}\left(\frac{i\,b}{|b|}+1\right)}-\frac{\sqrt{\pi}\,\mathrm{erf}\left(i\,\sqrt{b}x\left(-\frac{i\,b}{|b|}+1\right)\right)e^{(-2i\,a)}}{32\,b^{\frac{3}{2}}\left(-\frac{i\,b}{|b|}+1\right)}$$

```
[In] integrate(x^2*cos(b*x^2+a)^2,x, algorithm="giac")
[Out] 1/6*x^3 - 1/16*I*x*e^(2*I*b*x^2 + 2*I*a)/b + 1/16*I*x*e^(-2*I*b*x^2 - 2*I*a
)/b - 1/32*sqrt(pi)*erf(-I*sqrt(b)*x*(I*b/abs(b) + 1))*e^(2*I*a)/(b^(3/2)*(
I*b/abs(b) + 1)) - 1/32*sqrt(pi)*erf(I*sqrt(b)*x*(-I*b/abs(b) + 1))*e^(-2*I
*a)/(b^(3/2)*(-I*b/abs(b) + 1))
```

Mupad [F(-1)]

Timed out.

$$\int x^2\cos^2\left(a+bx^2\right)\,dx=\int x^2\cos\left(b\,x^2+a\right)^2\,dx$$

```
[In] int(x^2*cos(a + b*x^2)^2,x)
[Out] int(x^2*cos(a + b*x^2)^2, x)
```

3.10 $\int x \cos^2(a+bx^2)\, dx$

Optimal result

Integrand size = 12, antiderivative size = 31

$$\int x \cos^2(a+bx^2)\, dx = \frac{x^2}{4} + \frac{\cos(a+bx^2)\sin(a+bx^2)}{4b}$$

```
[Out] 1/4*x^2+1/4*cos(b*x^2+a)*sin(b*x^2+a)/b
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3461, 2715, 8}

$$\int x \cos^2(a+bx^2)\, dx = \frac{\sin(a+bx^2)\cos(a+bx^2)}{4b} + \frac{x^2}{4}$$

```
[In] Int[x*Cos[a + b*x^2]^2,x]
```

```
[Out] x^2/4 + (Cos[a + b*x^2]*Sin[a + b*x^2])/(4*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \cos^2(a+bx)\,dx, x, x^2\right)\\ &= \frac{\cos(a+bx^2)\sin(a+bx^2)}{4b} + \frac{1}{4}\text{Subst}\left(\int 1\,dx, x, x^2\right)\\ &= \frac{x^2}{4} + \frac{\cos(a+bx^2)\sin(a+bx^2)}{4b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int x\cos^2\left(a+bx^2\right)\,dx = \frac{2(a+bx^2)+\sin\left(2(a+bx^2)\right)}{8b}$$

```
[In] Integrate[x*Cos[a + b*x^2]^2,x]
[Out] (2*(a + b*x^2) + Sin[2*(a + b*x^2)])/(8*b)
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.74

method	result	size
risch	$\frac{x^2}{4}+\frac{\sin(2bx^2+2a)}{8b}$	23
parallelrisch	$\frac{2bx^2+\sin(2bx^2+2a)}{8b}$	24
derivativedivides	$\frac{\frac{\cos(bx^2+a)\sin(bx^2+a)}{2}+\frac{bx^2}{2}+\frac{a}{2}}{2b}$	34
default	$\frac{\frac{\cos(bx^2+a)\sin(bx^2+a)}{2}+\frac{bx^2}{2}+\frac{a}{2}}{2b}$	34
norman	$\frac{\frac{x^2}{4}+\frac{\tan\left(\frac{a}{2}+\frac{bx^2}{2}\right)}{2b}-\frac{\tan^3\left(\frac{a}{2}+\frac{bx^2}{2}\right)}{2b}+\frac{x^2\left(\tan^2\left(\frac{a}{2}+\frac{bx^2}{2}\right)\right)}{2}+\frac{x^2\left(\tan^4\left(\frac{a}{2}+\frac{bx^2}{2}\right)\right)}{4}}{\left(1+\tan^2\left(\frac{a}{2}+\frac{bx^2}{2}\right)\right)^2}$	95

```
[In] int(x*cos(b*x^2+a)^2,x,method=_RETURNVERBOSE)
[Out] 1/4*x^2+1/8*sin(2*b*x^2+2*a)/b
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.90

$$\int x\cos^2\left(a+bx^2\right)\,dx=\frac{bx^2+\cos\left(bx^2+a\right)\sin\left(bx^2+a\right)}{4\,b}$$

```
[In] integrate(x*cos(b*x^2+a)^2,x, algorithm="fricas")
[Out] 1/4*(b*x^2 + cos(b*x^2 + a)*sin(b*x^2 + a))/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(24) = 48$.

Time = 0.12 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.94

$$\int x\cos^2\left(a+bx^2\right)\,dx=\begin{cases}\frac{x^2\sin^2\left(a+bx^2\right)}{4}+\frac{x^2\cos^2\left(a+bx^2\right)}{4}+\frac{\sin\left(a+bx^2\right)\cos\left(a+bx^2\right)}{4b} & \text{for } b\neq 0\\ \frac{x^2\cos^2\left(a\right)}{2} & \text{otherwise}\end{cases}$$

```
[In] integrate(x*cos(b*x**2+a)**2,x)
[Out] Piecewise((x**2*sin(a + b*x**2)**2/4 + x**2*cos(a + b*x**2)**2/4 + sin(a +
b*x**2)*cos(a + b*x**2)/(4*b), Ne(b, 0)), (x**2*cos(a)**2/2, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.74

$$\int x\cos^2\left(a+bx^2\right)\,dx=\frac{2\,bx^2+\sin\left(2\,bx^2+2\,a\right)}{8\,b}$$

```
[In] integrate(x*cos(b*x^2+a)^2,x, algorithm="maxima")
[Out] 1/8*(2*b*x^2 + sin(2*b*x^2 + 2*a))/b
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int x\cos^2\left(a+bx^2\right)\,dx=\frac{2\,bx^2+2\,a+\sin\left(2\,bx^2+2\,a\right)}{8\,b}$$

```
[In] integrate(x*cos(b*x^2+a)^2,x, algorithm="giac")
[Out] 1/8*(2*b*x^2 + 2*a + sin(2*b*x^2 + 2*a))/b
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.71

$$\int x\cos^2\left(a+bx^2\right)\,dx=\frac{\sin\left(2\,b\,x^2+2\,a\right)}{8\,b}+\frac{x^2}{4}$$

```
[In] int(x*cos(a + b*x^2)^2,x)
[Out] sin(2*a + 2*b*x^2)/(8*b) + x^2/4
```

3.11 $\int \cos^2(a+bx^2)\, dx$

Optimal result

Integrand size = 10, antiderivative size = 70

$$\int \cos^2(a+bx^2)\, dx = \frac{x}{2} + \frac{\sqrt{\pi}\cos(2a)\operatorname{FresnelC}\left(\frac{2\sqrt{b}x}{\sqrt{\pi}}\right)}{4\sqrt{b}} - \frac{\sqrt{\pi}\operatorname{FresnelS}\left(\frac{2\sqrt{b}x}{\sqrt{\pi}}\right)\sin(2a)}{4\sqrt{b}}$$

```
[Out] 1/2*x+1/4*cos(2*a)*FresnelC(2*x*b^(1/2)/Pi^(1/2))*Pi^(1/2)/b^(1/2)-1/4*Fres
nelS(2*x*b^(1/2)/Pi^(1/2))*sin(2*a)*Pi^(1/2)/b^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3439, 3435, 3433, 3432}

$$\int \cos^2(a+bx^2)\, dx = \frac{\sqrt{\pi}\cos(2a)\operatorname{FresnelC}\left(\frac{2\sqrt{b}x}{\sqrt{\pi}}\right)}{4\sqrt{b}} - \frac{\sqrt{\pi}\sin(2a)\operatorname{FresnelS}\left(\frac{2\sqrt{b}x}{\sqrt{\pi}}\right)}{4\sqrt{b}} + \frac{x}{2}$$

```
[In] Int[Cos[a + b*x^2]^2,x]
```

```
[Out] x/2 + (Sqrt[Pi]*Cos[2*a]*FresnelC[(2*Sqrt[b]*x)/Sqrt[Pi]])/(4*Sqrt[b]) - (S
qrt[Pi]*FresnelS[(2*Sqrt[b]*x)/Sqrt[Pi]]*Sin[2*a])/(4*Sqrt[b])
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3435

```
Int[Cos[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Cos[c], Int
[Cos[d*(e + f*x)^2], x], x] - Dist[Sin[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3439

```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_), x_Sy
mbol] :> Int[ExpandTrigReduce[(a + b*Cos[c + d*(e + f*x)^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 1] && IGtQ[n, 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{1}{2}+\frac{1}{2}\cos\left(2a+2bx^2\right)\right)\,dx\\
&= \frac{x}{2}+\frac{1}{2}\int \cos\left(2a+2bx^2\right)\,dx\\
&= \frac{x}{2}+\frac{1}{2}\cos(2a)\int \cos\left(2bx^2\right)\,dx-\frac{1}{2}\sin(2a)\int \sin\left(2bx^2\right)\,dx\\
&= \frac{x}{2}+\frac{\sqrt{\pi}\cos(2a)\,\text{FresnelC}\left(\frac{2\sqrt{b}x}{\sqrt{\pi}}\right)}{4\sqrt{b}}-\frac{\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{b}x}{\sqrt{\pi}}\right)\sin(2a)}{4\sqrt{b}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.96

$$\int \cos^2\left(a+bx^2\right)\,dx=\frac{2\sqrt{b}x+\sqrt{\pi}\cos(2a)\,\text{FresnelC}\left(\frac{2\sqrt{b}x}{\sqrt{\pi}}\right)-\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{b}x}{\sqrt{\pi}}\right)\sin(2a)}{4\sqrt{b}}$$

```
[In] Integrate[Cos[a + b*x^2]^2,x]
[Out] (2*Sqrt[b]*x + Sqrt[Pi]*Cos[2*a]*FresnelC[(2*Sqrt[b]*x)/Sqrt[Pi]] - Sqrt[Pi
]*FresnelS[(2*Sqrt[b]*x)/Sqrt[Pi]]*Sin[2*a])/(4*Sqrt[b])
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.64

method	result	size
default	$\frac{x}{2}+\frac{\sqrt{\pi}\,\left(\cos(2a)\,\mathrm{C}\left(\frac{2x\sqrt{b}}{\sqrt{\pi}}\right)-\sin(2a)\,\mathrm{S}\left(\frac{2x\sqrt{b}}{\sqrt{\pi}}\right)\right)}{4\sqrt{b}}$	45
risch	$\frac{x}{2}+\frac{\mathrm{e}^{-2ia}\sqrt{\pi}\,\sqrt{2}\,\mathrm{erf}\left(\sqrt{2}\,\sqrt{ib}\,x\right)}{16\sqrt{ib}}+\frac{\mathrm{e}^{2ia}\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{-2ib}\,x\right)}{8\sqrt{-2ib}}$	61

```
[In] int(cos(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x+1/4*Pi^(1/2)/b^(1/2)*(cos(2*a)*FresnelC(2*x*b^(1/2)/Pi^(1/2))-sin(2*a
)*FresnelS(2*x*b^(1/2)/Pi^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.84

$$\int \cos^2\left(a+bx^2\right)\,dx=\frac{\pi\sqrt{\frac{b}{\pi}}\cos\left(2\,a\right)\mathrm{C}\left(2\,x\sqrt{\frac{b}{\pi}}\right)-\pi\sqrt{\frac{b}{\pi}}\,\mathrm{S}\left(2\,x\sqrt{\frac{b}{\pi}}\right)\sin\left(2\,a\right)+2\,bx}{4\,b}$$

```
[In] integrate(cos(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(pi*sqrt(b/pi)*cos(2*a)*fresnel_cos(2*x*sqrt(b/pi)) - pi*sqrt(b/pi)*fre
snel_sin(2*x*sqrt(b/pi))*sin(2*a) + 2*b*x)/b
```

Sympy [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.80

$$\int \cos^2\left(a+bx^2\right)\,dx=\frac{x}{2}+\frac{\sqrt{\pi}\left(-\sin\left(2a\right)S\left(\frac{2\sqrt{b}x}{\sqrt{\pi}}\right)+\cos\left(2a\right)C\left(\frac{2\sqrt{b}x}{\sqrt{\pi}}\right)\right)\sqrt{\frac{1}{b}}}{4}$$

```
[In] integrate(cos(b*x**2+a)**2,x)
```

```
[Out] x/2 + sqrt(pi)*(-sin(2*a)*fresnels(2*sqrt(b)*x/sqrt(pi)) + cos(2*a)*fresnel
c(2*sqrt(b)*x/sqrt(pi)))*sqrt(1/b)/4
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00

$$\int \cos^2\left(a+bx^2\right)\,dx = -\frac{4^{\frac{1}{4}}\sqrt{2}\sqrt{\pi}\left(\left((i-1)\cos\left(2a\right)+(i+1)\sin\left(2a\right)\right)\operatorname{erf}\left(\sqrt{2i\,b}x\right)+\left(-(i+1)\cos\left(2a\right)-(i-1)\sin\left(2a\right)\right)\operatorname{er}\right.}{32\,b^2}$$

```
[In] integrate(cos(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/32*(4^(1/4)*sqrt(2)*sqrt(pi)*(((I - 1)*cos(2*a) + (I + 1)*sin(2*a))*erf(
sqrt(2*I*b)*x) + (-(I + 1)*cos(2*a) - (I - 1)*sin(2*a))*erf(sqrt(-2*I*b)*x)
)*b^(3/2) - 16*b^2*x)/b^2
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.17

$$\int \cos^2\left(a+bx^2\right)\,dx = \frac{1}{2}x + \frac{i\sqrt{\pi}\operatorname{erf}\left(-i\sqrt{b}x\left(\frac{i\,b}{|b|}+1\right)\right)e^{(2i\,a)}}{8\sqrt{b}\left(\frac{i\,b}{|b|}+1\right)} - \frac{i\sqrt{\pi}\operatorname{erf}\left(i\sqrt{b}x\left(-\frac{i\,b}{|b|}+1\right)\right)e^{(-2i\,a)}}{8\sqrt{b}\left(-\frac{i\,b}{|b|}+1\right)}$$

```
[In] integrate(cos(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*x + 1/8*I*sqrt(pi)*erf(-I*sqrt(b)*x*(I*b/abs(b) + 1))*e^(2*I*a)/(sqrt(b
)*(I*b/abs(b) + 1)) - 1/8*I*sqrt(pi)*erf(I*sqrt(b)*x*(-I*b/abs(b) + 1))*e^(
-2*I*a)/(sqrt(b)*(-I*b/abs(b) + 1))
```

Mupad [F(-1)]

Timed out.

$$\int \cos^2\left(a+bx^2\right)\,dx = \int \cos\left(b\,x^2+a\right)^2\,dx$$

```
[In] int(cos(a + b*x^2)^2,x)
```

```
[Out] int(cos(a + b*x^2)^2, x)
```

3.12 $\int \frac{\cos^2(a+bx^2)}{x} dx$

Optimal result

Integrand size = 14, antiderivative size = 37

$$\int \frac{\cos^2(a+bx^2)}{x} dx = \frac{1}{4}\cos(2a)\,\text{CosIntegral}\left(2bx^2\right) + \frac{\log(x)}{2} - \frac{1}{4}\sin(2a)\text{Si}\left(2bx^2\right)$$

```
[Out] 1/4*Ci(2*b*x^2)*cos(2*a)+1/2*ln(x)-1/4*Si(2*b*x^2)*sin(2*a)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3485, 3459, 3457, 3456}

$$\int \frac{\cos^2(a+bx^2)}{x} dx = \frac{1}{4}\cos(2a)\,\text{CosIntegral}\left(2bx^2\right) - \frac{1}{4}\sin(2a)\text{Si}\left(2bx^2\right) + \frac{\log(x)}{2}$$

```
[In] Int[Cos[a + b*x^2]^2/x,x]
```

```
[Out] (Cos[2*a]*CosIntegral[2*b*x^2])/4 + Log[x]/2 - (Sin[2*a]*SinIntegral[2*b*x^
2])/4
```

Rule 3456

```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3457

```
Int[Cos[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[CosIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3459

```
Int[Cos[(c_) + (d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Dist[Cos[c], Int[Cos[d*x
^n]/x, x], x] - Dist[Sin[c], Int[Sin[d*x^n]/x, x], x] /; FreeQ[{c, d, n}, x
]
```

Rule 3485

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_)*((e_.)*(x_))^(m_.), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Cos[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{1}{2x} + \frac{\cos(2a + 2bx^2)}{2x}\right) dx \\
&= \frac{\log(x)}{2} + \frac{1}{2}\int \frac{\cos(2a + 2bx^2)}{x}\, dx \\
&= \frac{\log(x)}{2} + \frac{1}{2}\cos(2a)\int \frac{\cos(2bx^2)}{x}\, dx - \frac{1}{2}\sin(2a)\int \frac{\sin(2bx^2)}{x}\, dx \\
&= \frac{1}{4}\cos(2a)\,\text{CosIntegral}\left(2bx^2\right) + \frac{\log(x)}{2} - \frac{1}{4}\sin(2a)\text{Si}\left(2bx^2\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.92

$$\int \frac{\cos^2(a + bx^2)}{x}\, dx = \frac{1}{4}\left(\cos(2a)\,\text{CosIntegral}\left(2bx^2\right) + 2\log(x) - \sin(2a)\text{Si}\left(2bx^2\right)\right)$$

```
[In] Integrate[Cos[a + b*x^2]^2/x,x]
[Out] (Cos[2*a]*CosIntegral[2*b*x^2] + 2*Log[x] - Sin[2*a]*SinIntegral[2*b*x^2])/
4
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.62 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.84

method	result	size
risch	$\frac{\ln(x)}{2} + \frac{i\mathrm{e}^{-2ia}\pi\ \mathrm{csgn}(b\,x^2)}{8} - \frac{i\mathrm{e}^{-2ia}\,\mathrm{Si}(2b\,x^2)}{4} - \frac{\mathrm{e}^{-2ia}\,\mathrm{Ei}_1(-2ib\,x^2)}{8} - \frac{\mathrm{e}^{2ia}\,\mathrm{Ei}_1(-2ib\,x^2)}{8}$	68

```
[In] int(cos(b*x^2+a)^2/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*ln(x)+1/8*I*exp(-2*I*a)*Pi*csgn(b*x^2)-1/4*I*exp(-2*I*a)*Si(2*b*x^2)-1/
8*exp(-2*I*a)*Ei(1,-2*I*b*x^2)-1/8*exp(2*I*a)*Ei(1,-2*I*b*x^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.84

$$\int \frac{\cos^2(a+bx^2)}{x}\,dx = \frac{1}{4}\cos(2a)\operatorname{Ci}\left(2bx^2\right) - \frac{1}{4}\sin(2a)\operatorname{Si}\left(2bx^2\right) + \frac{1}{2}\log(x)$$

```
[In] integrate(cos(b*x^2+a)^2/x,x, algorithm="fricas")
```

```
[Out] 1/4*cos(2*a)*cos_integral(2*b*x^2) - 1/4*sin(2*a)*sin_integral(2*b*x^2) + 1
/2*log(x)
```

Sympy [F]

$$\int \frac{\cos^2(a+bx^2)}{x}\,dx = \int \frac{\cos^2(a+bx^2)}{x}\,dx$$

```
[In] integrate(cos(b*x**2+a)**2/x,x)
```

```
[Out] Integral(cos(a + b*x**2)**2/x, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.37 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.38

$$\int \frac{\cos^2(a+bx^2)}{x}\,dx = \frac{1}{8}\left(\operatorname{Ei}\left(2i\,bx^2\right) + \operatorname{Ei}\left(-2i\,bx^2\right)\right)\cos(2a) + \frac{1}{8}\left(i\operatorname{Ei}\left(2i\,bx^2\right) - i\operatorname{Ei}\left(-2i\,bx^2\right)\right)\sin(2a) + \frac{1}{2}\log(x)$$

```
[In] integrate(cos(b*x^2+a)^2/x,x, algorithm="maxima")
```

```
[Out] 1/8*(Ei(2*I*b*x^2) + Ei(-2*I*b*x^2))*cos(2*a) + 1/8*(I*Ei(2*I*b*x^2) - I*Ei
(-2*I*b*x^2))*sin(2*a) + 1/2*log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.95

$$\int \frac{\cos^2(a+bx^2)}{x}\,dx = \frac{1}{4}\cos(2a)\operatorname{Ci}(2bx^2) + \frac{1}{4}\sin(2a)\operatorname{Si}(-2bx^2) + \frac{1}{4}\log(bx^2)$$

```
[In] integrate(cos(b*x^2+a)^2/x,x, algorithm="giac")
[Out] 1/4*cos(2*a)*cos_integral(2*b*x^2) + 1/4*sin(2*a)*sin_integral(-2*b*x^2) +
1/4*log(b*x^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(a+bx^2)}{x}\,dx = \int \frac{\cos(b\,x^2+a)^2}{x}\,dx$$

```
[In] int(cos(a + b*x^2)^2/x,x)
[Out] int(cos(a + b*x^2)^2/x, x)
```

3.13 $\int \frac{\cos^2(a+bx^2)}{x^2} dx$

Optimal result

Integrand size = 14, antiderivative size = 76

$$\int \frac{\cos^2(a+bx^2)}{x^2} dx = -\frac{\cos^2(a+bx^2)}{x} - \sqrt{b}\sqrt{\pi}\cos(2a)\,\text{FresnelS}\left(\frac{2\sqrt{b}x}{\sqrt{\pi}}\right) - \sqrt{b}\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{b}x}{\sqrt{\pi}}\right)\sin(2a)$$

```
[Out] -cos(b*x^2+a)^2/x-cos(2*a)*FresnelS(2*x*b^(1/2)/Pi^(1/2))*b^(1/2)*Pi^(1/2)-
FresnelC(2*x*b^(1/2)/Pi^(1/2))*sin(2*a)*b^(1/2)*Pi^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3475, 4669, 3454, 3434, 3433, 3432}

$$\int \frac{\cos^2(a+bx^2)}{x^2} dx = -\sqrt{\pi}\sqrt{b}\sin(2a)\,\text{FresnelC}\left(\frac{2\sqrt{b}x}{\sqrt{\pi}}\right) - \sqrt{\pi}\sqrt{b}\cos(2a)\,\text{FresnelS}\left(\frac{2\sqrt{b}x}{\sqrt{\pi}}\right) - \frac{\cos^2(a+bx^2)}{x}$$

```
[In] Int[Cos[a + b*x^2]^2/x^2,x]
```

```
[Out] -(Cos[a + b*x^2]^2/x) - Sqrt[b]*Sqrt[Pi]*Cos[2*a]*FresnelS[(2*Sqrt[b]*x)/Sq
rt[Pi]] - Sqrt[b]*Sqrt[Pi]*FresnelC[(2*Sqrt[b]*x)/Sqrt[Pi]]*Sin[2*a]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3434

```
Int[Sin[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Sin[c], Int
[Cos[d*(e + f*x)^2], x], x] + Dist[Cos[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3454

```
Int[((a_.) + (b_.)*Sin[u_])^(p_.), x_Symbol] :> Int[(a + b*Sin[ExpandToSum[
u, x]])^p, x] /; FreeQ[{a, b, p}, x] && BinomialQ[u, x] &&  !BinomialMatchQ
[u, x]
```

Rule 3475

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_)]^(p_)*(x_)^(m_.), x_Symbol] :> Simp[x^(m +
1)*(Cos[a + b*x^n]^p/(m + 1)), x] + Dist[b*n*(p/(m + 1)), Int[Cos[a + b*x^n
]^(p - 1)*Sin[a + b*x^n], x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 1] && EqQ[m
 + n, 0] && NeQ[n, 1] && IntegerQ[n]
```

Rule 4669

```
Int[Cos[w_]^(p_.)*(u_.)*Sin[v_]^(p_.), x_Symbol] :> Dist[1/2^p, Int[u*Sin[2
*v]^p, x], x] /; EqQ[w, v] && IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\cos^2\left(a+bx^2\right)}{x} - (4b)\int \cos\left(a+bx^2\right)\sin\left(a+bx^2\right)\,dx \\
&= -\frac{\cos^2\left(a+bx^2\right)}{x} - (2b)\int \sin\left(2\left(a+bx^2\right)\right)\,dx \\
&= -\frac{\cos^2\left(a+bx^2\right)}{x} - (2b)\int \sin\left(2a+2bx^2\right)\,dx \\
&= -\frac{\cos^2\left(a+bx^2\right)}{x} - (2b\cos(2a))\int \sin\left(2bx^2\right)\,dx - (2b\sin(2a))\int \cos\left(2bx^2\right)\,dx \\
&= -\frac{\cos^2\left(a+bx^2\right)}{x} - \sqrt{b}\sqrt{\pi}\cos(2a)\,\text{FresnelS}\left(\frac{2\sqrt{b}x}{\sqrt{\pi}}\right) - \sqrt{b}\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{b}x}{\sqrt{\pi}}\right)\sin(2a)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00

$$\int \frac{\cos^2(a+bx^2)}{x^2}\,dx = -\frac{\cos^2(a+bx^2)+\sqrt{b}\sqrt{\pi}x\cos(2a)\,\text{FresnelS}\left(\frac{2\sqrt{b}x}{\sqrt{\pi}}\right)+\sqrt{b}\sqrt{\pi}x\,\text{FresnelC}\left(\frac{2\sqrt{b}x}{\sqrt{\pi}}\right)\sin(2a)}{x}$$

```
[In] Integrate[Cos[a + b*x^2]^2/x^2,x]
```

```
[Out] -((Cos[a + b*x^2]^2 + Sqrt[b]*Sqrt[Pi]*x*Cos[2*a]*FresnelS[(2*Sqrt[b]*x)/Sq
rt[Pi]] + Sqrt[b]*Sqrt[Pi]*x*FresnelC[(2*Sqrt[b]*x)/Sqrt[Pi]]*Sin[2*a])/x)
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.82

method	result	size
default	$-\frac{1}{2x}-\frac{\cos(2b\,x^2+2a)}{2x}-\sqrt{b}\,\sqrt{\pi}\left(\cos(2a)\,\mathrm{S}\left(\frac{2x\sqrt{b}}{\sqrt{\pi}}\right)+\sin(2a)\,\mathrm{C}\left(\frac{2x\sqrt{b}}{\sqrt{\pi}}\right)\right)$	62
risch	$-\frac{1}{2x}-\frac{i\mathrm{e}^{-2ia}b\sqrt{\pi}\,\sqrt{2}\,\mathrm{erf}\left(\sqrt{2}\,\sqrt{ib}\,x\right)}{4\sqrt{ib}}+\frac{i\mathrm{e}^{2ia}b\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{-2ib}\,x\right)}{2\sqrt{-2ib}}-\frac{\cos(2b\,x^2+2a)}{2x}$	83

```
[In] int(cos(b*x^2+a)^2/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/x-1/2/x*cos(2*b*x^2+2*a)-b^(1/2)*Pi^(1/2)*(cos(2*a)*FresnelS(2*x*b^(1/
2)/Pi^(1/2))+sin(2*a)*FresnelC(2*x*b^(1/2)/Pi^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.87

$$\int \frac{\cos^2(a+bx^2)}{x^2}\,dx = -\frac{\pi x\sqrt{\frac{b}{\pi}}\cos(2\,a)\,\mathrm{S}\left(2\,x\sqrt{\frac{b}{\pi}}\right)+\pi x\sqrt{\frac{b}{\pi}}\,\mathrm{C}\left(2\,x\sqrt{\frac{b}{\pi}}\right)\sin(2\,a)+\cos\left(bx^2+a\right)^2}{x}$$

```
[In] integrate(cos(b*x^2+a)^2/x^2,x, algorithm="fricas")
```

```
[Out] -(pi*x*sqrt(b/pi)*cos(2*a)*fresnel_sin(2*x*sqrt(b/pi)) + pi*x*sqrt(b/pi)*fr
esnel_cos(2*x*sqrt(b/pi))*sin(2*a) + cos(b*x^2 + a)^2)/x
```

Sympy [F]

$$\int \frac{\cos^2(a+bx^2)}{x^2}\,dx = \int \frac{\cos^2(a+bx^2)}{x^2}\,dx$$

```
[In] integrate(cos(b*x**2+a)**2/x**2,x)
[Out] Integral(cos(a + b*x**2)**2/x**2, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.63 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.09

$$\int \frac{\cos^2(a+bx^2)}{x^2}\,dx = \frac{\sqrt{2}\sqrt{bx^2}\left(\left(-(i+1)\sqrt{2}\Gamma\left(-\frac{1}{2},2i\,bx^2\right)+(i-1)\sqrt{2}\Gamma\left(-\frac{1}{2},-2i\,bx^2\right)\right)\cos(2\,a)+\left((i-1)\sqrt{2}\Gamma\left(-\frac{1}{2},2i\,bx^2\right)\right.}{16\,x}$$

```
[In] integrate(cos(b*x^2+a)^2/x^2,x, algorithm="maxima")
[Out] 1/16*(sqrt(2)*sqrt(b*x^2)*((-(I + 1)*sqrt(2)*gamma(-1/2, 2*I*b*x^2) + (I -
1)*sqrt(2)*gamma(-1/2, -2*I*b*x^2))*cos(2*a) + ((I - 1)*sqrt(2)*gamma(-1/2,
 2*I*b*x^2) - (I + 1)*sqrt(2)*gamma(-1/2, -2*I*b*x^2))*sin(2*a)) - 8)/x
```

Giac [F]

$$\int \frac{\cos^2(a+bx^2)}{x^2}\,dx = \int \frac{\cos(bx^2+a)^2}{x^2}\,dx$$

```
[In] integrate(cos(b*x^2+a)^2/x^2,x, algorithm="giac")
[Out] integrate(cos(b*x^2 + a)^2/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(a+bx^2)}{x^2}\,dx = \int \frac{\cos\left(b\,x^2+a\right)^2}{x^2}\,dx$$

```
[In] int(cos(a + b*x^2)^2/x^2,x)
[Out] int(cos(a + b*x^2)^2/x^2, x)
```

3.14 $\int \frac{\cos^2(a+bx^2)}{x^3}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 57

$$\int \frac{\cos^2(a+bx^2)}{x^3}\,dx = -\frac{1}{4x^2}-\frac{\cos(2(a+bx^2))}{4x^2} -\frac{1}{2}b\,\text{CosIntegral}(2bx^2)\sin(2a)-\frac{1}{2}b\cos(2a)\text{Si}(2bx^2)$$

```
[Out] -1/4/x^2-1/4*cos(2*b*x^2+2*a)/x^2-1/2*b*cos(2*a)*Si(2*b*x^2)-1/2*b*Ci(2*b*x
^2)*sin(2*a)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3485, 3461, 3378, 3384, 3380, 3383}

$$\int \frac{\cos^2(a+bx^2)}{x^3}\,dx = -\frac{1}{2}b\sin(2a)\,\text{CosIntegral}(2bx^2) -\frac{1}{2}b\cos(2a)\text{Si}(2bx^2)-\frac{\cos(2(a+bx^2))}{4x^2}-\frac{1}{4x^2}$$

```
[In] Int[Cos[a + b*x^2]^2/x^3,x]
```

```
[Out] -1/4*1/x^2 - Cos[2*(a + b*x^2)]/(4*x^2) - (b*CosIntegral[2*b*x^2]*Sin[2*a])
/2 - (b*Cos[2*a]*SinIntegral[2*b*x^2])/2
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
```

```
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3485

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_)*((e_.)*(x_))^(m_.), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Cos[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{1}{2x^3}+\frac{\cos\left(2a+2bx^2\right)}{2x^3}\right) dx\\
&= -\frac{1}{4x^2}+\frac{1}{2}\int \frac{\cos\left(2a+2bx^2\right)}{x^3}\, dx\\
&= -\frac{1}{4x^2}+\frac{1}{4}\text{Subst}\left(\int \frac{\cos(2a+2bx)}{x^2}\, dx, x, x^2\right)
\end{aligned}$$

$$= -\frac{1}{4x^2} - \frac{\cos(2(a+bx^2))}{4x^2} - \frac{1}{2}b\text{Subst}\left(\int \frac{\sin(2a+2bx)}{x}\,dx, x, x^2\right)$$

$$= -\frac{1}{4x^2} - \frac{\cos(2(a+bx^2))}{4x^2} - \frac{1}{2}(b\cos(2a))\text{Subst}\left(\int \frac{\sin(2bx)}{x}\,dx, x, x^2\right) - \frac{1}{2}(b\sin(2a))\text{Subst}\left(\int \frac{\cos(2bx)}{x}\,dx, x, x^2\right)$$

$$= -\frac{1}{4x^2} - \frac{\cos(2(a+bx^2))}{4x^2} - \frac{1}{2}b\,\text{CosIntegral}\left(2bx^2\right)\sin(2a) - \frac{1}{2}b\cos(2a)\text{Si}\left(2bx^2\right)$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.88

$$\int \frac{\cos^2(a+bx^2)}{x^3}\,dx = -\frac{\cos^2(a+bx^2) + bx^2\,\text{CosIntegral}(2bx^2)\sin(2a) + bx^2\cos(2a)\text{Si}(2bx^2)}{2x^2}$$

```
[In] Integrate[Cos[a + b*x^2]^2/x^3,x]
[Out] -1/2*(Cos[a + b*x^2]^2 + b*x^2*CosIntegral[2*b*x^2]*Sin[2*a] + b*x^2*Cos[2*
a]*SinIntegral[2*b*x^2])/x^2
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.64 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.72

method	result	size
risch	$\frac{i\mathrm{e}^{-2ia}\,\mathrm{Ei}_1(-2ib\,x^2)b\,x^2+\mathrm{e}^{-2ia}\pi\,\mathrm{csgn}(b\,x^2)b\,x^2-i\mathrm{e}^{2ia}b\,\mathrm{Ei}_1(-2ib\,x^2)x^2-2\,\mathrm{e}^{-2ia}\,\mathrm{Si}(2b\,x^2)b\,x^2-\cos(2b\,x^2+2a)-1}{4x^2}$	98

```
[In] int(cos(b*x^2+a)^2/x^3,x,method=_RETURNVERBOSE)
[Out] 1/4*(I*exp(-2*I*a)*Ei(1,-2*I*b*x^2)*b*x^2+exp(-2*I*a)*Pi*csgn(b*x^2)*b*x^2-
I*exp(2*I*a)*b*Ei(1,-2*I*b*x^2)*x^2-2*exp(-2*I*a)*Si(2*b*x^2)*b*x^2-cos(2*b
*x^2+2*a)-1)/x^2
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.84

$$\int \frac{\cos^2(a+bx^2)}{x^3}\,dx = -\frac{bx^2\operatorname{Ci}(2\,bx^2)\sin(2\,a)+bx^2\cos(2\,a)\operatorname{Si}(2\,bx^2)+\cos(bx^2+a)^2}{2\,x^2}$$

```
[In] integrate(cos(b*x^2+a)^2/x^3,x, algorithm="fricas")
[Out] -1/2*(b*x^2*cos_integral(2*b*x^2)*sin(2*a) + b*x^2*cos(2*a)*sin_integral(2*
b*x^2) + cos(b*x^2 + a)^2)/x^2
```

Sympy [F]

$$\int \frac{\cos^2(a+bx^2)}{x^3}\,dx = \int \frac{\cos^2(a+bx^2)}{x^3}\,dx$$

```
[In] integrate(cos(b*x**2+a)**2/x**3,x)
[Out] Integral(cos(a + b*x**2)**2/x**3, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.36 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.07

$$\int \frac{\cos^2(a+bx^2)}{x^3}\,dx = -\frac{((i\,\Gamma(-1,2i\,bx^2)-i\,\Gamma(-1,-2i\,bx^2))\cos(2\,a)+(\Gamma(-1,2i\,bx^2)+\Gamma(-1,-2i\,bx^2))\sin(2\,a))bx^2+1}{4\,x^2}$$

```
[In] integrate(cos(b*x^2+a)^2/x^3,x, algorithm="maxima")
[Out] -1/4*(((I*gamma(-1, 2*I*b*x^2) - I*gamma(-1, -2*I*b*x^2))*cos(2*a) + (gamma
(-1, 2*I*b*x^2) + gamma(-1, -2*I*b*x^2))*sin(2*a))*b*x^2 + 1)/x^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 107 vs. $2(50) = 100$.

Time = 0.31 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.88

$$\int \frac{\cos^2(a+bx^2)}{x^3}\,dx = -\frac{2\,(bx^2+a)b^2\,\mathrm{Ci}\,(2\,bx^2)\sin(2\,a) - 2\,ab^2\,\mathrm{Ci}\,(2\,bx^2)\sin(2\,a) - 2\,(bx^2+a)b^2\cos(2\,a)\,\mathrm{Si}\,(-2\,bx^2) + 2\,ab^2\,\mathrm{co}}{4\,b^2x^2}$$

```
[In] integrate(cos(b*x^2+a)^2/x^3,x, algorithm="giac")
```

```
[Out] -1/4*(2*(b*x^2 + a)*b^2*cos_integral(2*b*x^2)*sin(2*a) - 2*a*b^2*cos_integr
al(2*b*x^2)*sin(2*a) - 2*(b*x^2 + a)*b^2*cos(2*a)*sin_integral(-2*b*x^2) +
2*a*b^2*cos(2*a)*sin_integral(-2*b*x^2) + b^2*cos(2*b*x^2 + 2*a) + b^2)/(b^
2*x^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(a+bx^2)}{x^3}\,dx = \int \frac{\cos\left(b\,x^2+a\right)^2}{x^3}\,dx$$

```
[In] int(cos(a + b*x^2)^2/x^3,x)
```

```
[Out] int(cos(a + b*x^2)^2/x^3, x)
```

3.15 $\int x^3 \cos^3(a+bx^2)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 79

$$\int x^3 \cos^3\left(a+bx^2\right)\,dx = \frac{\cos\left(a+bx^2\right)}{3b^2}+\frac{\cos^3\left(a+bx^2\right)}{18b^2}+\frac{x^2\sin\left(a+bx^2\right)}{3b}+\frac{x^2\cos^2\left(a+bx^2\right)\sin\left(a+bx^2\right)}{6b}$$

```
[Out] 1/3*cos(b*x^2+a)/b^2+1/18*cos(b*x^2+a)^3/b^2+1/3*x^2*sin(b*x^2+a)/b+1/6*x^2
*cos(b*x^2+a)^2*sin(b*x^2+a)/b
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3461, 3391, 3377, 2718}

$$\int x^3 \cos^3\left(a+bx^2\right)\,dx = \frac{\cos^3\left(a+bx^2\right)}{18b^2}+\frac{\cos\left(a+bx^2\right)}{3b^2}+\frac{x^2\sin\left(a+bx^2\right)}{3b}+\frac{x^2\sin\left(a+bx^2\right)\cos^2\left(a+bx^2\right)}{6b}$$

```
[In] Int[x^3*Cos[a + b*x^2]^3,x]
```

```
[Out] Cos[a + b*x^2]/(3*b^2) + Cos[a + b*x^2]^3/(18*b^2) + (x^2*Sin[a + b*x^2])/(
3*b) + (x^2*Cos[a + b*x^2]^2*Sin[a + b*x^2])/(6*b)
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int x\cos^3(a+bx)\,dx, x, x^2\right)\\
&= \frac{\cos^3\left(a+bx^2\right)}{18b^2}+\frac{x^2\cos^2\left(a+bx^2\right)\sin\left(a+bx^2\right)}{6b}+\frac{1}{3}\text{Subst}\left(\int x\cos(a+bx)\,dx, x, x^2\right)\\
&= \frac{\cos^3\left(a+bx^2\right)}{18b^2}+\frac{x^2\sin\left(a+bx^2\right)}{3b}+\frac{x^2\cos^2\left(a+bx^2\right)\sin\left(a+bx^2\right)}{6b}\\
&\quad -\frac{\text{Subst}\left(\int\sin(a+bx)\,dx, x, x^2\right)}{3b}\\
&= \frac{\cos\left(a+bx^2\right)}{3b^2}+\frac{\cos^3\left(a+bx^2\right)}{18b^2}+\frac{x^2\sin\left(a+bx^2\right)}{3b}+\frac{x^2\cos^2\left(a+bx^2\right)\sin\left(a+bx^2\right)}{6b}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.70

$$\int x^3\cos^3\left(a+bx^2\right)\,dx = \frac{27\cos\left(a+bx^2\right)+\cos\left(3\left(a+bx^2\right)\right)+3bx^2\left(9\sin\left(a+bx^2\right)+\sin\left(3\left(a+bx^2\right)\right)\right)}{72b^2}$$

```
[In] Integrate[x^3*Cos[a + b*x^2]^3,x]
```

```
[Out] (27*Cos[a + b*x^2] + Cos[3*(a + b*x^2)] + 3*b*x^2*(9*Sin[a + b*x^2] + Sin[3
*(a + b*x^2)]))/(72*b^2)
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.84

method	result	size
default	$\frac{3x^2\sin(b\,x^2+a)}{8b}+\frac{3\cos(b\,x^2+a)}{8b^2}+\frac{x^2\sin(3b\,x^2+3a)}{24b}+\frac{\cos(3b\,x^2+3a)}{72b^2}$	66
risch	$\frac{3x^2\sin(b\,x^2+a)}{8b}+\frac{3\cos(b\,x^2+a)}{8b^2}+\frac{x^2\sin(3b\,x^2+3a)}{24b}+\frac{\cos(3b\,x^2+3a)}{72b^2}$	66
parallelrisch	$\frac{7+9\left(\tan^5\left(\frac{a}{2}+\frac{b\,x^2}{2}\right)\right)x^2b+6\left(\tan^3\left(\frac{a}{2}+\frac{b\,x^2}{2}\right)\right)x^2b+9\tan\left(\frac{a}{2}+\frac{b\,x^2}{2}\right)x^2b+9\left(\tan^4\left(\frac{a}{2}+\frac{b\,x^2}{2}\right)\right)+12\left(\tan^2\left(\frac{a}{2}+\frac{b\,x^2}{2}\right)\right)}{9b^2\left(1+\tan^2\left(\frac{a}{2}+\frac{b\,x^2}{2}\right)\right)^3}$	110
norman	$\frac{\frac{x^2\tan\left(\frac{a}{2}+\frac{b\,x^2}{2}\right)}{b}+\frac{x^2\left(\tan^5\left(\frac{a}{2}+\frac{b\,x^2}{2}\right)\right)}{b}+\frac{\tan^4\left(\frac{a}{2}+\frac{b\,x^2}{2}\right)}{b^2}+\frac{7}{9b^2}+\frac{2x^2\left(\tan^3\left(\frac{a}{2}+\frac{b\,x^2}{2}\right)\right)}{3b}+\frac{4\left(\tan^2\left(\frac{a}{2}+\frac{b\,x^2}{2}\right)\right)}{3b^2}}{\left(1+\tan^2\left(\frac{a}{2}+\frac{b\,x^2}{2}\right)\right)^3}$	119

```
[In] int(x^3*cos(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 3/8*x^2*sin(b*x^2+a)/b+3/8*cos(b*x^2+a)/b^2+1/24/b*x^2*sin(3*b*x^2+3*a)+1/7
2/b^2*cos(3*b*x^2+3*a)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.73

$$\int x^3\cos^3\left(a+bx^2\right)\,dx = \frac{\cos\left(bx^2+a\right)^3+3\left(bx^2\cos\left(bx^2+a\right)^2+2\,bx^2\right)\sin\left(bx^2+a\right)+6\,\cos\left(bx^2+a\right)}{18\,b^2}$$

```
[In] integrate(x^3*cos(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/18*(cos(b*x^2 + a)^3 + 3*(b*x^2*cos(b*x^2 + a)^2 + 2*b*x^2)*sin(b*x^2 + a
) + 6*cos(b*x^2 + a))/b^2
```

Sympy [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.16

$$\int x^3\cos^3\left(a+bx^2\right)\,dx = \begin{cases} \frac{x^2\sin^3\left(a+bx^2\right)}{3b}+\frac{x^2\sin\left(a+bx^2\right)\cos^2\left(a+bx^2\right)}{2b}+\frac{\sin^2\left(a+bx^2\right)\cos\left(a+bx^2\right)}{3b^2}+\frac{7\cos^3\left(a+bx^2\right)}{18b^2} & \text{for } b\neq 0 \\ \frac{x^4\cos^3(a)}{4} & \text{otherwise} \end{cases}$$

```
[In] integrate(x**3*cos(b*x**2+a)**3,x)
```

```
[Out] Piecewise((x**2*sin(a + b*x**2)**3/(3*b) + x**2*sin(a + b*x**2)*cos(a + b*x
**2)**2/(2*b) + sin(a + b*x**2)**2*cos(a + b*x**2)/(3*b**2) + 7*cos(a + b*x
**2)**3/(18*b**2), Ne(b, 0)), (x**4*cos(a)**3/4, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.73

$$\int x^3\cos^3\left(a+bx^2\right)\,dx = \frac{3\,bx^2\sin\left(3\,bx^2+3\,a\right)+27\,bx^2\sin\left(bx^2+a\right)+\cos\left(3\,bx^2+3\,a\right)+27\,\cos\left(bx^2+a\right)}{72\,b^2}$$

```
[In] integrate(x^3*cos(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/72*(3*b*x^2*sin(3*b*x^2 + 3*a) + 27*b*x^2*sin(b*x^2 + a) + cos(3*b*x^2 +
3*a) + 27*cos(b*x^2 + a))/b^2
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.16

$$\int x^3\cos^3\left(a+bx^2\right)\,dx = \frac{\left(\sin\left(bx^2+a\right)^3-3\,\sin\left(bx^2+a\right)\right)a}{6\,b^2} + \frac{3\left(bx^2+a\right)\sin\left(3\,bx^2+3\,a\right)+27\left(bx^2+a\right)\sin\left(bx^2+a\right)+\cos\left(3\,bx^2+3\,a\right)+27\,\cos\left(bx^2+a\right)}{72\,b^2}$$

```
[In] integrate(x^3*cos(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/6*(sin(b*x^2 + a)^3 - 3*sin(b*x^2 + a))*a/b^2 + 1/72*(3*(b*x^2 + a)*sin(3
*b*x^2 + 3*a) + 27*(b*x^2 + a)*sin(b*x^2 + a) + cos(3*b*x^2 + 3*a) + 27*cos
(b*x^2 + a))/b^2
```

Mupad [B] (verification not implemented)

Time = 13.93 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.84

$$\int x^3 \cos^3\left(a+bx^2\right)\,dx=\frac{\frac{\cos\left(b\,x^2+a\right)}{3}+\frac{{\cos\left(b\,x^2+a\right)}^3}{18}+b\,\left(\frac{x^2\,\sin\left(b\,x^2+a\right)}{3}+\frac{x^2\,{\cos\left(b\,x^2+a\right)}^2\,\sin\left(b\,x^2+a\right)}{6}\right)}{b^2}$$

```
[In] int(x^3*cos(a + b*x^2)^3,x)
```

```
[Out] (cos(a + b*x^2)/3 + cos(a + b*x^2)^3/18 + b*((x^2*sin(a + b*x^2))/3 + (x^2*
cos(a + b*x^2)^2*sin(a + b*x^2))/6))/b^2
```

3.16 $\int x^2 \cos^3(a+bx^2)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 188

$$\int x^2\cos^3\left(a+bx^2\right)dx = -\frac{3\sqrt{\frac{\pi}{2}}\cos(a)\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)}{8b^{3/2}} - \frac{\sqrt{\frac{\pi}{6}}\cos(3a)\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{6}{\pi}}x\right)}{24b^{3/2}} - \frac{3\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)\sin(a)}{8b^{3/2}} - \frac{\sqrt{\frac{\pi}{6}}\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{6}{\pi}}x\right)\sin(3a)}{24b^{3/2}} + \frac{3x\sin\left(a+bx^2\right)}{8b} + \frac{x\sin\left(3a+3bx^2\right)}{24b}$$

```
[Out] 3/8*x*sin(b*x^2+a)/b+1/24*x*sin(3*b*x^2+3*a)/b-1/144*cos(3*a)*FresnelS(x*b^
(1/2)*6^(1/2)/Pi^(1/2))*6^(1/2)*Pi^(1/2)/b^(3/2)-1/144*FresnelC(x*b^(1/2)*6
^(1/2)/Pi^(1/2))*sin(3*a)*6^(1/2)*Pi^(1/2)/b^(3/2)-3/16*cos(a)*FresnelS(x*b
^(1/2)*2^(1/2)/Pi^(1/2))*2^(1/2)*Pi^(1/2)/b^(3/2)-3/16*FresnelC(x*b^(1/2)*2
^(1/2)/Pi^(1/2))*sin(a)*2^(1/2)*Pi^(1/2)/b^(3/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {3485, 3467, 3434, 3433, 3432}

$$\begin{aligned}\int x^2\cos^3\left(a+bx^2\right)\,dx = &-\frac{3\sqrt{\frac{\pi}{2}}\sin(a)\operatorname{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)}{8b^{3/2}}\\&-\frac{\sqrt{\frac{\pi}{6}}\sin(3a)\operatorname{FresnelC}\left(\sqrt{b}\sqrt{\frac{6}{\pi}}x\right)}{24b^{3/2}}\\&-\frac{3\sqrt{\frac{\pi}{2}}\cos(a)\operatorname{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)}{8b^{3/2}}\\&-\frac{\sqrt{\frac{\pi}{6}}\cos(3a)\operatorname{FresnelS}\left(\sqrt{b}\sqrt{\frac{6}{\pi}}x\right)}{24b^{3/2}}\\&+\frac{3x\sin\left(a+bx^2\right)}{8b}+\frac{x\sin\left(3a+3bx^2\right)}{24b}\end{aligned}$$

```
[In] Int[x^2*Cos[a + b*x^2]^3,x]
```

```
[Out] (-3*Sqrt[Pi/2]*Cos[a]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*x])/(8*b^(3/2)) - (Sqrt[P
i/6]*Cos[3*a]*FresnelS[Sqrt[b]*Sqrt[6/Pi]*x])/(24*b^(3/2)) - (3*Sqrt[Pi/2]*
FresnelC[Sqrt[b]*Sqrt[2/Pi]*x]*Sin[a])/(8*b^(3/2)) - (Sqrt[Pi/6]*FresnelC[S
qrt[b]*Sqrt[6/Pi]*x]*Sin[3*a])/(24*b^(3/2)) + (3*x*Sin[a + b*x^2])/(8*b) +
(x*Sin[3*a + 3*b*x^2])/(24*b)
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3434

```
Int[Sin[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Sin[c], Int
[Cos[d*(e + f*x)^2], x], x] + Dist[Cos[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3467

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[e^(n
 - 1)*(e*x)^(m - n + 1)*(Sin[c + d*x^n]/(d*n)), x] - Dist[e^n*((m - n + 1)/
```

```
(d*n)), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3485

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_)*((e_.)*(x_))^(m_.), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Cos[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{3}{4}x^2\cos\left(a+bx^2\right)+\frac{1}{4}x^2\cos\left(3a+3bx^2\right)\right)\,dx\\
&= \frac{1}{4}\int x^2\cos\left(3a+3bx^2\right)\,dx+\frac{3}{4}\int x^2\cos\left(a+bx^2\right)\,dx\\
&= \frac{3x\sin\left(a+bx^2\right)}{8b}+\frac{x\sin\left(3a+3bx^2\right)}{24b}-\frac{\int\sin\left(3a+3bx^2\right)\,dx}{24b}-\frac{3\int\sin\left(a+bx^2\right)\,dx}{8b}\\
&= \frac{3x\sin\left(a+bx^2\right)}{8b}+\frac{x\sin\left(3a+3bx^2\right)}{24b}-\frac{(3\cos(a))\int\sin\left(bx^2\right)\,dx}{8b}\\
&\quad-\frac{\cos(3a)\int\sin\left(3bx^2\right)\,dx}{24b}-\frac{(3\sin(a))\int\cos\left(bx^2\right)\,dx}{8b}-\frac{\sin(3a)\int\cos\left(3bx^2\right)\,dx}{24b}\\
&= -\frac{3\sqrt{\frac{\pi}{2}}\cos(a)\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)}{8b^{3/2}}-\frac{\sqrt{\frac{\pi}{6}}\cos(3a)\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{6}{\pi}}x\right)}{24b^{3/2}}\\
&\quad-\frac{3\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)\sin(a)}{8b^{3/2}}-\frac{\sqrt{\frac{\pi}{6}}\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{6}{\pi}}x\right)\sin(3a)}{24b^{3/2}}\\
&\quad+\frac{3x\sin\left(a+bx^2\right)}{8b}+\frac{x\sin\left(3a+3bx^2\right)}{24b}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.85

$$
\int x^2\cos^3\left(a+bx^2\right)\,dx
$$

$$
=\frac{-27\sqrt{2\pi}\cos(a)\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)-\sqrt{6\pi}\cos(3a)\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{6}{\pi}}x\right)-27\sqrt{2\pi}\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)\text{si}}{144b^{3/2}}
$$

```
[In] Integrate[x^2*Cos[a + b*x^2]^3,x]
```

```
[Out] (-27*Sqrt[2*Pi]*Cos[a]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*x] - Sqrt[6*Pi]*Cos[3*a]
*FresnelS[Sqrt[b]*Sqrt[6/Pi]*x] - 27*Sqrt[2*Pi]*FresnelC[Sqrt[b]*Sqrt[2/Pi]
*x]*Sin[a] - Sqrt[6*Pi]*FresnelC[Sqrt[b]*Sqrt[6/Pi]*x]*Sin[3*a] + 54*Sqrt[b
]*x*Sin[a + b*x^2] + 6*Sqrt[b]*x*Sin[3*(a + b*x^2)])/(144*b^(3/2))
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.69

method	result
default	$\frac{3x\sin(b\,x^2+a)}{8b}-\frac{3\sqrt{2}\,\sqrt{\pi}\,\left(\cos(a)\,\mathrm{S}\left(\frac{x\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}}\right)+\sin(a)\,\mathrm{C}\left(\frac{x\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}}\right)\right)}{16b^{\frac{3}{2}}}+\frac{x\sin(3b\,x^2+3a)}{24b}-\frac{\sqrt{2}\,\sqrt{\pi}\,\sqrt{3}\,\left(\cos(3a)\,\mathrm{S}\left(\frac{\sqrt{2}\,\sqrt{3}\,\sqrt{b}\,x}{\sqrt{\pi}}\right)+\mathrm{sin}\right.}{144b^{\frac{3}{2}}}$
risch	$-\frac{i\mathrm{e}^{-3ia}\sqrt{\pi}\,\sqrt{3}\,\mathrm{erf}\left(\sqrt{3}\,\sqrt{ib}\,x\right)}{288b\sqrt{ib}}-\frac{3i\mathrm{e}^{-ia}\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{ib}\,x\right)}{32b\sqrt{ib}}+\frac{3i\mathrm{e}^{ia}\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{-ib}\,x\right)}{32b\sqrt{-ib}}+\frac{i\mathrm{e}^{3ia}\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{-3ib}\,x\right)}{96b\sqrt{-3ib}}+\frac{3x\sin(b\,x^2+a)}{8b}+$

```
[In] int(x^2*cos(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 3/8*x*sin(b*x^2+a)/b-3/16/b^(3/2)*2^(1/2)*Pi^(1/2)*(cos(a)*FresnelS(x*b^(1/
2)*2^(1/2)/Pi^(1/2))+sin(a)*FresnelC(x*b^(1/2)*2^(1/2)/Pi^(1/2)))+1/24*x*si
n(3*b*x^2+3*a)/b-1/144/b^(3/2)*2^(1/2)*Pi^(1/2)*3^(1/2)*(cos(3*a)*FresnelS(
2^(1/2)/Pi^(1/2)*3^(1/2)*b^(1/2)*x)+sin(3*a)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1
/2)*b^(1/2)*x))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.79

$$\int x^2\cos^3\left(a+bx^2\right)\,dx = -\frac{\sqrt{6}\pi\sqrt{\frac{b}{\pi}}\cos\left(3\,a\right)\mathrm{S}\left(\sqrt{6}x\sqrt{\frac{b}{\pi}}\right)+27\,\sqrt{2}\pi\sqrt{\frac{b}{\pi}}\cos\left(a\right)\mathrm{S}\left(\sqrt{2}x\sqrt{\frac{b}{\pi}}\right)+\sqrt{6}\pi\sqrt{\frac{b}{\pi}}\,\mathrm{C}\left(\sqrt{6}x\sqrt{\frac{b}{\pi}}\right)\sin\left(3\,a\right)+27}{144\,b^2}$$

```
[In] integrate(x^2*cos(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] -1/144*(sqrt(6)*pi*sqrt(b/pi)*cos(3*a)*fresnel_sin(sqrt(6)*x*sqrt(b/pi)) +
27*sqrt(2)*pi*sqrt(b/pi)*cos(a)*fresnel_sin(sqrt(2)*x*sqrt(b/pi)) + sqrt(6)
*pi*sqrt(b/pi)*fresnel_cos(sqrt(6)*x*sqrt(b/pi))*sin(3*a) + 27*sqrt(2)*pi*s
qrt(b/pi)*fresnel_cos(sqrt(2)*x*sqrt(b/pi))*sin(a) - 24*(b*x*cos(b*x^2 + a)
^2 + 2*b*x)*sin(b*x^2 + a))/b^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 439 vs. 2(194) = 388.

Time = 2.18 (sec) , antiderivative size = 439, normalized size of antiderivative = 2.34

$$\int x^2\cos^3\left(a+bx^2\right)\,dx = \frac{3b^{\frac{3}{2}}x^5\sqrt{\frac{1}{b}}\sin{(a)}\Gamma\left(\frac{3}{4}\right)\Gamma\left(\frac{5}{4}\right){}_{2}F_{3}\left(\begin{matrix}\frac{3}{4},\frac{5}{4}\\ \frac{3}{2},\frac{7}{4},\frac{9}{4}\end{matrix}\middle|-\frac{b^2x^4}{4}\right)}{32\Gamma\left(\frac{7}{4}\right)\Gamma\left(\frac{9}{4}\right)} + \frac{3b^{\frac{3}{2}}x^5\sqrt{\frac{1}{b}}\sin{(3a)}\Gamma\left(\frac{3}{4}\right)\Gamma\left(\frac{5}{4}\right){}_{2}F_{3}\left(\begin{matrix}\frac{3}{4},\frac{5}{4}\\ \frac{3}{2},\frac{7}{4},\frac{9}{4}\end{matrix}\middle|-\frac{9b^2x^4}{4}\right)}{32\Gamma\left(\frac{7}{4}\right)\Gamma\left(\frac{9}{4}\right)} - \frac{3\sqrt{b}x^3\sqrt{\frac{1}{b}}\cos{(a)}\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right){}_{2}F_{3}\left(\begin{matrix}\frac{1}{4},\frac{3}{4}\\ \frac{1}{2},\frac{5}{4},\frac{7}{4}\end{matrix}\middle|-\frac{b^2x^4}{4}\right)}{32\Gamma\left(\frac{5}{4}\right)\Gamma\left(\frac{7}{4}\right)} - \frac{\sqrt{b}x^3\sqrt{\frac{1}{b}}\cos{(3a)}\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right){}_{2}F_{3}\left(\begin{matrix}\frac{1}{4},\frac{3}{4}\\ \frac{1}{2},\frac{5}{4},\frac{7}{4}\end{matrix}\middle|-\frac{9b^2x^4}{4}\right)}{32\Gamma\left(\frac{5}{4}\right)\Gamma\left(\frac{7}{4}\right)} - \frac{3\sqrt{2}\sqrt{\pi}x^2\sqrt{\frac{1}{b}}\sin{(a)}S\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{\pi}}\right)}{8} - \frac{\sqrt{6}\sqrt{\pi}x^2\sqrt{\frac{1}{b}}\sin{(3a)}S\left(\frac{\sqrt{6}\sqrt{b}x}{\sqrt{\pi}}\right)}{24} + \frac{3\sqrt{2}\sqrt{\pi}x^2\sqrt{\frac{1}{b}}\cos{(a)}C\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{\pi}}\right)}{8} + \frac{\sqrt{6}\sqrt{\pi}x^2\sqrt{\frac{1}{b}}\cos{(3a)}C\left(\frac{\sqrt{6}\sqrt{b}x}{\sqrt{\pi}}\right)}{24}$$

```
[In] integrate(x**2*cos(b*x**2+a)**3,x)
```

```
[Out] 3*b**(3/2)*x**5*sqrt(1/b)*sin(a)*gamma(3/4)*gamma(5/4)*hyper((3/4, 5/4), (3
/2, 7/4, 9/4), -b**2*x**4/4)/(32*gamma(7/4)*gamma(9/4)) + 3*b**(3/2)*x**5*s
qrt(1/b)*sin(3*a)*gamma(3/4)*gamma(5/4)*hyper((3/4, 5/4), (3/2, 7/4, 9/4),
-9*b**2*x**4/4)/(32*gamma(7/4)*gamma(9/4)) - 3*sqrt(b)*x**3*sqrt(1/b)*cos(a
)*gamma(1/4)*gamma(3/4)*hyper((1/4, 3/4), (1/2, 5/4, 7/4), -b**2*x**4/4)/(3
2*gamma(5/4)*gamma(7/4)) - sqrt(b)*x**3*sqrt(1/b)*cos(3*a)*gamma(1/4)*gamma
(3/4)*hyper((1/4, 3/4), (1/2, 5/4, 7/4), -9*b**2*x**4/4)/(32*gamma(5/4)*gam
ma(7/4)) - 3*sqrt(2)*sqrt(pi)*x**2*sqrt(1/b)*sin(a)*fresnels(sqrt(2)*sqrt(b
)*x/sqrt(pi))/8 - sqrt(6)*sqrt(pi)*x**2*sqrt(1/b)*sin(3*a)*fresnels(sqrt(6)
```

```
*sqrt(b)*x/sqrt(pi))/24 + 3*sqrt(2)*sqrt(pi)*x**2*sqrt(1/b)*cos(a)*fresnelc
(sqrt(2)*sqrt(b)*x/sqrt(pi))/8 + sqrt(6)*sqrt(pi)*x**2*sqrt(1/b)*cos(3*a)*f
resnelc(sqrt(6)*sqrt(b)*x/sqrt(pi))/24
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.37 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.76

$$\int x^2\cos^3\left(a+bx^2\right)\,dx = \frac{24\,b^2x\sin\left(3\,bx^2+3\,a\right)+216\,b^2x\sin\left(bx^2+a\right)+9^{\frac{1}{4}}\sqrt{2}\sqrt{\pi}\Big(\left(-(i+1)\,\cos\left(3\,a\right)+(i-1)\,\sin\left(3\,a\right)\right)\mathrm{erf}\left(\sqrt{3i}\right.}{}$$

```
[In] integrate(x^2*cos(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/576*(24*b^2*x*sin(3*b*x^2 + 3*a) + 216*b^2*x*sin(b*x^2 + a) + 9^(1/4)*sqr
t(2)*sqrt(pi)*((-(I + 1)*cos(3*a) + (I - 1)*sin(3*a))*erf(sqrt(3*I*b)*x) +
((I - 1)*cos(3*a) - (I + 1)*sin(3*a))*erf(sqrt(-3*I*b)*x))*b^(3/2) - 27*sqr
t(2)*sqrt(pi)*(((I + 1)*cos(a) - (I - 1)*sin(a))*erf(sqrt(I*b)*x) + (-(I -
1)*cos(a) + (I + 1)*sin(a))*erf(sqrt(-I*b)*x))*b^(3/2))/b^3
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.38

$$\begin{aligned}\int x^2\cos^3\left(a+bx^2\right)\,dx &= -\frac{i\,xe^{\left(3i\,bx^2+3i\,a\right)}}{48\,b}-\frac{3i\,xe^{\left(i\,bx^2+i\,a\right)}}{16\,b}+\frac{3i\,xe^{\left(-i\,bx^2-i\,a\right)}}{16\,b}\\ &\quad+\frac{i\,xe^{\left(-3i\,bx^2-3i\,a\right)}}{48\,b}-\frac{\sqrt{6}\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}i\,\sqrt{6}\sqrt{b}x\left(\frac{i\,b}{|b|}+1\right)\right)e^{\left(3i\,a\right)}}{288\,b^{\frac{3}{2}}\left(\frac{i\,b}{|b|}+1\right)}\\ &\quad-\frac{3\,\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}i\,\sqrt{2}x\left(\frac{i\,b}{|b|}+1\right)\sqrt{|b|}\right)e^{\left(i\,a\right)}}{32\,b\left(\frac{i\,b}{|b|}+1\right)\sqrt{|b|}}\\ &\quad-\frac{3\,\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(\frac{1}{2}i\,\sqrt{2}x\left(-\frac{i\,b}{|b|}+1\right)\sqrt{|b|}\right)e^{\left(-i\,a\right)}}{32\,b\left(-\frac{i\,b}{|b|}+1\right)\sqrt{|b|}}\\ &\quad-\frac{\sqrt{6}\sqrt{\pi}\,\mathrm{erf}\left(\frac{1}{2}i\,\sqrt{6}\sqrt{b}x\left(-\frac{i\,b}{|b|}+1\right)\right)e^{\left(-3i\,a\right)}}{288\,b^{\frac{3}{2}}\left(-\frac{i\,b}{|b|}+1\right)}\end{aligned}$$

```
[In] integrate(x^2*cos(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -1/48*I*x*e^(3*I*b*x^2 + 3*I*a)/b - 3/16*I*x*e^(I*b*x^2 + I*a)/b + 3/16*I*x
*e^(-I*b*x^2 - I*a)/b + 1/48*I*x*e^(-3*I*b*x^2 - 3*I*a)/b - 1/288*sqrt(6)*s
qrt(pi)*erf(-1/2*I*sqrt(6)*sqrt(b)*x*(I*b/abs(b) + 1))*e^(3*I*a)/(b^(3/2)*(
I*b/abs(b) + 1)) - 3/32*sqrt(2)*sqrt(pi)*erf(-1/2*I*sqrt(2)*x*(I*b/abs(b) +
 1)*sqrt(abs(b)))*e^(I*a)/(b*(I*b/abs(b) + 1)*sqrt(abs(b))) - 3/32*sqrt(2)*
sqrt(pi)*erf(1/2*I*sqrt(2)*x*(-I*b/abs(b) + 1)*sqrt(abs(b)))*e^(-I*a)/(b*(-
I*b/abs(b) + 1)*sqrt(abs(b))) - 1/288*sqrt(6)*sqrt(pi)*erf(1/2*I*sqrt(6)*sq
rt(b)*x*(-I*b/abs(b) + 1))*e^(-3*I*a)/(b^(3/2)*(-I*b/abs(b) + 1))
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \cos^3\left(a+bx^2\right)\,dx=\int x^2\cos\left(b\,x^2+a\right)^3\,dx$$

```
[In] int(x^2*cos(a + b*x^2)^3,x)
```

```
[Out] int(x^2*cos(a + b*x^2)^3, x)
```

3.17 $\int x \cos^3(a+bx^2)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 33

$$\int x \cos^3(a+bx^2)\,dx = \frac{\sin(a+bx^2)}{2b} - \frac{\sin^3(a+bx^2)}{6b}$$

```
[Out] 1/2*sin(b*x^2+a)/b-1/6*sin(b*x^2+a)^3/b
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3461, 2713}

$$\int x \cos^3(a+bx^2)\,dx = \frac{\sin(a+bx^2)}{2b} - \frac{\sin^3(a+bx^2)}{6b}$$

```
[In] Int[x*Cos[a + b*x^2]^3,x]
```

```
[Out] Sin[a + b*x^2]/(2*b) - Sin[a + b*x^2]^3/(6*b)
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
```

```
m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}\text{integral}&=\frac{1}{2}\text{Subst}\left(\int \cos^3(a+bx)\,dx,x,x^2\right)\\&=-\frac{\text{Subst}\left(\int \left(1-x^2\right)\,dx,x,-\sin\left(a+bx^2\right)\right)}{2b}\\&=\frac{\sin\left(a+bx^2\right)}{2b}-\frac{\sin^3\left(a+bx^2\right)}{6b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int x\cos^3\left(a+bx^2\right)\,dx=\frac{\sin\left(a+bx^2\right)}{2b}-\frac{\sin^3\left(a+bx^2\right)}{6b}$$

```
[In] Integrate[x*Cos[a + b*x^2]^3,x]
[Out] Sin[a + b*x^2]/(2*b) - Sin[a + b*x^2]^3/(6*b)
```

Maple [A] (verified)

Time = 1.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.79

method	result	size
derivativedivides	$\frac{\left(2+\cos^2\left(b\,x^2+a\right)\right)\sin\left(b\,x^2+a\right)}{6b}$	26
default	$\frac{\left(2+\cos^2\left(b\,x^2+a\right)\right)\sin\left(b\,x^2+a\right)}{6b}$	26
parallelrisch	$\frac{9\sin\left(b\,x^2+a\right)+\sin\left(3b\,x^2+3a\right)}{24b}$	28
risch	$\frac{3\sin\left(b\,x^2+a\right)}{8b}+\frac{\sin\left(3b\,x^2+3a\right)}{24b}$	31
norman	$\frac{\frac{\tan\left(\frac{a}{2}+\frac{b\,x^2}{2}\right)}{b}+\frac{\tan^5\left(\frac{a}{2}+\frac{b\,x^2}{2}\right)}{b}+\frac{2\left(\tan^3\left(\frac{a}{2}+\frac{b\,x^2}{2}\right)\right)}{3b}}{\left(1+\tan^2\left(\frac{a}{2}+\frac{b\,x^2}{2}\right)\right)^3}$	70

```
[In] int(x*cos(b*x^2+a)^3,x,method=_RETURNVERBOSE)
[Out] 1/6/b*(2+cos(b*x^2+a)^2)*sin(b*x^2+a)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int x\cos^3\left(a+bx^2\right)\,dx=\frac{\left(\cos\left(bx^2+a\right)^2+2\right)\sin\left(bx^2+a\right)}{6\,b}$$

```
[In] integrate(x*cos(b*x^2+a)^3,x, algorithm="fricas")
[Out] 1/6*(cos(b*x^2 + a)^2 + 2)*sin(b*x^2 + a)/b
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.33

$$\int x\cos^3\left(a+bx^2\right)\,dx=\begin{cases}\frac{\sin^3\left(a+bx^2\right)}{3b}+\frac{\sin\left(a+bx^2\right)\cos^2\left(a+bx^2\right)}{2b} & \text{for } b\neq 0\\ \frac{x^2\cos^3\left(a\right)}{2} & \text{otherwise}\end{cases}$$

```
[In] integrate(x*cos(b*x**2+a)**3,x)
[Out] Piecewise((sin(a + b*x**2)**3/(3*b) + sin(a + b*x**2)*cos(a + b*x**2)**2/(2
*b), Ne(b, 0)), (x**2*cos(a)**3/2, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int x\cos^3\left(a+bx^2\right)\,dx=\frac{\sin\left(3\,bx^2+3\,a\right)+9\,\sin\left(bx^2+a\right)}{24\,b}$$

```
[In] integrate(x*cos(b*x^2+a)^3,x, algorithm="maxima")
[Out] 1/24*(sin(3*b*x^2 + 3*a) + 9*sin(b*x^2 + a))/b
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.79

$$\int x\cos^3\left(a+bx^2\right)\,dx=-\frac{\sin\left(bx^2+a\right)^3-3\,\sin\left(bx^2+a\right)}{6\,b}$$

```
[In] integrate(x*cos(b*x^2+a)^3,x, algorithm="giac")
[Out] -1/6*(sin(b*x^2 + a)^3 - 3*sin(b*x^2 + a))/b
```

Mupad [B] (verification not implemented)

Time = 13.87 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85

$$\int x\cos^3\left(a+bx^2\right)\,dx=\frac{3\,\sin\left(b\,x^2+a\right)-\sin\left(b\,x^2+a\right)^3}{6\,b}$$

```
[In] int(x*cos(a + b*x^2)^3,x)
[Out] (3*sin(a + b*x^2) - sin(a + b*x^2)^3)/(6*b)
```

3.18 $\int \cos^3(a+bx^2)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 153

$$\int \cos^3(a+bx^2)\,dx = \frac{3\sqrt{\frac{\pi}{2}}\cos(a)\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)}{4\sqrt{b}} + \frac{\sqrt{\frac{\pi}{6}}\cos(3a)\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{6}{\pi}}x\right)}{4\sqrt{b}} - \frac{3\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)\sin(a)}{4\sqrt{b}} - \frac{\sqrt{\frac{\pi}{6}}\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{6}{\pi}}x\right)\sin(3a)}{4\sqrt{b}}$$

```
[Out] 1/24*cos(3*a)*FresnelC(x*b^(1/2)*6^(1/2)/Pi^(1/2))*6^(1/2)*Pi^(1/2)/b^(1/2)
-1/24*FresnelS(x*b^(1/2)*6^(1/2)/Pi^(1/2))*sin(3*a)*6^(1/2)*Pi^(1/2)/b^(1/2
)+3/8*cos(a)*FresnelC(x*b^(1/2)*2^(1/2)/Pi^(1/2))*2^(1/2)*Pi^(1/2)/b^(1/2)-
3/8*FresnelS(x*b^(1/2)*2^(1/2)/Pi^(1/2))*sin(a)*2^(1/2)*Pi^(1/2)/b^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3439, 3435, 3433, 3432}

$$\int \cos^3(a+bx^2)\,dx = \frac{3\sqrt{\frac{\pi}{2}}\cos(a)\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)}{4\sqrt{b}} + \frac{\sqrt{\frac{\pi}{6}}\cos(3a)\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{6}{\pi}}x\right)}{4\sqrt{b}} - \frac{3\sqrt{\frac{\pi}{2}}\sin(a)\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)}{4\sqrt{b}} - \frac{\sqrt{\frac{\pi}{6}}\sin(3a)\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{6}{\pi}}x\right)}{4\sqrt{b}}$$

```
[In] Int[Cos[a + b*x^2]^3,x]
```

```
[Out] (3*Sqrt[Pi/2]*Cos[a]*FresnelC[Sqrt[b]*Sqrt[2/Pi]*x])/(4*Sqrt[b]) + (Sqrt[Pi
/6]*Cos[3*a]*FresnelC[Sqrt[b]*Sqrt[6/Pi]*x])/(4*Sqrt[b]) - (3*Sqrt[Pi/2]*Fr
```

```
esnelS[Sqrt[b]*Sqrt[2/Pi]*x]*Sin[a])/(4*Sqrt[b]) - (Sqrt[Pi/6]*FresnelS[Sqr
t[b]*Sqrt[6/Pi]*x]*Sin[3*a])/(4*Sqrt[b])
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3435

```
Int[Cos[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Cos[c], Int
[Cos[d*(e + f*x)^2], x], x] - Dist[Sin[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3439

```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_), x_Sy
mbol] :> Int[ExpandTrigReduce[(a + b*Cos[c + d*(e + f*x)^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 1] && IGtQ[n, 1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{3}{4}\cos\left(a+bx^2\right)+\frac{1}{4}\cos\left(3a+3bx^2\right)\right)\,dx\\
&= \frac{1}{4}\int \cos\left(3a+3bx^2\right)\,dx+\frac{3}{4}\int \cos\left(a+bx^2\right)\,dx\\
&= \frac{1}{4}(3\cos(a))\int \cos\left(bx^2\right)\,dx+\frac{1}{4}\cos(3a)\int \cos\left(3bx^2\right)\,dx\\
&\quad -\frac{1}{4}(3\sin(a))\int \sin\left(bx^2\right)\,dx-\frac{1}{4}\sin(3a)\int \sin\left(3bx^2\right)\,dx\\
&= \frac{3\sqrt{\frac{\pi}{2}}\cos(a)\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)}{4\sqrt{b}}+\frac{\sqrt{\frac{\pi}{6}}\cos(3a)\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{6}{\pi}}x\right)}{4\sqrt{b}}\\
&\quad -\frac{3\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)\sin(a)}{4\sqrt{b}}-\frac{\sqrt{\frac{\pi}{6}}\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{6}{\pi}}x\right)\sin(3a)}{4\sqrt{b}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.76

$$\int \cos^3\left(a+bx^2\right)\,dx$$
$$=\frac{\sqrt{\frac{\pi}{6}}\left(3\sqrt{3}\cos(a)\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)+\cos(3a)\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{6}{\pi}}x\right)-3\sqrt{3}\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)\sin(a)-\text{F}\right.}{4\sqrt{b}}$$

```
[In] Integrate[Cos[a + b*x^2]^3,x]
```

```
[Out] (Sqrt[Pi/6]*(3*Sqrt[3]*Cos[a]*FresnelC[Sqrt[b]*Sqrt[2/Pi]*x] + Cos[3*a]*Fre
snelC[Sqrt[b]*Sqrt[6/Pi]*x] - 3*Sqrt[3]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*x]*Sin[
a] - FresnelS[Sqrt[b]*Sqrt[6/Pi]*x]*Sin[3*a]))/(4*Sqrt[b])
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.66

method	result	size
default	$\frac{3\sqrt{2}\,\sqrt{\pi}\,\left(\cos(a)\,\mathrm{C}\left(\frac{x\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}}\right)-\sin(a)\,\mathrm{S}\left(\frac{x\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}}\right)\right)}{8\sqrt{b}}+\frac{\sqrt{2}\,\sqrt{\pi}\,\sqrt{3}\,\left(\cos(3a)\,\mathrm{C}\left(\frac{\sqrt{2}\,\sqrt{3}\,\sqrt{b}\,x}{\sqrt{\pi}}\right)-\sin(3a)\,\mathrm{S}\left(\frac{\sqrt{2}\,\sqrt{3}\,\sqrt{b}\,x}{\sqrt{\pi}}\right)\right)}{24\sqrt{b}}$	101
risch	$\frac{\mathrm{e}^{-3ia}\sqrt{\pi}\,\sqrt{3}\,\mathrm{erf}\left(\sqrt{3}\,\sqrt{ib}\,x\right)}{48\sqrt{ib}}+\frac{3\,\mathrm{e}^{-ia}\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{ib}\,x\right)}{16\sqrt{ib}}+\frac{\mathrm{e}^{3ia}\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{-3ib}\,x\right)}{16\sqrt{-3ib}}+\frac{3\,\mathrm{e}^{ia}\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{-ib}\,x\right)}{16\sqrt{-ib}}$	108

```
[In] int(cos(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 3/8*2^(1/2)*Pi^(1/2)/b^(1/2)*(cos(a)*FresnelC(x*b^(1/2)*2^(1/2)/Pi^(1/2))-s
in(a)*FresnelS(x*b^(1/2)*2^(1/2)/Pi^(1/2)))+1/24*2^(1/2)*Pi^(1/2)*3^(1/2)/b
^(1/2)*(cos(3*a)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)*b^(1/2)*x)-sin(3*a)*Fres
nelS(2^(1/2)/Pi^(1/2)*3^(1/2)*b^(1/2)*x))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.79

$$\int \cos^3\left(a+bx^2\right)\,dx$$
$$=\frac{\sqrt{6}\pi\sqrt{\frac{b}{\pi}}\cos\left(3\,a\right)\mathrm{C}\left(\sqrt{6}x\sqrt{\frac{b}{\pi}}\right)+9\,\sqrt{2}\pi\sqrt{\frac{b}{\pi}}\cos\left(a\right)\mathrm{C}\left(\sqrt{2}x\sqrt{\frac{b}{\pi}}\right)-\sqrt{6}\pi\sqrt{\frac{b}{\pi}}\,\mathrm{S}\left(\sqrt{6}x\sqrt{\frac{b}{\pi}}\right)\sin\left(3\,a\right)-9\,\sqrt{2}\pi}{24\,b}$$

```
[In] integrate(cos(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/24*(sqrt(6)*pi*sqrt(b/pi)*cos(3*a)*fresnel_cos(sqrt(6)*x*sqrt(b/pi)) + 9*
sqrt(2)*pi*sqrt(b/pi)*cos(a)*fresnel_cos(sqrt(2)*x*sqrt(b/pi)) - sqrt(6)*pi
*sqrt(b/pi)*fresnel_sin(sqrt(6)*x*sqrt(b/pi))*sin(3*a) - 9*sqrt(2)*pi*sqrt(
b/pi)*fresnel_sin(sqrt(2)*x*sqrt(b/pi))*sin(a))/b
```

Sympy [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.84

$$\int \cos^3(a+bx^2)\,dx = \frac{3\sqrt{2}\sqrt{\pi}\left(-\sin(a)S\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{\pi}}\right)+\cos(a)C\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{\pi}}\right)\right)\sqrt{\frac{1}{b}}}{8} + \frac{\sqrt{6}\sqrt{\pi}\left(-\sin(3a)S\left(\frac{\sqrt{6}\sqrt{b}x}{\sqrt{\pi}}\right)+\cos(3a)C\left(\frac{\sqrt{6}\sqrt{b}x}{\sqrt{\pi}}\right)\right)\sqrt{\frac{1}{b}}}{24}$$

```
[In] integrate(cos(b*x**2+a)**3,x)
```

```
[Out] 3*sqrt(2)*sqrt(pi)*(-sin(a)*fresnels(sqrt(2)*sqrt(b)*x/sqrt(pi)) + cos(a)*f
resnelc(sqrt(2)*sqrt(b)*x/sqrt(pi)))*sqrt(1/b)/8 + sqrt(6)*sqrt(pi)*(-sin(3
*a)*fresnels(sqrt(6)*sqrt(b)*x/sqrt(pi)) + cos(3*a)*fresnelc(sqrt(6)*sqrt(b
)*x/sqrt(pi)))*sqrt(1/b)/24
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.39 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.73

$$\int \cos^3(a+bx^2)\,dx = -\frac{9^{\frac{1}{4}}\sqrt{2}\sqrt{\pi}\left(((i-1)\cos(3a)+(i+1)\sin(3a))\operatorname{erf}\left(\sqrt{3i\,b}x\right)+(-(i+1)\cos(3a)-(i-1)\sin(3a))\operatorname{er}\right.}{}$$

```
[In] integrate(cos(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/96*(9^(1/4)*sqrt(2)*sqrt(pi)*(((I - 1)*cos(3*a) + (I + 1)*sin(3*a))*erf(
sqrt(3*I*b)*x) + (-(I + 1)*cos(3*a) - (I - 1)*sin(3*a))*erf(sqrt(-3*I*b)*x)
)*b^(3/2) - 9*sqrt(2)*sqrt(pi)*((-(I - 1)*cos(a) - (I + 1)*sin(a))*erf(sqrt
(I*b)*x) + ((I + 1)*cos(a) + (I - 1)*sin(a))*erf(sqrt(-I*b)*x))*b^(3/2))/b^
2
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.34 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.21

$$\int \cos^3\left(a+bx^2\right)\,dx = \frac{i\sqrt{6}\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}i\sqrt{6}\sqrt{b}x\left(\frac{ib}{|b|}+1\right)\right)e^{(3ia)}}{48\sqrt{b}\left(\frac{ib}{|b|}+1\right)} + \frac{3i\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}i\sqrt{2}x\left(\frac{ib}{|b|}+1\right)\sqrt{|b|}\right)e^{(ia)}}{16\left(\frac{ib}{|b|}+1\right)\sqrt{|b|}} - \frac{3i\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(\frac{1}{2}i\sqrt{2}x\left(-\frac{ib}{|b|}+1\right)\sqrt{|b|}\right)e^{(-ia)}}{16\left(-\frac{ib}{|b|}+1\right)\sqrt{|b|}} - \frac{i\sqrt{6}\sqrt{\pi}\,\mathrm{erf}\left(\frac{1}{2}i\sqrt{6}\sqrt{b}x\left(-\frac{ib}{|b|}+1\right)\right)e^{(-3ia)}}{48\sqrt{b}\left(-\frac{ib}{|b|}+1\right)}$$

```
[In] integrate(cos(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/48*I*sqrt(6)*sqrt(pi)*erf(-1/2*I*sqrt(6)*sqrt(b)*x*(I*b/abs(b) + 1))*e^(3
*I*a)/(sqrt(b)*(I*b/abs(b) + 1)) + 3/16*I*sqrt(2)*sqrt(pi)*erf(-1/2*I*sqrt(
2)*x*(I*b/abs(b) + 1)*sqrt(abs(b)))*e^(I*a)/((I*b/abs(b) + 1)*sqrt(abs(b)))
 - 3/16*I*sqrt(2)*sqrt(pi)*erf(1/2*I*sqrt(2)*x*(-I*b/abs(b) + 1)*sqrt(abs(b
)))*e^(-I*a)/((-I*b/abs(b) + 1)*sqrt(abs(b))) - 1/48*I*sqrt(6)*sqrt(pi)*erf
(1/2*I*sqrt(6)*sqrt(b)*x*(-I*b/abs(b) + 1))*e^(-3*I*a)/(sqrt(b)*(-I*b/abs(b
) + 1))
```

Mupad [F(-1)]

Timed out.

$$\int \cos^3\left(a+bx^2\right)\,dx = \int \cos\left(b\,x^2+a\right)^3\,dx$$

```
[In] int(cos(a + b*x^2)^3,x)
```

```
[Out] int(cos(a + b*x^2)^3, x)
```

3.19 $\int \frac{\cos^3(a+bx^2)}{x} dx$

Optimal result

Integrand size = 14, antiderivative size = 55

$$\int \frac{\cos^3(a+bx^2)}{x} dx = \frac{3}{8}\cos(a)\,\text{CosIntegral}\left(bx^2\right) + \frac{1}{8}\cos(3a)\,\text{CosIntegral}\left(3bx^2\right) - \frac{3}{8}\sin(a)\text{Si}\left(bx^2\right) - \frac{1}{8}\sin(3a)\text{Si}\left(3bx^2\right)$$

```
[Out] 3/8*Ci(b*x^2)*cos(a)+1/8*Ci(3*b*x^2)*cos(3*a)-3/8*Si(b*x^2)*sin(a)-1/8*Si(3
*b*x^2)*sin(3*a)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3485, 3459, 3457, 3456}

$$\int \frac{\cos^3(a+bx^2)}{x} dx = \frac{3}{8}\cos(a)\,\text{CosIntegral}\left(bx^2\right) + \frac{1}{8}\cos(3a)\,\text{CosIntegral}\left(3bx^2\right) - \frac{3}{8}\sin(a)\text{Si}\left(bx^2\right) - \frac{1}{8}\sin(3a)\text{Si}\left(3bx^2\right)$$

```
[In] Int[Cos[a + b*x^2]^3/x,x]
```

```
[Out] (3*Cos[a]*CosIntegral[b*x^2])/8 + (Cos[3*a]*CosIntegral[3*b*x^2])/8 - (3*Si
n[a]*SinIntegral[b*x^2])/8 - (Sin[3*a]*SinIntegral[3*b*x^2])/8
```

Rule 3456

```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3457

```
Int[Cos[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[CosIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3459

```
Int[Cos[(c_) + (d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Dist[Cos[c], Int[Cos[d*x
^n]/x, x], x] - Dist[Sin[c], Int[Sin[d*x^n]/x, x], x] /; FreeQ[{c, d, n}, x
]
```

Rule 3485

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_)*((e_.)*(x_))^(m_.), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Cos[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{3\cos(a+bx^2)}{4x}+\frac{\cos(3a+3bx^2)}{4x}\right)dx \\
&= \frac{1}{4}\int \frac{\cos(3a+3bx^2)}{x}\,dx+\frac{3}{4}\int \frac{\cos(a+bx^2)}{x}\,dx \\
&= \frac{1}{4}(3\cos(a))\int \frac{\cos(bx^2)}{x}\,dx+\frac{1}{4}\cos(3a)\int \frac{\cos(3bx^2)}{x}\,dx \\
&\quad -\frac{1}{4}(3\sin(a))\int \frac{\sin(bx^2)}{x}\,dx-\frac{1}{4}\sin(3a)\int \frac{\sin(3bx^2)}{x}\,dx \\
&= \frac{3}{8}\cos(a)\,\text{CosIntegral}\left(bx^2\right)+\frac{1}{8}\cos(3a)\,\text{CosIntegral}\left(3bx^2\right) \\
&\quad -\frac{3}{8}\sin(a)\text{Si}\left(bx^2\right)-\frac{1}{8}\sin(3a)\text{Si}\left(3bx^2\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.91

$$\int \frac{\cos^3(a+bx^2)}{x}\,dx=\frac{1}{8}\big(3\cos(a)\,\text{CosIntegral}\left(bx^2\right)+\cos(3a)\,\text{CosIntegral}\left(3bx^2\right) \\ -3\sin(a)\text{Si}\left(bx^2\right)-\sin(3a)\text{Si}\left(3bx^2\right)\big)$$

```
[In] Integrate[Cos[a + b*x^2]^3/x,x]
[Out] (3*Cos[a]*CosIntegral[b*x^2] + Cos[3*a]*CosIntegral[3*b*x^2] - 3*Sin[a]*Sin
Integral[b*x^2] - Sin[3*a]*SinIntegral[3*b*x^2])/8
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.07 (sec) , antiderivative size = 125, normalized size of antiderivative = 2.27

method	result
risch	$\frac{i\mathrm{e}^{-3ia}\pi\,\mathrm{csgn}(b\,x^2)}{16}-\frac{i\mathrm{e}^{-3ia}\,\mathrm{Si}(3b\,x^2)}{8}-\frac{\mathrm{e}^{-3ia}\,\mathrm{Ei}_1(-3ib\,x^2)}{16}+\frac{3i\mathrm{e}^{-ia}\pi\,\mathrm{csgn}(b\,x^2)}{16}-\frac{3i\mathrm{e}^{-ia}\,\mathrm{Si}(b\,x^2)}{8}-\frac{3\,\mathrm{e}^{-ia}\,\mathrm{Ei}_1(-ib\,x^2)}{16}$

```
[In] int(cos(b*x^2+a)^3/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/16*I*exp(-3*I*a)*Pi*csgn(b*x^2)-1/8*I*exp(-3*I*a)*Si(3*b*x^2)-1/16*exp(-3
*I*a)*Ei(1,-3*I*b*x^2)+3/16*I*exp(-I*a)*Pi*csgn(b*x^2)-3/8*I*exp(-I*a)*Si(b
*x^2)-3/16*exp(-I*a)*Ei(1,-I*b*x^2)-3/16*exp(I*a)*Ei(1,-I*b*x^2)-1/16*exp(3
*I*a)*Ei(1,-3*I*b*x^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.85

$$\int \frac{\cos^3(a+bx^2)}{x}\,dx = \frac{1}{8}\cos(3a)\,\mathrm{Ci}\left(3bx^2\right)+\frac{3}{8}\cos(a)\,\mathrm{Ci}\left(bx^2\right) - \frac{1}{8}\sin(3a)\,\mathrm{Si}\left(3bx^2\right)-\frac{3}{8}\sin(a)\,\mathrm{Si}\left(bx^2\right)$$

```
[In] integrate(cos(b*x^2+a)^3/x,x, algorithm="fricas")
```

```
[Out] 1/8*cos(3*a)*cos_integral(3*b*x^2) + 3/8*cos(a)*cos_integral(b*x^2) - 1/8*s
in(3*a)*sin_integral(3*b*x^2) - 3/8*sin(a)*sin_integral(b*x^2)
```

Sympy [F]

$$\int \frac{\cos^3(a+bx^2)}{x}\,dx = \int \frac{\cos^3(a+bx^2)}{x}\,dx$$

```
[In] integrate(cos(b*x**2+a)**3/x,x)
```

```
[Out] Integral(cos(a + b*x**2)**3/x, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.40 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.62

$$\int \frac{\cos^3(a+bx^2)}{x}\,dx = \frac{1}{16}\left(\mathrm{Ei}\left(3i\,bx^2\right)+\mathrm{Ei}\left(-3i\,bx^2\right)\right)\cos(3\,a) + \frac{3}{16}\left(\mathrm{Ei}\left(i\,bx^2\right)+\mathrm{Ei}\left(-i\,bx^2\right)\right)\cos(a) + \frac{1}{16}\left(i\,\mathrm{Ei}\left(3i\,bx^2\right)-i\,\mathrm{Ei}\left(-3i\,bx^2\right)\right)\sin(3\,a) - \frac{3}{16}\left(-i\,\mathrm{Ei}\left(i\,bx^2\right)+i\,\mathrm{Ei}\left(-i\,bx^2\right)\right)\sin(a)$$

```
[In] integrate(cos(b*x^2+a)^3/x,x, algorithm="maxima")
[Out] 1/16*(Ei(3*I*b*x^2) + Ei(-3*I*b*x^2))*cos(3*a) + 3/16*(Ei(I*b*x^2) + Ei(-I*
b*x^2))*cos(a) + 1/16*(I*Ei(3*I*b*x^2) - I*Ei(-3*I*b*x^2))*sin(3*a) - 3/16*
(-I*Ei(I*b*x^2) + I*Ei(-I*b*x^2))*sin(a)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.85

$$\int \frac{\cos^3(a+bx^2)}{x}\,dx = \frac{1}{8}\cos(3\,a)\,\mathrm{Ci}\left(3\,bx^2\right) + \frac{3}{8}\cos(a)\,\mathrm{Ci}\left(bx^2\right) - \frac{3}{8}\sin(a)\,\mathrm{Si}\left(bx^2\right) + \frac{1}{8}\sin(3\,a)\,\mathrm{Si}\left(-3\,bx^2\right)$$

```
[In] integrate(cos(b*x^2+a)^3/x,x, algorithm="giac")
[Out] 1/8*cos(3*a)*cos_integral(3*b*x^2) + 3/8*cos(a)*cos_integral(b*x^2) - 3/8*s
in(a)*sin_integral(b*x^2) + 1/8*sin(3*a)*sin_integral(-3*b*x^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^3(a+bx^2)}{x}\,dx = \int \frac{\cos\left(b\,x^2+a\right)^3}{x}\,dx$$

```
[In] int(cos(a + b*x^2)^3/x,x)
[Out] int(cos(a + b*x^2)^3/x, x)
```

3.20 $\int \frac{\cos^3(a+bx^2)}{x^2} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 168

$$\int \frac{\cos^3(a+bx^2)}{x^2} \, dx = -\frac{\cos^3(a+bx^2)}{x} - \frac{3}{2}\sqrt{b}\sqrt{\frac{\pi}{2}}\cos(a)\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right) - \frac{1}{2}\sqrt{b}\sqrt{\frac{3\pi}{2}}\cos(3a)\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{6}{\pi}}x\right) - \frac{3}{2}\sqrt{b}\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)\sin(a) - \frac{1}{2}\sqrt{b}\sqrt{\frac{3\pi}{2}}\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{6}{\pi}}x\right)\sin(3a)$$

```
[Out] -cos(b*x^2+a)^3/x-3/4*cos(a)*FresnelS(x*b^(1/2)*2^(1/2)/Pi^(1/2))*b^(1/2)*2
^(1/2)*Pi^(1/2)-3/4*FresnelC(x*b^(1/2)*2^(1/2)/Pi^(1/2))*sin(a)*b^(1/2)*2^(
1/2)*Pi^(1/2)-1/4*cos(3*a)*FresnelS(x*b^(1/2)*6^(1/2)/Pi^(1/2))*b^(1/2)*6^(
1/2)*Pi^(1/2)-1/4*FresnelC(x*b^(1/2)*6^(1/2)/Pi^(1/2))*sin(3*a)*b^(1/2)*6^(
1/2)*Pi^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used

= {3475, 4670, 3434, 3433, 3432}

$$\int \frac{\cos^3(a+bx^2)}{x^2}\,dx = -\frac{3}{2}\sqrt{\frac{\pi}{2}}\sqrt{b}\sin(a)\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right) - \frac{1}{2}\sqrt{\frac{3\pi}{2}}\sqrt{b}\sin(3a)\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{6}{\pi}}x\right) - \frac{3}{2}\sqrt{\frac{\pi}{2}}\sqrt{b}\cos(a)\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right) - \frac{1}{2}\sqrt{\frac{3\pi}{2}}\sqrt{b}\cos(3a)\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{6}{\pi}}x\right) - \frac{\cos^3(a+bx^2)}{x}$$

```
[In] Int[Cos[a + b*x^2]^3/x^2,x]
```

```
[Out] -(Cos[a + b*x^2]^3/x) - (3*Sqrt[b]*Sqrt[Pi/2]*Cos[a]*FresnelS[Sqrt[b]*Sqrt[
2/Pi]*x])/2 - (Sqrt[b]*Sqrt[(3*Pi)/2]*Cos[3*a]*FresnelS[Sqrt[b]*Sqrt[6/Pi]*
x])/2 - (3*Sqrt[b]*Sqrt[Pi/2]*FresnelC[Sqrt[b]*Sqrt[2/Pi]*x]*Sin[a])/2 - (S
qrt[b]*Sqrt[(3*Pi)/2]*FresnelC[Sqrt[b]*Sqrt[6/Pi]*x]*Sin[3*a])/2
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3434

```
Int[Sin[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Sin[c], Int
[Cos[d*(e + f*x)^2], x], x] + Dist[Cos[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3475

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_)]^(p_)*(x_)^(m_.), x_Symbol] :> Simp[x^(m +
1)*(Cos[a + b*x^n]^p/(m + 1)), x] + Dist[b*n*(p/(m + 1)), Int[Cos[a + b*x^n
]^(p - 1)*Sin[a + b*x^n], x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 1] && EqQ[m
 + n, 0] && NeQ[n, 1] && IntegerQ[n]
```

Rule 4670

```
Int[Cos[w_]^(q_.)*Sin[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Cos[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] && Pol
```

```
ynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x]))
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\cos^3(a+bx^2)}{x} - (6b)\int \cos^2(a+bx^2)\sin(a+bx^2)\,dx\\ &= -\frac{\cos^3(a+bx^2)}{x} - (6b)\int \left(\frac{1}{4}\sin(a+bx^2) + \frac{1}{4}\sin(3a+3bx^2)\right)dx\\ &= -\frac{\cos^3(a+bx^2)}{x} - \frac{1}{2}(3b)\int \sin(a+bx^2)\,dx - \frac{1}{2}(3b)\int \sin(3a+3bx^2)\,dx\\ &= -\frac{\cos^3(a+bx^2)}{x} - \frac{1}{2}(3b\cos(a))\int \sin(bx^2)\,dx - \frac{1}{2}(3b\cos(3a))\int \sin(3bx^2)\,dx\\ &\quad - \frac{1}{2}(3b\sin(a))\int \cos(bx^2)\,dx - \frac{1}{2}(3b\sin(3a))\int \cos(3bx^2)\,dx\\ &= -\frac{\cos^3(a+bx^2)}{x} - \frac{3}{2}\sqrt{b}\sqrt{\frac{\pi}{2}}\cos(a)\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)\\ &\quad - \frac{1}{2}\sqrt{b}\sqrt{\frac{3\pi}{2}}\cos(3a)\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{6}{\pi}}x\right) - \frac{3}{2}\sqrt{b}\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)\sin(a)\\ &\quad - \frac{1}{2}\sqrt{b}\sqrt{\frac{3\pi}{2}}\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{6}{\pi}}x\right)\sin(3a)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.99

$$\int \frac{\cos^3(a+bx^2)}{x^2}\,dx = -\frac{3\cos(a+bx^2) + \cos(3(a+bx^2)) + 3\sqrt{b}\sqrt{2\pi}x\cos(a)\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right) + \sqrt{b}\sqrt{6\pi}x\cos(3a)\,\text{FresnelS}}{4x}$$

```
[In] Integrate[Cos[a + b*x^2]^3/x^2,x]
```

```
[Out] -1/4*(3*Cos[a + b*x^2] + Cos[3*(a + b*x^2)] + 3*Sqrt[b]*Sqrt[2*Pi]*x*Cos[a]
*FresnelS[Sqrt[b]*Sqrt[2/Pi]*x] + Sqrt[b]*Sqrt[6*Pi]*x*Cos[3*a]*FresnelS[Sq
rt[b]*Sqrt[6/Pi]*x] + 3*Sqrt[b]*Sqrt[2*Pi]*x*FresnelC[Sqrt[b]*Sqrt[2/Pi]*x]
*Sin[a] + Sqrt[b]*Sqrt[6*Pi]*x*FresnelC[Sqrt[b]*Sqrt[6/Pi]*x]*Sin[3*a])/x
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.76

method	result
default	$-\frac{3\cos\left(b\,x^{2}+a\right)}{4x}-\frac{3\sqrt{b}\,\sqrt{2}\,\sqrt{\pi}\,\left(\cos\left(a\right)\mathrm{S}\left(\frac{x\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}}\right)+\sin\left(a\right)\mathrm{C}\left(\frac{x\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}}\right)\right)}{4}-\frac{\cos\left(3b\,x^{2}+3a\right)}{4x}-\frac{\sqrt{b}\,\sqrt{2}\,\sqrt{\pi}\,\sqrt{3}\,\left(\cos\left(3a\right)\mathrm{S}\left(\frac{\sqrt{2}\,\sqrt{3}\,\sqrt{b}}{\sqrt{\pi}}\right.\right.}{4}$
risch	$-\frac{i\mathrm{e}^{-3ia}b\sqrt{\pi}\,\sqrt{3}\,\mathrm{erf}\left(\sqrt{3}\,\sqrt{ib}\,x\right)}{8\sqrt{ib}}-\frac{3i\mathrm{e}^{-ia}b\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{ib}\,x\right)}{8\sqrt{ib}}+\frac{3i\mathrm{e}^{ia}b\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{-ib}\,x\right)}{8\sqrt{-ib}}+\frac{3i\mathrm{e}^{3ia}b\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{-3ib}\,x\right)}{8\sqrt{-3ib}}-\frac{3\cos\left(b\,x^{2}+a\right.}{4x}$

```
[In] int(cos(b*x^2+a)^3/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -3/4*cos(b*x^2+a)/x-3/4*b^(1/2)*2^(1/2)*Pi^(1/2)*(cos(a)*FresnelS(x*b^(1/2)
*2^(1/2)/Pi^(1/2))+sin(a)*FresnelC(x*b^(1/2)*2^(1/2)/Pi^(1/2)))-1/4*cos(3*b
*x^2+3*a)/x-1/4*b^(1/2)*2^(1/2)*Pi^(1/2)*3^(1/2)*(cos(3*a)*FresnelS(2^(1/2)
/Pi^(1/2)*3^(1/2)*b^(1/2)*x)+sin(3*a)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)*b^(
1/2)*x))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.81

$$\int \frac{\cos^3\left(a+bx^2\right)}{x^2}\,dx =$$
$$-\frac{\sqrt{6}\pi x\sqrt{\frac{b}{\pi}}\cos\left(3\,a\right)\mathrm{S}\left(\sqrt{6}x\sqrt{\frac{b}{\pi}}\right)+3\,\sqrt{2}\pi x\sqrt{\frac{b}{\pi}}\cos\left(a\right)\mathrm{S}\left(\sqrt{2}x\sqrt{\frac{b}{\pi}}\right)+\sqrt{6}\pi x\sqrt{\frac{b}{\pi}}\,\mathrm{C}\left(\sqrt{6}x\sqrt{\frac{b}{\pi}}\right)\sin\left(3\,a\right)+}{4\,x}$$

```
[In] integrate(cos(b*x^2+a)^3/x^2,x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(6)*pi*x*sqrt(b/pi)*cos(3*a)*fresnel_sin(sqrt(6)*x*sqrt(b/pi)) +
3*sqrt(2)*pi*x*sqrt(b/pi)*cos(a)*fresnel_sin(sqrt(2)*x*sqrt(b/pi)) + sqrt(6
)*pi*x*sqrt(b/pi)*fresnel_cos(sqrt(6)*x*sqrt(b/pi))*sin(3*a) + 3*sqrt(2)*pi
*x*sqrt(b/pi)*fresnel_cos(sqrt(2)*x*sqrt(b/pi))*sin(a) + 4*cos(b*x^2 + a)^3
)/x
```

Sympy [F]

$$\int \frac{\cos^3(a+bx^2)}{x^2}\,dx = \int \frac{\cos^3(a+bx^2)}{x^2}\,dx$$

```
[In] integrate(cos(b*x**2+a)**3/x**2,x)
```

```
[Out] Integral(cos(a + b*x**2)**3/x**2, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.66 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.90

$$\int \frac{\cos^3(a+bx^2)}{x^2}\,dx = \frac{\sqrt{3}\sqrt{bx^2}\left(\left(-(i+1)\,\sqrt{2}\Gamma\left(-\frac{1}{2},3i\,bx^2\right)+(i-1)\,\sqrt{2}\Gamma\left(-\frac{1}{2},-3i\,bx^2\right)\right)\cos\left(3\,a\right)+\left((i-1)\,\sqrt{2}\Gamma\left(-\frac{1}{2},3i\,bx^2\right)\right.}{}$$

```
[In] integrate(cos(b*x^2+a)^3/x^2,x, algorithm="maxima")
```

```
[Out] 1/32*(sqrt(3)*sqrt(b*x^2)*((-(I + 1)*sqrt(2)*gamma(-1/2, 3*I*b*x^2) + (I -
1)*sqrt(2)*gamma(-1/2, -3*I*b*x^2))*cos(3*a) + ((I - 1)*sqrt(2)*gamma(-1/2,
 3*I*b*x^2) - (I + 1)*sqrt(2)*gamma(-1/2, -3*I*b*x^2))*sin(3*a)) - 3*sqrt(b
*x^2)*(((I + 1)*sqrt(2)*gamma(-1/2, I*b*x^2) - (I - 1)*sqrt(2)*gamma(-1/2,
-I*b*x^2))*cos(a) + (-(I - 1)*sqrt(2)*gamma(-1/2, I*b*x^2) + (I + 1)*sqrt(2
)*gamma(-1/2, -I*b*x^2))*sin(a)))/x
```

Giac [F]

$$\int \frac{\cos^3(a+bx^2)}{x^2}\,dx = \int \frac{\cos\left(bx^2+a\right)^3}{x^2}\,dx$$

```
[In] integrate(cos(b*x^2+a)^3/x^2,x, algorithm="giac")
```

```
[Out] integrate(cos(b*x^2 + a)^3/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^3(a+bx^2)}{x^2}\,dx=\int \frac{\cos\left(b\,x^2+a\right)^3}{x^2}\,dx$$

```
[In] int(cos(a + b*x^2)^3/x^2,x)
[Out] int(cos(a + b*x^2)^3/x^2, x)
```

3.21 $\int \frac{\cos^3(a+bx^2)}{x^3} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 91

$$\int \frac{\cos^3(a+bx^2)}{x^3} \, dx = -\frac{3\cos(a+bx^2)}{8x^2} - \frac{\cos(3(a+bx^2))}{8x^2} - \frac{3}{8}b\,\text{CosIntegral}(bx^2)\sin(a) - \frac{3}{8}b\,\text{CosIntegral}(3bx^2)\sin(3a) - \frac{3}{8}b\cos(a)\text{Si}(bx^2) - \frac{3}{8}b\cos(3a)\text{Si}(3bx^2)$$

```
[Out] -3/8*cos(b*x^2+a)/x^2-1/8*cos(3*b*x^2+3*a)/x^2-3/8*b*cos(a)*Si(b*x^2)-3/8*b
*cos(3*a)*Si(3*b*x^2)-3/8*b*Ci(b*x^2)*sin(a)-3/8*b*Ci(3*b*x^2)*sin(3*a)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3485, 3461, 3378, 3384, 3380, 3383}

$$\int \frac{\cos^3(a+bx^2)}{x^3} \, dx = -\frac{3}{8}b\sin(a)\,\text{CosIntegral}(bx^2) - \frac{3}{8}b\sin(3a)\,\text{CosIntegral}(3bx^2) - \frac{3}{8}b\cos(a)\text{Si}(bx^2) - \frac{3}{8}b\cos(3a)\text{Si}(3bx^2) - \frac{3\cos(a+bx^2)}{8x^2} - \frac{\cos(3(a+bx^2))}{8x^2}$$

```
[In] Int[Cos[a + b*x^2]^3/x^3,x]
```

```
[Out] (-3*Cos[a + b*x^2])/(8*x^2) - Cos[3*(a + b*x^2)]/(8*x^2) - (3*b*CosIntegral
[b*x^2]*Sin[a])/8 - (3*b*CosIntegral[3*b*x^2]*Sin[3*a])/8 - (3*b*Cos[a]*Sin
Integral[b*x^2])/8 - (3*b*Cos[3*a]*SinIntegral[3*b*x^2])/8
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3485

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_)*((e_.)*(x_))^(m_.), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Cos[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \left(\frac{3\cos(a+bx^2)}{4x^3} + \frac{\cos(3a+3bx^2)}{4x^3} \right) dx \\ &= \frac{1}{4} \int \frac{\cos(3a+3bx^2)}{x^3} \, dx + \frac{3}{4} \int \frac{\cos(a+bx^2)}{x^3} \, dx \end{aligned}$$

$$= \frac{1}{8}\text{Subst}\left(\int \frac{\cos(3a+3bx)}{x^2}\,dx,x,x^2\right)+\frac{3}{8}\text{Subst}\left(\int \frac{\cos(a+bx)}{x^2}\,dx,x,x^2\right)$$

$$= -\frac{3\cos(a+bx^2)}{8x^2}-\frac{\cos(3(a+bx^2))}{8x^2}-\frac{1}{8}(3b)\text{Subst}\left(\int \frac{\sin(a+bx)}{x}\,dx,x,x^2\right) - \frac{1}{8}(3b)\text{Subst}\left(\int \frac{\sin(3a+3bx)}{x}\,dx,x,x^2\right)$$

$$= -\frac{3\cos(a+bx^2)}{8x^2}-\frac{\cos(3(a+bx^2))}{8x^2}-\frac{1}{8}(3b\cos(a))\text{Subst}\left(\int \frac{\sin(bx)}{x}\,dx,x,x^2\right) - \frac{1}{8}(3b\cos(3a))\text{Subst}\left(\int \frac{\sin(3bx)}{x}\,dx,x,x^2\right) - \frac{1}{8}(3b\sin(a))\text{Subst}\left(\int \frac{\cos(bx)}{x}\,dx,x,x^2\right) - \frac{1}{8}(3b\sin(3a))\text{Subst}\left(\int \frac{\cos(3bx)}{x}\,dx,x,x^2\right)$$

$$= -\frac{3\cos(a+bx^2)}{8x^2}-\frac{\cos(3(a+bx^2))}{8x^2}-\frac{3}{8}b\,\text{CosIntegral}(bx^2)\sin(a) - \frac{3}{8}b\,\text{CosIntegral}(3bx^2)\sin(3a)-\frac{3}{8}b\cos(a)\text{Si}(bx^2)-\frac{3}{8}b\cos(3a)\text{Si}(3bx^2)$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.99

$$\int \frac{\cos^3(a+bx^2)}{x^3}\,dx = -\frac{3\cos(a+bx^2)+\cos(3(a+bx^2))+3bx^2\,\text{CosIntegral}(bx^2)\sin(a)+3bx^2\,\text{CosIntegral}(3bx^2)\sin(3a)+3}{8x^2}$$

```
[In] Integrate[Cos[a + b*x^2]^3/x^3,x]
```

```
[Out] -1/8*(3*Cos[a + b*x^2] + Cos[3*(a + b*x^2)] + 3*b*x^2*CosIntegral[b*x^2]*Si
n[a] + 3*b*x^2*CosIntegral[3*b*x^2]*Sin[3*a] + 3*b*x^2*Cos[a]*SinIntegral[b
*x^2] + 3*b*x^2*Cos[3*a]*SinIntegral[3*b*x^2])/x^2
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.97 (sec) , antiderivative size = 185, normalized size of antiderivative = 2.03

method	result
risch	$-\frac{3ie^{3ia}b\,\mathrm{Ei}_1(-3ib\,x^2)x^2-3ie^{-ia}\,\mathrm{Ei}_1(-ib\,x^2)b\,x^2-3\,\mathrm{e}^{-ia}\pi\,\mathrm{csgn}(b\,x^2)b\,x^2+3ie^{ia}b\,\mathrm{Ei}_1(-ib\,x^2)x^2-3\pi\,\mathrm{csgn}(b\,x^2)\mathrm{e}^{-3ia}b\,x^2-3ie^{-3ia}}{16x^2}$

```
[In] int(cos(b*x^2+a)^3/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/16*(3*I*exp(3*I*a)*b*Ei(1,-3*I*b*x^2)*x^2-3*I*exp(-I*a)*Ei(1,-I*b*x^2)*b
*x^2-3*exp(-I*a)*Pi*csgn(b*x^2)*b*x^2+3*I*exp(I*a)*b*Ei(1,-I*b*x^2)*x^2-3*P
i*csgn(b*x^2)*exp(-3*I*a)*b*x^2-3*I*exp(-3*I*a)*Ei(1,-3*I*b*x^2)*b*x^2+6*ex
p(-I*a)*Si(b*x^2)*b*x^2+6*Si(3*b*x^2)*exp(-3*I*a)*b*x^2+6*cos(b*x^2+a)+2*co
s(3*b*x^2+3*a))/x^2
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.88

$$\int \frac{\cos^3(a+bx^2)}{x^3}\,dx = -\frac{3\,bx^2\,\mathrm{Ci}\left(3\,bx^2\right)\sin\left(3\,a\right)+3\,bx^2\,\mathrm{Ci}\left(bx^2\right)\sin\left(a\right)+3\,bx^2\cos\left(3\,a\right)\mathrm{Si}\left(3\,bx^2\right)+3\,bx^2\cos\left(a\right)\mathrm{Si}\left(bx^2\right)+4\,\cos\left(\right.}{8\,x^2}$$

```
[In] integrate(cos(b*x^2+a)^3/x^3,x, algorithm="fricas")
```

```
[Out] -1/8*(3*b*x^2*cos_integral(3*b*x^2)*sin(3*a) + 3*b*x^2*cos_integral(b*x^2)*
sin(a) + 3*b*x^2*cos(3*a)*sin_integral(3*b*x^2) + 3*b*x^2*cos(a)*sin_integr
al(b*x^2) + 4*cos(b*x^2 + a)^3)/x^2
```

Sympy [F]

$$\int \frac{\cos^3(a+bx^2)}{x^3}\,dx = \int \frac{\cos^3(a+bx^2)}{x^3}\,dx$$

```
[In] integrate(cos(b*x**2+a)**3/x**3,x)
```

```
[Out] Integral(cos(a + b*x**2)**3/x**3, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.38 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.08

$$\int \frac{\cos^3(a+bx^2)}{x^3}\,dx$$
$$= \frac{3}{16}\left(\left(-i\,\Gamma\left(-1,3i\,bx^2\right)+i\,\Gamma\left(-1,-3i\,bx^2\right)\right)\cos(3\,a)+\left(-i\,\Gamma\left(-1,i\,bx^2\right)+i\,\Gamma\left(-1,-i\,bx^2\right)\right)\cos(a)-(\Gamma(-$$

```
[In] integrate(cos(b*x^2+a)^3/x^3,x, algorithm="maxima")
```

```
[Out] 3/16*((-I*gamma(-1, 3*I*b*x^2) + I*gamma(-1, -3*I*b*x^2))*cos(3*a) + (-I*ga
mma(-1, I*b*x^2) + I*gamma(-1, -I*b*x^2))*cos(a) - (gamma(-1, 3*I*b*x^2) +
gamma(-1, -3*I*b*x^2))*sin(3*a) - (gamma(-1, I*b*x^2) + gamma(-1, -I*b*x^2)
)*sin(a))*b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 185 vs. 2(80) = 160.

Time = 0.31 (sec) , antiderivative size = 185, normalized size of antiderivative = 2.03

$$\int \frac{\cos^3(a+bx^2)}{x^3}\,dx =$$
$$-\frac{3\,(bx^2+a)b^2\,\mathrm{Ci}\left(3\,bx^2\right)\sin(3\,a)-3\,ab^2\,\mathrm{Ci}\left(3\,bx^2\right)\sin(3\,a)+3\,(bx^2+a)b^2\,\mathrm{Ci}\left(bx^2\right)\sin(a)-3\,ab^2\,\mathrm{Ci}\left(bx^2\right.}{}$$

```
[In] integrate(cos(b*x^2+a)^3/x^3,x, algorithm="giac")
```

```
[Out] -1/8*(3*(b*x^2 + a)*b^2*cos_integral(3*b*x^2)*sin(3*a) - 3*a*b^2*cos_integr
al(3*b*x^2)*sin(3*a) + 3*(b*x^2 + a)*b^2*cos_integral(b*x^2)*sin(a) - 3*a*b
^2*cos_integral(b*x^2)*sin(a) + 3*(b*x^2 + a)*b^2*cos(a)*sin_integral(b*x^2
) - 3*a*b^2*cos(a)*sin_integral(b*x^2) - 3*(b*x^2 + a)*b^2*cos(3*a)*sin_int
egral(-3*b*x^2) + 3*a*b^2*cos(3*a)*sin_integral(-3*b*x^2) + b^2*cos(3*b*x^2
 + 3*a) + 3*b^2*cos(b*x^2 + a))/(b^2*x^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^3(a+bx^2)}{x^3}\,dx = \int \frac{\cos\left(b\,x^2+a\right)^3}{x^3}\,dx$$

```
[In] int(cos(a + b*x^2)^3/x^3,x)
```

```
[Out] int(cos(a + b*x^2)^3/x^3, x)
```

3.22 $\int x \cos^7\left(a + bx^2\right) \, dx$

Optimal result

Integrand size = 12, antiderivative size = 67

$$\int x \cos^7\left(a + bx^2\right) \, dx = \frac{\sin\left(a + bx^2\right)}{2b} - \frac{\sin^3\left(a + bx^2\right)}{2b} + \frac{3\sin^5\left(a + bx^2\right)}{10b} - \frac{\sin^7\left(a + bx^2\right)}{14b}$$

```
[Out] 1/2*sin(b*x^2+a)/b-1/2*sin(b*x^2+a)^3/b+3/10*sin(b*x^2+a)^5/b-1/14*sin(b*x^
2+a)^7/b
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3461, 2713}

$$\int x \cos^7\left(a + bx^2\right) \, dx = -\frac{\sin^7\left(a + bx^2\right)}{14b} + \frac{3\sin^5\left(a + bx^2\right)}{10b} - \frac{\sin^3\left(a + bx^2\right)}{2b} + \frac{\sin\left(a + bx^2\right)}{2b}$$

```
[In] Int[x*Cos[a + b*x^2]^7,x]
```

```
[Out] Sin[a + b*x^2]/(2*b) - Sin[a + b*x^2]^3/(2*b) + (3*Sin[a + b*x^2]^5)/(10*b)
 - Sin[a + b*x^2]^7/(14*b)
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
```

```
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \cos^7(a+bx)\,dx, x, x^2\right)\\
&= -\frac{\text{Subst}\left(\int \left(1-3x^2+3x^4-x^6\right)\,dx, x, -\sin\left(a+bx^2\right)\right)}{2b}\\
&= \frac{\sin\left(a+bx^2\right)}{2b} - \frac{\sin^3\left(a+bx^2\right)}{2b} + \frac{3\sin^5\left(a+bx^2\right)}{10b} - \frac{\sin^7\left(a+bx^2\right)}{14b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.81

$$\int x\cos^7\left(a+bx^2\right)\,dx = \frac{35\sin\left(a+bx^2\right)-35\sin^3\left(a+bx^2\right)+21\sin^5\left(a+bx^2\right)-5\sin^7\left(a+bx^2\right)}{70b}$$

```
[In] Integrate[x*Cos[a + b*x^2]^7,x]
[Out] (35*Sin[a + b*x^2] - 35*Sin[a + b*x^2]^3 + 21*Sin[a + b*x^2]^5 - 5*Sin[a +
b*x^2]^7)/(70*b)
```

Maple [A] (verified)

Time = 1.44 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.75

method	result	size
derivativedivides	$\frac{\left(\frac{16}{5}+\cos^6\left(b\,x^2+a\right)+\frac{6\left(\cos^4\left(b\,x^2+a\right)\right)}{5}+\frac{8\left(\cos^2\left(b\,x^2+a\right)\right)}{5}\right)\sin\left(b\,x^2+a\right)}{14b}$	50
default	$\frac{\left(\frac{16}{5}+\cos^6\left(b\,x^2+a\right)+\frac{6\left(\cos^4\left(b\,x^2+a\right)\right)}{5}+\frac{8\left(\cos^2\left(b\,x^2+a\right)\right)}{5}\right)\sin\left(b\,x^2+a\right)}{14b}$	50
parallelrisch	$\frac{1225\sin\left(b\,x^2+a\right)+5\sin\left(7b\,x^2+7a\right)+49\sin\left(5b\,x^2+5a\right)+245\sin\left(3b\,x^2+3a\right)}{4480b}$	56
risch	$\frac{35\sin\left(b\,x^2+a\right)}{128b}+\frac{\sin\left(7b\,x^2+7a\right)}{896b}+\frac{7\sin\left(5b\,x^2+5a\right)}{640b}+\frac{7\sin\left(3b\,x^2+3a\right)}{128b}$	63

```
[In] int(x*cos(b*x^2+a)^7,x,method=_RETURNVERBOSE)
```

```
[Out] 1/14/b*(16/5+cos(b*x^2+a)^6+6/5*cos(b*x^2+a)^4+8/5*cos(b*x^2+a)^2)*sin(b*x^
2+a)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.76

$$\int x\cos^7\left(a+bx^2\right)\,dx = \frac{\left(5\,\cos\left(bx^2+a\right)^6+6\,\cos\left(bx^2+a\right)^4+8\,\cos\left(bx^2+a\right)^2+16\right)\sin\left(bx^2+a\right)}{70\,b}$$

```
[In] integrate(x*cos(b*x^2+a)^7,x, algorithm="fricas")
```

```
[Out] 1/70*(5*cos(b*x^2 + a)^6 + 6*cos(b*x^2 + a)^4 + 8*cos(b*x^2 + a)^2 + 16)*si
n(b*x^2 + a)/b
```

Sympy [A] (verification not implemented)

Time = 0.70 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.40

$$\int x\cos^7\left(a+bx^2\right)\,dx = \begin{cases} \frac{8\sin^7\left(a+bx^2\right)}{35b}+\frac{4\sin^5\left(a+bx^2\right)\cos^2\left(a+bx^2\right)}{5b}+\frac{\sin^3\left(a+bx^2\right)\cos^4\left(a+bx^2\right)}{b}+\frac{\sin\left(a+bx^2\right)\cos^6\left(a+bx^2\right)}{2b} & \text{for } b\neq 0 \\ \frac{x^2\cos^7(a)}{2} & \text{otherwise} \end{cases}$$

```
[In] integrate(x*cos(b*x**2+a)**7,x)
```

```
[Out] Piecewise((8*sin(a + b*x**2)**7/(35*b) + 4*sin(a + b*x**2)**5*cos(a + b*x**
2)**2/(5*b) + sin(a + b*x**2)**3*cos(a + b*x**2)**4/b + sin(a + b*x**2)*cos
(a + b*x**2)**6/(2*b), Ne(b, 0)), (x**2*cos(a)**7/2, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.82

$$\int x\cos^7\left(a+bx^2\right)\,dx = \frac{5\,\sin\left(7\,bx^2+7\,a\right)+49\,\sin\left(5\,bx^2+5\,a\right)+245\,\sin\left(3\,bx^2+3\,a\right)+1225\,\sin\left(bx^2+a\right)}{4480\,b}$$

```
[In] integrate(x*cos(b*x^2+a)^7,x, algorithm="maxima")
```

```
[Out] 1/4480*(5*sin(7*b*x^2 + 7*a) + 49*sin(5*b*x^2 + 5*a) + 245*sin(3*b*x^2 + 3*
a) + 1225*sin(b*x^2 + a))/b
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.78

$$\int x\cos^7\left(a+bx^2\right)\,dx = -\frac{5\,\sin\left(bx^2+a\right)^7-21\,\sin\left(bx^2+a\right)^5+35\,\sin\left(bx^2+a\right)^3-35\,\sin\left(bx^2+a\right)}{70\,b}$$

```
[In] integrate(x*cos(b*x^2+a)^7,x, algorithm="giac")
```

```
[Out] -1/70*(5*sin(b*x^2 + a)^7 - 21*sin(b*x^2 + a)^5 + 35*sin(b*x^2 + a)^3 - 35*
sin(b*x^2 + a))/b
```

Mupad [B] (verification not implemented)

Time = 14.41 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.82

$$\int x\cos^7\left(a+bx^2\right)\,dx = \frac{245\,\sin\left(3\,b\,x^2+3\,a\right)+49\,\sin\left(5\,b\,x^2+5\,a\right)+5\,\sin\left(7\,b\,x^2+7\,a\right)+1225\,\sin\left(b\,x^2+a\right)}{4480\,b}$$

```
[In] int(x*cos(a + b*x^2)^7,x)
```

```
[Out] (245*sin(3*a + 3*b*x^2) + 49*sin(5*a + 5*b*x^2) + 5*sin(7*a + 7*b*x^2) + 12
25*sin(a + b*x^2))/(4480*b)
```

3.23 $\int x^{5/2} \cos(a+bx^2)\, dx$

Optimal result

Integrand size = 14, antiderivative size = 111

$$\int x^{5/2}\cos\left(a+bx^2\right)\,dx=-\frac{3ie^{ia}x^{3/2}\Gamma\left(\frac{3}{4},-ibx^2\right)}{16b\left(-ibx^2\right)^{3/4}}+\frac{3ie^{-ia}x^{3/2}\Gamma\left(\frac{3}{4},ibx^2\right)}{16b\left(ibx^2\right)^{3/4}}+\frac{x^{3/2}\sin\left(a+bx^2\right)}{2b}$$

```
[Out] -3/16*I*exp(I*a)*x^(3/2)*GAMMA(3/4,-I*b*x^2)/b/(-I*b*x^2)^(3/4)+3/16*I*x^(3
/2)*GAMMA(3/4,I*b*x^2)/b/exp(I*a)/(I*b*x^2)^(3/4)+1/2*x^(3/2)*sin(b*x^2+a)/
b
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3467, 3470, 2250}

$$\int x^{5/2}\cos\left(a+bx^2\right)\,dx=\frac{x^{3/2}\sin\left(a+bx^2\right)}{2b}-\frac{3ie^{ia}x^{3/2}\Gamma\left(\frac{3}{4},-ibx^2\right)}{16b\left(-ibx^2\right)^{3/4}}+\frac{3ie^{-ia}x^{3/2}\Gamma\left(\frac{3}{4},ibx^2\right)}{16b\left(ibx^2\right)^{3/4}}$$

```
[In] Int[x^(5/2)*Cos[a + b*x^2],x]
```

```
[Out] (((-3*I)/16)*E^(I*a)*x^(3/2)*Gamma[3/4, (-I)*b*x^2])/(b*((-I)*b*x^2)^(3/4))
 + (((3*I)/16)*x^(3/2)*Gamma[3/4, I*b*x^2])/(b*E^(I*a)*(I*b*x^2)^(3/4)) + (
x^(3/2)*Sin[a + b*x^2])/(2*b)
```

Rule 2250

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 3467

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[e^(n
 - 1)*(e*x)^(m - n + 1)*(Sin[c + d*x^n]/(d*n)), x] - Dist[e^n*((m - n + 1)/
(d*n)), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3470

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[I/2,
 Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] - Dist[I/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{x^{3/2}\sin\left(a+bx^2\right)}{2b} - \frac{3\int \sqrt{x}\sin\left(a+bx^2\right)\,dx}{4b}\\ &= \frac{x^{3/2}\sin\left(a+bx^2\right)}{2b} - \frac{(3i)\int e^{-ia-ibx^2}\sqrt{x}\,dx}{8b} + \frac{(3i)\int e^{ia+ibx^2}\sqrt{x}\,dx}{8b}\\ &= -\frac{3ie^{ia}x^{3/2}\Gamma\left(\frac{3}{4},-ibx^2\right)}{16b\left(-ibx^2\right)^{3/4}} + \frac{3ie^{-ia}x^{3/2}\Gamma\left(\frac{3}{4},ibx^2\right)}{16b\left(ibx^2\right)^{3/4}} + \frac{x^{3/2}\sin\left(a+bx^2\right)}{2b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.02

$$\int x^{5/2}\cos\left(a+bx^2\right)\,dx = \frac{bx^{11/2}\left(3\left(ibx^2\right)^{3/4}\Gamma\left(\frac{3}{4},-ibx^2\right)\left(-i\cos(a)+\sin(a)\right)+3\left(-ibx^2\right)^{3/4}\Gamma\left(\frac{3}{4},ibx^2\right)\left(i\cos(a)+\sin(a)\right)+\right.}{16\left(b^2x^4\right)^{7/4}}$$

```
[In] Integrate[x^(5/2)*Cos[a + b*x^2],x]
```

```
[Out] (b*x^(11/2)*(3*(I*b*x^2)^(3/4)*Gamma[3/4, (-I)*b*x^2]*((-I)*Cos[a] + Sin[a]
) + 3*((-I)*b*x^2)^(3/4)*Gamma[3/4, I*b*x^2]*(I*Cos[a] + Sin[a]) + 8*(b^2*x
^4)^(3/4)*Sin[a + b*x^2]))/(16*(b^2*x^4)^(7/4))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.57 (sec) , antiderivative size = 229, normalized size of antiderivative = 2.06

method	result
meijerg	$\frac{2^{\frac{3}{4}}\cos(a)\sqrt{\pi}\left(\frac{2x^{\frac{3}{2}}2^{\frac{1}{4}}(b^2)^{\frac{7}{8}}\sin(bx^2)}{7\sqrt{\pi}\,b}+\frac{3x^{\frac{7}{2}}(b^2)^{\frac{7}{8}}2^{\frac{1}{4}}\sin(bx^2)s_{\frac{5}{4},\frac{3}{2}}^{(+)}(bx^2)}{14\sqrt{\pi}\,(bx^2)^{\frac{5}{4}}}+\frac{3x^{\frac{7}{2}}(b^2)^{\frac{7}{8}}2^{\frac{1}{4}}(\cos(bx^2)x^2b-\sin(bx^2))s_{\frac{1}{4},\frac{1}{2}}^{(+)}(bx^2)}{8\sqrt{\pi}\,(bx^2)^{\frac{9}{4}}}\right)}{2(b^2)^{\frac{7}{8}}}-\frac{2^{\frac{3}{4}}}{}$

[In] int(x^(5/2)*cos(b*x^2+a),x,method=_RETURNVERBOSE)

[Out] 1/2*2^(3/4)/(b^2)^(7/8)*cos(a)*Pi^(1/2)*(2/7/Pi^(1/2)*x^(3/2)*2^(1/4)*(b^2)^(7/8)/b*sin(b*x^2)+3/14/Pi^(1/2)*x^(7/2)*(b^2)^(7/8)*2^(1/4)/(b*x^2)^(5/4)*sin(b*x^2)*LommelS1(5/4,3/2,b*x^2)+3/8/Pi^(1/2)*x^(7/2)*(b^2)^(7/8)*2^(1/4)/(b*x^2)^(9/4)*(cos(b*x^2)*x^2*b-sin(b*x^2))*LommelS1(1/4,1/2,b*x^2))-1/2*2^(3/4)/b^(7/4)*sin(a)*Pi^(1/2)*(-1/8/Pi^(1/2)*x^(7/2)*b^(7/4)*2^(1/4)/(b*x^2)^(5/4)*sin(b*x^2)*LommelS1(1/4,3/2,b*x^2)-1/2/Pi^(1/2)*x^(7/2)*b^(7/4)*2^(1/4)/(b*x^2)^(9/4)*(cos(b*x^2)*x^2*b-sin(b*x^2))*LommelS1(5/4,1/2,b*x^2))

Fricas [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.58

$$\int x^{5/2}\cos\left(a+bx^2\right)\,dx=\frac{8\,bx^{\frac{3}{2}}\sin\left(bx^2+a\right)+3\,(i\,b)^{\frac{1}{4}}\left(\cos\left(a\right)-i\,\sin\left(a\right)\right)\Gamma\left(\frac{3}{4},i\,bx^2\right)+3\,(-i\,b)^{\frac{1}{4}}\left(\cos\left(a\right)+i\,\sin\left(a\right)\right)\Gamma\left(\frac{3}{4},-\right.}{16\,b^2}$$

[In] integrate(x^(5/2)*cos(b*x^2+a),x, algorithm="fricas")

[Out] 1/16*(8*b*x^(3/2)*sin(b*x^2 + a) + 3*(I*b)^(1/4)*(cos(a) - I*sin(a))*gamma(3/4, I*b*x^2) + 3*(-I*b)^(1/4)*(cos(a) + I*sin(a))*gamma(3/4, -I*b*x^2))/b^2

Sympy [F]

$$\int x^{5/2}\cos\left(a+bx^2\right)\,dx=\int x^{\frac{5}{2}}\cos\left(a+bx^2\right)dx$$

```
[In] integrate(x**(5/2)*cos(b*x**2+a),x)
[Out] Integral(x**(5/2)*cos(a + b*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int x^{5/2}\cos\left(a+bx^2\right)\,dx=\text{Exception raised: RuntimeError}$$

```
[In] integrate(x^(5/2)*cos(b*x^2+a),x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> Encountered operator mismatch in maxima-t
o-sr translation
```

Giac [F]

$$\int x^{5/2}\cos\left(a+bx^2\right)\,dx=\int x^{\frac{5}{2}}\cos\left(bx^2+a\right)\,dx$$

```
[In] integrate(x^(5/2)*cos(b*x^2+a),x, algorithm="giac")
[Out] integrate(x^(5/2)*cos(b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{5/2}\cos\left(a+bx^2\right)\,dx=\int x^{5/2}\,\cos\left(b\,x^2+a\right)\,dx$$

```
[In] int(x^(5/2)*cos(a + b*x^2),x)
[Out] int(x^(5/2)*cos(a + b*x^2), x)
```

3.24 $\int x^{3/2} \cos(a + bx^2) \, dx$

Optimal result

Integrand size = 14, antiderivative size = 111

$$\int x^{3/2} \cos(a + bx^2) \, dx = -\frac{ie^{ia}\sqrt{x}\Gamma\left(\frac{1}{4}, -ibx^2\right)}{16b\sqrt[4]{-ibx^2}} + \frac{ie^{-ia}\sqrt{x}\Gamma\left(\frac{1}{4}, ibx^2\right)}{16b\sqrt[4]{ibx^2}} + \frac{\sqrt{x}\sin(a + bx^2)}{2b}$$

```
[Out] -1/16*I*exp(I*a)*GAMMA(1/4,-I*b*x^2)*x^(1/2)/b/(-I*b*x^2)^(1/4)+1/16*I*GAMM
A(1/4,I*b*x^2)*x^(1/2)/b/exp(I*a)/(I*b*x^2)^(1/4)+1/2*sin(b*x^2+a)*x^(1/2)/
b
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3467, 3470, 2250}

$$\int x^{3/2} \cos(a + bx^2) \, dx = \frac{\sqrt{x}\sin(a + bx^2)}{2b} - \frac{ie^{ia}\sqrt{x}\Gamma\left(\frac{1}{4}, -ibx^2\right)}{16b\sqrt[4]{-ibx^2}} + \frac{ie^{-ia}\sqrt{x}\Gamma\left(\frac{1}{4}, ibx^2\right)}{16b\sqrt[4]{ibx^2}}$$

```
[In] Int[x^(3/2)*Cos[a + b*x^2],x]
```

```
[Out] ((-1/16*I)*E^(I*a)*Sqrt[x]*Gamma[1/4, (-I)*b*x^2])/(b*((-I)*b*x^2)^(1/4)) +
 ((I/16)*Sqrt[x]*Gamma[1/4, I*b*x^2])/(b*E^(I*a)*(I*b*x^2)^(1/4)) + (Sqrt[x
]*Sin[a + b*x^2])/(2*b)
```

Rule 2250

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 3467

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[e^(n
 - 1)*(e*x)^(m - n + 1)*(Sin[c + d*x^n]/(d*n)), x] - Dist[e^n*((m - n + 1)/
(d*n)), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3470

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[I/2,
 Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] - Dist[I/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\sqrt{x}\sin\left(a+bx^2\right)}{2b} - \frac{\int \frac{\sin\left(a+bx^2\right)}{\sqrt{x}}\,dx}{4b} \\
&= \frac{\sqrt{x}\sin\left(a+bx^2\right)}{2b} - \frac{i\int \frac{e^{-ia-ibx^2}}{\sqrt{x}}\,dx}{8b} + \frac{i\int \frac{e^{ia+ibx^2}}{\sqrt{x}}\,dx}{8b} \\
&= -\frac{ie^{ia}\sqrt{x}\Gamma\left(\frac{1}{4},-ibx^2\right)}{16b\sqrt[4]{-ibx^2}} + \frac{ie^{-ia}\sqrt{x}\Gamma\left(\frac{1}{4},ibx^2\right)}{16b\sqrt[4]{ibx^2}} + \frac{\sqrt{x}\sin\left(a+bx^2\right)}{2b}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00

$$
\int x^{3/2}\cos\left(a+bx^2\right)\,dx = \frac{bx^{9/2}\left(\sqrt[4]{ibx^2}\Gamma\left(\frac{1}{4},-ibx^2\right)\left(-i\cos(a)+\sin(a)\right)+\sqrt[4]{-ibx^2}\Gamma\left(\frac{1}{4},ibx^2\right)\left(i\cos(a)+\sin(a)\right)+8\sqrt[4]{b^2x^4}
$$

$$
16\left(b^2x^4\right)^{5/4}
$$

```
[In] Integrate[x^(3/2)*Cos[a + b*x^2],x]
```

```
[Out] (b*x^(9/2)*((I*b*x^2)^(1/4)*Gamma[1/4, (-I)*b*x^2]*((-I)*Cos[a] + Sin[a]) +
 ((-I)*b*x^2)^(1/4)*Gamma[1/4, I*b*x^2]*(I*Cos[a] + Sin[a]) + 8*(b^2*x^4)^(
1/4)*Sin[a + b*x^2]))/(16*(b^2*x^4)^(5/4))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.44 (sec) , antiderivative size = 290, normalized size of antiderivative = 2.61

method	result
meijerg	$\frac{2^{\frac{1}{4}} \cos(a)\sqrt{\pi}\left(\frac{2\sqrt{x}\, 2^{\frac{3}{4}} \left(b^{2}\right)^{\frac{5}{8}} \sin\left(b\, x^{2}\right)}{5\sqrt{\pi}\, b}+\frac{2\sqrt{x}\, 2^{\frac{3}{4}} \left(b^{2}\right)^{\frac{5}{8}} \left(\cos\left(b\, x^{2}\right) x^{2} b-\sin\left(b\, x^{2}\right)\right)}{5\sqrt{\pi}\, b}+\frac{x^{\frac{9}{2}} \left(b^{2}\right)^{\frac{5}{8}} 2^{\frac{3}{4}} b \sin\left(b\, x^{2}\right) s_{\frac{3}{4},\frac{3}{2}}^{(+)}\left(b\, x^{2}\right)}{10\sqrt{\pi}\left(b\, x^{2}\right)^{\frac{7}{4}}}-\frac{2x^{\frac{9}{2}} \left(b^{2}\right)^{\frac{5}{8}} 2^{\frac{3}{4}} b\left(\cos\left(\right.\right.}{5}\right.}{2\left(b^{2}\right)^{\frac{5}{8}}}$

[In] int(x^(3/2)*cos(b*x^2+a),x,method=_RETURNVERBOSE)

[Out] 1/2*2^(1/4)/(b^2)^(5/8)*cos(a)*Pi^(1/2)*(2/5/Pi^(1/2)*x^(1/2)*2^(3/4)*(b^2)^(5/8)/b*sin(b*x^2)+2/5/Pi^(1/2)*x^(1/2)*2^(3/4)*(b^2)^(5/8)/b*(cos(b*x^2)*x^2*b-sin(b*x^2))+1/10/Pi^(1/2)*x^(9/2)*(b^2)^(5/8)*2^(3/4)*b/(b*x^2)^(7/4)*sin(b*x^2)*LommelS1(3/4,3/2,b*x^2)-2/5/Pi^(1/2)*x^(9/2)*(b^2)^(5/8)*2^(3/4)*b/(b*x^2)^(11/4)*(cos(b*x^2)*x^2*b-sin(b*x^2))*LommelS1(7/4,1/2,b*x^2))-1/2*2^(1/4)/b^(5/4)*sin(a)*Pi^(1/2)*(2/9/Pi^(1/2)*x^(5/2)*2^(3/4)*b^(5/4)*sin(b*x^2)-2/9/Pi^(1/2)*x^(9/2)*b^(9/4)*2^(3/4)/(b*x^2)^(7/4)*sin(b*x^2)*LommelS1(7/4,3/2,b*x^2)-1/2/Pi^(1/2)*x^(9/2)*b^(9/4)*2^(3/4)/(b*x^2)^(11/4)*(cos(b*x^2)*x^2*b-sin(b*x^2))*LommelS1(3/4,1/2,b*x^2))

Fricas [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.56

$$\int x^{3/2}\cos\left(a+bx^2\right)\,dx=\frac{(i\,b)^{\frac{3}{4}}\left(\cos\left(a\right)-i\,\sin\left(a\right)\right)\Gamma\left(\frac{1}{4},i\,bx^2\right)+(-i\,b)^{\frac{3}{4}}\left(\cos\left(a\right)+i\,\sin\left(a\right)\right)\Gamma\left(\frac{1}{4},-i\,bx^2\right)+8\,b\sqrt{x}\sin\left(bx^2+\right.}{16\,b^2}$$

[In] integrate(x^(3/2)*cos(b*x^2+a),x, algorithm="fricas")

[Out] 1/16*((I*b)^(3/4)*(cos(a) - I*sin(a))*gamma(1/4, I*b*x^2) + (-I*b)^(3/4)*(cos(a) + I*sin(a))*gamma(1/4, -I*b*x^2) + 8*b*sqrt(x)*sin(b*x^2 + a))/b^2

Sympy [F]

$$\int x^{3/2} \cos\left(a+bx^2\right)\,dx = \int x^{\frac{3}{2}} \cos\left(a+bx^2\right)dx$$

```
[In] integrate(x**(3/2)*cos(b*x**2+a),x)
[Out] Integral(x**(3/2)*cos(a + b*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int x^{3/2} \cos\left(a+bx^2\right)\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(x^(3/2)*cos(b*x^2+a),x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> Encountered operator mismatch in maxima-t
o-sr translation
```

Giac [F]

$$\int x^{3/2} \cos\left(a+bx^2\right)\,dx = \int x^{\frac{3}{2}} \cos\left(bx^2+a\right)\,dx$$

```
[In] integrate(x^(3/2)*cos(b*x^2+a),x, algorithm="giac")
[Out] integrate(x^(3/2)*cos(b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{3/2} \cos\left(a+bx^2\right)\,dx = \int x^{3/2}\,\cos\left(b\,x^2+a\right)\,dx$$

```
[In] int(x^(3/2)*cos(a + b*x^2),x)
[Out] int(x^(3/2)*cos(a + b*x^2), x)
```

3.25 $\int \sqrt{x} \cos(a+bx^2)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 81

$$\int \sqrt{x}\cos(a+bx^2)\,dx = -\frac{e^{ia}x^{3/2}\Gamma\left(\frac{3}{4},-ibx^2\right)}{4(-ibx^2)^{3/4}} - \frac{e^{-ia}x^{3/2}\Gamma\left(\frac{3}{4},ibx^2\right)}{4(ibx^2)^{3/4}}$$

```
[Out] -1/4*exp(I*a)*x^(3/2)*GAMMA(3/4,-I*b*x^2)/(-I*b*x^2)^(3/4)-1/4*x^(3/2)*GAMM
A(3/4,I*b*x^2)/exp(I*a)/(I*b*x^2)^(3/4)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3471, 2250}

$$\int \sqrt{x}\cos(a+bx^2)\,dx = -\frac{e^{ia}x^{3/2}\Gamma\left(\frac{3}{4},-ibx^2\right)}{4(-ibx^2)^{3/4}} - \frac{e^{-ia}x^{3/2}\Gamma\left(\frac{3}{4},ibx^2\right)}{4(ibx^2)^{3/4}}$$

```
[In] Int[Sqrt[x]*Cos[a + b*x^2],x]
```

```
[Out] -1/4*(E^(I*a)*x^(3/2)*Gamma[3/4, (-I)*b*x^2])/((-I)*b*x^2)^(3/4) - (x^(3/2)
*Gamma[3/4, I*b*x^2])/(4*E^(I*a)*(I*b*x^2)^(3/4))
```

Rule 2250

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 3471

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Dist[1/2,
 Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] + Dist[1/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{1}{2}\int e^{-ia-ibx^2}\sqrt{x}\,dx+\frac{1}{2}\int e^{ia+ibx^2}\sqrt{x}\,dx \\ &= -\frac{e^{ia}x^{3/2}\Gamma\left(\frac{3}{4},-ibx^2\right)}{4\left(-ibx^2\right)^{3/4}}-\frac{e^{-ia}x^{3/2}\Gamma\left(\frac{3}{4},ibx^2\right)}{4\left(ibx^2\right)^{3/4}} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.10

$$\int \sqrt{x}\cos\left(a+bx^2\right)\,dx = -\frac{x^{3/2}\left(\left(-ibx^2\right)^{3/4}\Gamma\left(\frac{3}{4},ibx^2\right)\left(\cos(a)-i\sin(a)\right)+\left(ibx^2\right)^{3/4}\Gamma\left(\frac{3}{4},-ibx^2\right)\left(\cos(a)+i\sin(a)\right)\right)}{4\left(b^2x^4\right)^{3/4}}$$

```
[In] Integrate[Sqrt[x]*Cos[a + b*x^2],x]
[Out] -1/4*(x^(3/2)*(((-I)*b*x^2)^(3/4)*Gamma[3/4, I*b*x^2]*(Cos[a] - I*Sin[a]) +
 (I*b*x^2)^(3/4)*Gamma[3/4, (-I)*b*x^2]*(Cos[a] + I*Sin[a])))/(b^2*x^4)^(3/
4)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.43 (sec) , antiderivative size = 290, normalized size of antiderivative = 3.58

method	result
meijerg	$\frac{2^{\frac{3}{4}}\cos(a)\sqrt{\pi}\left(\frac{4\,2^{\frac{1}{4}}\left(b^2\right)^{\frac{3}{8}}\sin\left(b\,x^2\right)}{3\sqrt{\pi}\,\sqrt{x}\,b}+\frac{4\,2^{\frac{1}{4}}\left(b^2\right)^{\frac{3}{8}}\left(\cos\left(b\,x^2\right)x^2b-\sin\left(b\,x^2\right)\right)}{3\sqrt{\pi}\,\sqrt{x}\,b}-\frac{x^{\frac{7}{2}}\left(b^2\right)^{\frac{3}{8}}2^{\frac{1}{4}}b\sin\left(b\,x^2\right)s_{\frac{1}{4},\frac{3}{2}}^{(+)}\left(b\,x^2\right)}{3\sqrt{\pi}\left(b\,x^2\right)^{\frac{5}{4}}}-\frac{4x^{\frac{7}{2}}\left(b^2\right)^{\frac{3}{8}}2^{\frac{1}{4}}b\left(\cos\left(b\,x^2\right.\right.}{3\sqrt{\pi}}}{4\left(b^2\right)^{\frac{3}{8}}}$

```
[In] int(x^(1/2)*cos(b*x^2+a),x,method=_RETURNVERBOSE)
[Out] 1/4*2^(3/4)/(b^2)^(3/8)*cos(a)*Pi^(1/2)*(4/3/Pi^(1/2)/x^(1/2)*2^(1/4)*(b^2)
^(3/8)/b*sin(b*x^2)+4/3/Pi^(1/2)/x^(1/2)*2^(1/4)*(b^2)^(3/8)/b*(cos(b*x^2)*
x^2*b-sin(b*x^2))-1/3/Pi^(1/2)*x^(7/2)*(b^2)^(3/8)*2^(1/4)*b/(b*x^2)^(5/4)*
sin(b*x^2)*LommelS1(1/4,3/2,b*x^2)-4/3/Pi^(1/2)*x^(7/2)*(b^2)^(3/8)*2^(1/4)
```

```
*b/(b*x^2)^(9/4)*(cos(b*x^2)*x^2*b-sin(b*x^2))*LommelS1(5/4,1/2,b*x^2))-1/4
*2^(3/4)/b^(3/4)*sin(a)*Pi^(1/2)*(4/7/Pi^(1/2)*x^(3/2)*2^(1/4)*b^(3/4)*sin(
b*x^2)-4/7/Pi^(1/2)*x^(7/2)*b^(7/4)*2^(1/4)/(b*x^2)^(5/4)*sin(b*x^2)*Lommel
S1(5/4,3/2,b*x^2)-1/Pi^(1/2)*x^(7/2)*b^(7/4)*2^(1/4)/(b*x^2)^(9/4)*(cos(b*x
^2)*x^2*b-sin(b*x^2))*LommelS1(1/4,1/2,b*x^2))
```

Fricas [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.59

$$\int \sqrt{x}\cos\left(a+bx^2\right)\,dx = \frac{(ib)^{\frac{1}{4}}\left(i\cos(a)+\sin(a)\right)\Gamma\left(\frac{3}{4},ibx^2\right)+(-ib)^{\frac{1}{4}}\left(-i\cos(a)+\sin(a)\right)\Gamma\left(\frac{3}{4},-ibx^2\right)}{4b}$$

```
[In] integrate(x^(1/2)*cos(b*x^2+a),x, algorithm="fricas")
[Out] 1/4*((I*b)^(1/4)*(I*cos(a) + sin(a))*gamma(3/4, I*b*x^2) + (-I*b)^(1/4)*(-I
*cos(a) + sin(a))*gamma(3/4, -I*b*x^2))/b
```

Sympy [F]

$$\int \sqrt{x}\cos\left(a+bx^2\right)\,dx = \int \sqrt{x}\cos\left(a+bx^2\right)dx$$

```
[In] integrate(x**(1/2)*cos(b*x**2+a),x)
[Out] Integral(sqrt(x)*cos(a + b*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \sqrt{x}\cos\left(a+bx^2\right)\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(x^(1/2)*cos(b*x^2+a),x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> Encountered operator mismatch in maxima-t
o-sr translation
```

Giac [F]

$$\int \sqrt{x}\cos\left(a+bx^2\right)\,dx=\int \sqrt{x}\cos\left(bx^2+a\right)\,dx$$

```
[In] integrate(x^(1/2)*cos(b*x^2+a),x, algorithm="giac")
[Out] integrate(sqrt(x)*cos(b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{x}\cos\left(a+bx^2\right)\,dx=\int \sqrt{x}\,\cos\left(b\,x^2+a\right)\,dx$$

```
[In] int(x^(1/2)*cos(a + b*x^2),x)
[Out] int(x^(1/2)*cos(a + b*x^2), x)
```

3.26 $\int \frac{\cos(a+bx^2)}{\sqrt{x}} dx$

Optimal result

Integrand size = 14, antiderivative size = 81

$$\int \frac{\cos(a+bx^2)}{\sqrt{x}} dx = -\frac{e^{ia}\sqrt{x}\Gamma\left(\frac{1}{4},-ibx^2\right)}{4\sqrt[4]{-ibx^2}} - \frac{e^{-ia}\sqrt{x}\Gamma\left(\frac{1}{4},ibx^2\right)}{4\sqrt[4]{ibx^2}}$$

```
[Out] -1/4*exp(I*a)*GAMMA(1/4,-I*b*x^2)*x^(1/2)/(-I*b*x^2)^(1/4)-1/4*GAMMA(1/4,I*
b*x^2)*x^(1/2)/exp(I*a)/(I*b*x^2)^(1/4)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3471, 2250}

$$\int \frac{\cos(a+bx^2)}{\sqrt{x}} dx = -\frac{e^{ia}\sqrt{x}\Gamma\left(\frac{1}{4},-ibx^2\right)}{4\sqrt[4]{-ibx^2}} - \frac{e^{-ia}\sqrt{x}\Gamma\left(\frac{1}{4},ibx^2\right)}{4\sqrt[4]{ibx^2}}$$

```
[In] Int[Cos[a + b*x^2]/Sqrt[x],x]
```

```
[Out] -1/4*(E^(I*a)*Sqrt[x]*Gamma[1/4, (-I)*b*x^2])/((-I)*b*x^2)^(1/4) - (Sqrt[x]
*Gamma[1/4, I*b*x^2])/(4*E^(I*a)*(I*b*x^2)^(1/4))
```

Rule 2250

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 3471

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Dist[1/2,
 Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] + Dist[1/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}\int \frac{e^{-ia-ibx^2}}{\sqrt{x}}\,dx + \frac{1}{2}\int \frac{e^{ia+ibx^2}}{\sqrt{x}}\,dx\\ &= -\frac{e^{ia}\sqrt{x}\,\Gamma\left(\frac{1}{4},-ibx^2\right)}{4\sqrt[4]{-ibx^2}} - \frac{e^{-ia}\sqrt{x}\,\Gamma\left(\frac{1}{4},ibx^2\right)}{4\sqrt[4]{ibx^2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.10

$$\int \frac{\cos(a+bx^2)}{\sqrt{x}}\,dx = -\frac{\sqrt{x}\left(\sqrt[4]{-ibx^2}\,\Gamma\left(\frac{1}{4},ibx^2\right)(\cos(a)-i\sin(a))+\sqrt[4]{ibx^2}\,\Gamma\left(\frac{1}{4},-ibx^2\right)(\cos(a)+i\sin(a))\right)}{4\sqrt[4]{b^2x^4}}$$

```
[In] Integrate[Cos[a + b*x^2]/Sqrt[x],x]
```

```
[Out] -1/4*(Sqrt[x]*(((-I)*b*x^2)^(1/4)*Gamma[1/4, I*b*x^2]*(Cos[a] - I*Sin[a]) +
 (I*b*x^2)^(1/4)*Gamma[1/4, (-I)*b*x^2]*(Cos[a] + I*Sin[a])))/(b^2*x^4)^(1/
4)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.39 (sec) , antiderivative size = 338, normalized size of antiderivative = 4.17

method	result
meijerg	$\frac{\cos(a)\sqrt{\pi}\,2^{\frac{1}{4}}\left(\frac{6\,2^{\frac{3}{4}}(b^2)^{\frac{1}{8}}\left(\frac{8x^4b^2}{27}+\frac{2}{3}\right)\sin(b\,x^2)}{\sqrt{\pi}\,x^{\frac{3}{2}}\,b}+\frac{4\,2^{\frac{3}{4}}(b^2)^{\frac{1}{8}}\left(\cos(b\,x^2)x^2b-\sin(b\,x^2)\right)}{\sqrt{\pi}\,x^{\frac{3}{2}}\,b}-\frac{16x^{\frac{9}{2}}(b^2)^{\frac{1}{8}}b^2 2^{\frac{3}{4}}\sin(b\,x^2)s^{(+)}_{\frac{7}{4},\frac{3}{2}}(b\,x^2)}{9\sqrt{\pi}\,(b\,x^2)^{\frac{7}{4}}}-\frac{4x^{\frac{9}{2}}(b^2}{}\right.}{4(b^2)^{\frac{1}{8}}}$

```
[In] int(cos(b*x^2+a)/x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*cos(a)*Pi^(1/2)*2^(1/4)/(b^2)^(1/8)*(6/Pi^(1/2)/x^(3/2)*2^(3/4)*(b^2)^(
1/8)*(8/27*x^4*b^2+2/3)/b*sin(b*x^2)+4/Pi^(1/2)/x^(3/2)*2^(3/4)*(b^2)^(1/8)
/b*(cos(b*x^2)*x^2*b-sin(b*x^2))-16/9/Pi^(1/2)*x^(9/2)*(b^2)^(1/8)*b^2*2^(3
/4)/(b*x^2)^(7/4)*sin(b*x^2)*LommelS1(7/4,3/2,b*x^2)-4/Pi^(1/2)*x^(9/2)*(b^
```

```
2)^(1/8)*b^2*2^(3/4)/(b*x^2)^(11/4)*(cos(b*x^2)*x^2*b-sin(b*x^2))*LommelS1(
3/4,1/2,b*x^2))-1/4*sin(a)*Pi^(1/2)*2^(1/4)/b^(1/4)*(4/5/Pi^(1/2)*x^(1/2)*2
^(3/4)*b^(1/4)*sin(b*x^2)-16/5/Pi^(1/2)*x^(1/2)*2^(3/4)*b^(1/4)*(cos(b*x^2)
*x^2*b-sin(b*x^2))-4/5/Pi^(1/2)*x^(9/2)*b^(9/4)*2^(3/4)/(b*x^2)^(7/4)*sin(b
*x^2)*LommelS1(3/4,3/2,b*x^2)+16/5/Pi^(1/2)*x^(9/2)*b^(9/4)*2^(3/4)/(b*x^2)
^(11/4)*(cos(b*x^2)*x^2*b-sin(b*x^2))*LommelS1(7/4,1/2,b*x^2))
```

Fricas [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.59

$$\int \frac{\cos(a+bx^2)}{\sqrt{x}}\,dx = \frac{(i\,b)^{\frac{3}{4}}\,(i\,\cos(a)+\sin(a))\Gamma\left(\frac{1}{4},i\,bx^2\right)+(-i\,b)^{\frac{3}{4}}\,(-i\,\cos(a)+\sin(a))\Gamma\left(\frac{1}{4},-i\,bx^2\right)}{4\,b}$$

```
[In] integrate(cos(b*x^2+a)/x^(1/2),x, algorithm="fricas")
[Out] 1/4*((I*b)^(3/4)*(I*cos(a) + sin(a))*gamma(1/4, I*b*x^2) + (-I*b)^(3/4)*(-I
*cos(a) + sin(a))*gamma(1/4, -I*b*x^2))/b
```

Sympy [F]

$$\int \frac{\cos(a+bx^2)}{\sqrt{x}}\,dx = \int \frac{\cos(a+bx^2)}{\sqrt{x}}\,dx$$

```
[In] integrate(cos(b*x**2+a)/x**(1/2),x)
[Out] Integral(cos(a + b*x**2)/sqrt(x), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos(a+bx^2)}{\sqrt{x}}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(cos(b*x^2+a)/x^(1/2),x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> Encountered operator mismatch in maxima-t
o-sr translation
```

Giac [F]

$$\int \frac{\cos(a+bx^2)}{\sqrt{x}}\,dx = \int \frac{\cos(bx^2+a)}{\sqrt{x}}\,dx$$

```
[In] integrate(cos(b*x^2+a)/x^(1/2),x, algorithm="giac")
[Out] integrate(cos(b*x^2 + a)/sqrt(x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(a+bx^2)}{\sqrt{x}}\,dx = \int \frac{\cos(b\,x^2+a)}{\sqrt{x}}\,dx$$

```
[In] int(cos(a + b*x^2)/x^(1/2),x)
[Out] int(cos(a + b*x^2)/x^(1/2), x)
```

3.27 $\int \frac{\cos(a+bx^2)}{x^{3/2}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 98

$$\int \frac{\cos(a+bx^2)}{x^{3/2}}\,dx = -\frac{2\cos(a+bx^2)}{\sqrt{x}} - \frac{ibe^{ia}x^{3/2}\Gamma\left(\frac{3}{4},-ibx^2\right)}{(-ibx^2)^{3/4}} + \frac{ibe^{-ia}x^{3/2}\Gamma\left(\frac{3}{4},ibx^2\right)}{(ibx^2)^{3/4}}$$

```
[Out] -I*b*exp(I*a)*x^(3/2)*GAMMA(3/4,-I*b*x^2)/(-I*b*x^2)^(3/4)+I*b*x^(3/2)*GAMM
A(3/4,I*b*x^2)/exp(I*a)/(I*b*x^2)^(3/4)-2*cos(b*x^2+a)/x^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3469, 3470, 2250}

$$\int \frac{\cos(a+bx^2)}{x^{3/2}}\,dx = -\frac{2\cos(a+bx^2)}{\sqrt{x}} - \frac{ie^{ia}bx^{3/2}\Gamma\left(\frac{3}{4},-ibx^2\right)}{(-ibx^2)^{3/4}} + \frac{ie^{-ia}bx^{3/2}\Gamma\left(\frac{3}{4},ibx^2\right)}{(ibx^2)^{3/4}}$$

```
[In] Int[Cos[a + b*x^2]/x^(3/2),x]
```

```
[Out] (-2*Cos[a + b*x^2])/Sqrt[x] - (I*b*E^(I*a)*x^(3/2)*Gamma[3/4, (-I)*b*x^2])/
((-I)*b*x^2)^(3/4) + (I*b*x^(3/2)*Gamma[3/4, I*b*x^2])/(E^(I*a)*(I*b*x^2)^(
3/4))
```

Rule 2250

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 3469

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_), x_Symbol] :> Simp[(e*x)
^(m + 1)*(Cos[c + d*x^n]/(e*(m + 1))), x] + Dist[d*(n/(e^n*(m + 1))), Int[(
e*x)^(m + n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3470

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[I/2,
 Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] - Dist[I/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{2\cos(a+bx^2)}{\sqrt{x}} - (4b)\int \sqrt{x}\sin\left(a+bx^2\right)\,dx \\ &= -\frac{2\cos(a+bx^2)}{\sqrt{x}} - (2ib)\int e^{-ia-ibx^2}\sqrt{x}\,dx + (2ib)\int e^{ia+ibx^2}\sqrt{x}\,dx \\ &= -\frac{2\cos(a+bx^2)}{\sqrt{x}} - \frac{ibe^{ia}x^{3/2}\Gamma\left(\frac{3}{4},-ibx^2\right)}{(-ibx^2)^{3/4}} + \frac{ibe^{-ia}x^{3/2}\Gamma\left(\frac{3}{4},ibx^2\right)}{(ibx^2)^{3/4}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.16

$$\int \frac{\cos(a+bx^2)}{x^{3/2}}\,dx = \frac{-2(b^2x^4)^{3/4}\cos(a+bx^2) + bx^2(ibx^2)^{3/4}\Gamma\left(\frac{3}{4},-ibx^2\right)(-i\cos(a)+\sin(a)) + i(-ibx^2)^{7/}}{\sqrt{x}\,(b^2x^4)^{3/4}}$$

```
[In] Integrate[Cos[a + b*x^2]/x^(3/2),x]
```

```
[Out] (-2*(b^2*x^4)^(3/4)*Cos[a + b*x^2] + b*x^2*(I*b*x^2)^(3/4)*Gamma[3/4, (-I)*
b*x^2]*((-I)*Cos[a] + Sin[a]) + I*((-I)*b*x^2)^(7/4)*Gamma[3/4, I*b*x^2]*(I
*Cos[a] + Sin[a]))/(Sqrt[x]*(b^2*x^4)^(3/4))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.43 (sec) , antiderivative size = 338, normalized size of antiderivative = 3.45

method	result
meijerg	$\frac{\cos(a)\sqrt{\pi}\,2^{\frac{3}{4}}\left(b^{2}\right)^{\frac{1}{8}}\left(-\frac{12\,2^{\frac{1}{4}}\left(\frac{8x^{4}b^{2}}{21}+\frac{2}{3}\right)\sin\left(b\,x^{2}\right)}{\sqrt{\pi}\,x^{\frac{5}{2}}\left(b^{2}\right)^{\frac{1}{8}}b}-\frac{8\,2^{\frac{1}{4}}\left(\cos\left(b\,x^{2}\right)x^{2}b-\sin\left(b\,x^{2}\right)\right)}{\sqrt{\pi}\,x^{\frac{5}{2}}\left(b^{2}\right)^{\frac{1}{8}}b}+\frac{32x^{\frac{7}{2}}b^{2}2^{\frac{1}{4}}\sin\left(b\,x^{2}\right)s_{\frac{5}{4},\frac{3}{2}}^{(+)}\left(b\,x^{2}\right)}{7\sqrt{\pi}\left(b^{2}\right)^{\frac{1}{8}}\left(b\,x^{2}\right)^{\frac{5}{4}}}+\frac{8x^{\frac{7}{2}}b^{2}2^{\frac{1}{4}}\left(\cos\left(b\,x^{2}\right)\right.}{\sqrt{\pi}\left(b\right.}\right.}{8}$

[In] int(cos(b*x^2+a)/x^(3/2),x,method=_RETURNVERBOSE)

[Out] 1/8*cos(a)*Pi^(1/2)*2^(3/4)*(b^2)^(1/8)*(-12/Pi^(1/2)/x^(5/2)*2^(1/4)/(b^2)^(1/8)*(8/21*x^4*b^2+2/3)/b*sin(b*x^2)-8/Pi^(1/2)/x^(5/2)*2^(1/4)/(b^2)^(1/8)/b*(cos(b*x^2)*x^2*b-sin(b*x^2))+32/7/Pi^(1/2)*x^(7/2)/(b^2)^(1/8)*b^2*2^(1/4)/(b*x^2)^(5/4)*sin(b*x^2)*LommelS1(5/4,3/2,b*x^2)+8/Pi^(1/2)*x^(7/2)/(b^2)^(1/8)*b^2*2^(1/4)/(b*x^2)^(9/4)*(cos(b*x^2)*x^2*b-sin(b*x^2))*LommelS1(1/4,1/2,b*x^2))-1/8*sin(a)*Pi^(1/2)*2^(3/4)*b^(1/4)*(8/3/Pi^(1/2)/x^(1/2)*2^(1/4)/b^(1/4)*sin(b*x^2)+32/3/Pi^(1/2)/x^(1/2)*2^(1/4)/b^(1/4)*(cos(b*x^2)*x^2*b-sin(b*x^2))-8/3/Pi^(1/2)*x^(7/2)*b^(7/4)*2^(1/4)/(b*x^2)^(5/4)*sin(b*x^2)*LommelS1(1/4,3/2,b*x^2)-32/3/Pi^(1/2)*x^(7/2)*b^(7/4)*2^(1/4)/(b*x^2)^(9/4)*(cos(b*x^2)*x^2*b-sin(b*x^2))*LommelS1(5/4,1/2,b*x^2))

Fricas [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.67

$$\int \frac{\cos\left(a+bx^{2}\right)}{x^{3/2}}\,dx = \frac{\left(x\cos\left(a\right)-i\,x\sin\left(a\right)\right)\left(i\,b\right)^{\frac{1}{4}}\Gamma\left(\frac{3}{4},i\,bx^{2}\right)+\left(x\cos\left(a\right)+i\,x\sin\left(a\right)\right)\left(-i\,b\right)^{\frac{1}{4}}\Gamma\left(\frac{3}{4},-i\,bx^{2}\right)-}{x}$$

[In] integrate(cos(b*x^2+a)/x^(3/2),x, algorithm="fricas")

[Out] ((x*cos(a) - I*x*sin(a))*(I*b)^(1/4)*gamma(3/4, I*b*x^2) + (x*cos(a) + I*x*sin(a))*(-I*b)^(1/4)*gamma(3/4, -I*b*x^2) - 2*sqrt(x)*cos(b*x^2 + a))/x

Sympy [F]

$$\int \frac{\cos(a+bx^2)}{x^{3/2}}\,dx = \int \frac{\cos(a+bx^2)}{x^{\frac{3}{2}}}\,dx$$

```
[In] integrate(cos(b*x**2+a)/x**(3/2),x)
[Out] Integral(cos(a + b*x**2)/x**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos(a+bx^2)}{x^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(cos(b*x^2+a)/x^(3/2),x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> Encountered operator mismatch in maxima-t
o-sr translation
```

Giac [F]

$$\int \frac{\cos(a+bx^2)}{x^{3/2}}\,dx = \int \frac{\cos(bx^2+a)}{x^{\frac{3}{2}}}\,dx$$

```
[In] integrate(cos(b*x^2+a)/x^(3/2),x, algorithm="giac")
[Out] integrate(cos(b*x^2 + a)/x^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(a+bx^2)}{x^{3/2}}\,dx = \int \frac{\cos(b\,x^2+a)}{x^{3/2}}\,dx$$

```
[In] int(cos(a + b*x^2)/x^(3/2),x)
[Out] int(cos(a + b*x^2)/x^(3/2), x)
```

3.28 $\int \frac{\cos(a+bx^2)}{x^{5/2}} dx$

Optimal result

Integrand size = 14, antiderivative size = 104

$$\int \frac{\cos(a+bx^2)}{x^{5/2}} dx = -\frac{2\cos(a+bx^2)}{3x^{3/2}} - \frac{ibe^{ia}\sqrt{x}\Gamma\left(\frac{1}{4},-ibx^2\right)}{3\sqrt[4]{-ibx^2}} + \frac{ibe^{-ia}\sqrt{x}\Gamma\left(\frac{1}{4},ibx^2\right)}{3\sqrt[4]{ibx^2}}$$

```
[Out] -2/3*cos(b*x^2+a)/x^(3/2)-1/3*I*b*exp(I*a)*GAMMA(1/4,-I*b*x^2)*x^(1/2)/(-I*
b*x^2)^(1/4)+1/3*I*b*GAMMA(1/4,I*b*x^2)*x^(1/2)/exp(I*a)/(I*b*x^2)^(1/4)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3469, 3470, 2250}

$$\int \frac{\cos(a+bx^2)}{x^{5/2}} dx = -\frac{ie^{ia}b\sqrt{x}\Gamma\left(\frac{1}{4},-ibx^2\right)}{3\sqrt[4]{-ibx^2}} + \frac{ie^{-ia}b\sqrt{x}\Gamma\left(\frac{1}{4},ibx^2\right)}{3\sqrt[4]{ibx^2}} - \frac{2\cos(a+bx^2)}{3x^{3/2}}$$

```
[In] Int[Cos[a + b*x^2]/x^(5/2),x]
```

```
[Out] (-2*Cos[a + b*x^2])/(3*x^(3/2)) - ((I/3)*b*E^(I*a)*Sqrt[x]*Gamma[1/4, (-I)*
b*x^2])/((-I)*b*x^2)^(1/4) + ((I/3)*b*Sqrt[x]*Gamma[1/4, I*b*x^2])/(E^(I*a)
*(I*b*x^2)^(1/4))
```

Rule 2250

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 3469

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_), x_Symbol] :> Simp[(e*x)
^(m + 1)*(Cos[c + d*x^n]/(e*(m + 1))), x] + Dist[d*(n/(e^n*(m + 1))), Int[(
e*x)^(m + n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3470

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[I/2,
 Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] - Dist[I/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2\cos\left(a+bx^2\right)}{3x^{3/2}} - \frac{1}{3}(4b)\int \frac{\sin\left(a+bx^2\right)}{\sqrt{x}}\,dx \\
&= -\frac{2\cos\left(a+bx^2\right)}{3x^{3/2}} - \frac{1}{3}(2ib)\int \frac{e^{-ia-ibx^2}}{\sqrt{x}}\,dx + \frac{1}{3}(2ib)\int \frac{e^{ia+ibx^2}}{\sqrt{x}}\,dx \\
&= -\frac{2\cos\left(a+bx^2\right)}{3x^{3/2}} - \frac{ibe^{ia}\sqrt{x}\Gamma\left(\frac{1}{4},-ibx^2\right)}{3\sqrt[4]{-ibx^2}} + \frac{ibe^{-ia}\sqrt{x}\Gamma\left(\frac{1}{4},ibx^2\right)}{3\sqrt[4]{ibx^2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.12

$$\int \frac{\cos\left(a+bx^2\right)}{x^{5/2}}\,dx = \frac{-2\sqrt[4]{b^2x^4}\cos\left(a+bx^2\right) + bx^2\sqrt[4]{ibx^2}\Gamma\left(\frac{1}{4},-ibx^2\right)\left(-i\cos(a)+\sin(a)\right) + i\left(-ibx^2\right)^{5/4}\Gamma\left(\frac{1}{4}\right.}{3x^{3/2}\sqrt[4]{b^2x^4}}$$

```
[In] Integrate[Cos[a + b*x^2]/x^(5/2),x]
```

```
[Out] (-2*(b^2*x^4)^(1/4)*Cos[a + b*x^2] + b*x^2*(I*b*x^2)^(1/4)*Gamma[1/4, (-I)*
b*x^2]*((-I)*Cos[a] + Sin[a]) + I*((-I)*b*x^2)^(5/4)*Gamma[1/4, I*b*x^2]*(I
*Cos[a] + Sin[a]))/(3*x^(3/2)*(b^2*x^4)^(1/4))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.46 (sec) , antiderivative size = 358, normalized size of antiderivative = 3.44

method	result
meijerg	$\frac{\cos(a)\sqrt{\pi}\,2^{\frac{1}{4}}(b^2)^{\frac{3}{8}}\left(-\frac{4\,2^{\frac{3}{4}}\left(\frac{8x^4b^2}{15}+\frac{2}{3}\right)\sin(b\,x^2)}{\sqrt{\pi}\,x^{\frac{7}{2}}(b^2)^{\frac{3}{8}}b}-\frac{8\,2^{\frac{3}{4}}(-16x^4b^2+5)(\cos(b\,x^2)x^2b-\sin(b\,x^2))}{15\sqrt{\pi}\,x^{\frac{7}{2}}(b^2)^{\frac{3}{8}}b}+\frac{32x^{\frac{9}{2}}2^{\frac{3}{4}}b^3\sin(b\,x^2)s_{\frac{3}{4},\frac{3}{2}}^{(+)}(b\,x^2)}{15\sqrt{\pi}(b^2)^{\frac{3}{8}}(b\,x^2)^{\frac{7}{4}}}-\frac{128x^{\frac{9}{2}}2\cdots}{\cdots}\cdots\right.}{8}$

[In] int(cos(b*x^2+a)/x^(5/2),x,method=_RETURNVERBOSE)

[Out] 1/8*cos(a)*Pi^(1/2)*2^(1/4)*(b^2)^(3/8)*(-4/Pi^(1/2)/x^(7/2)*2^(3/4)/(b^2)^
(3/8)*(8/15*x^4*b^2+2/3)/b*sin(b*x^2)-8/15/Pi^(1/2)/x^(7/2)*2^(3/4)/(b^2)^(
3/8)/b*(-16*b^2*x^4+5)*(cos(b*x^2)*x^2*b-sin(b*x^2))+32/15/Pi^(1/2)*x^(9/2)
/(b^2)^(3/8)*2^(3/4)*b^3/(b*x^2)^(7/4)*sin(b*x^2)*LommelS1(3/4,3/2,b*x^2)-1
28/15/Pi^(1/2)*x^(9/2)/(b^2)^(3/8)*2^(3/4)*b^3/(b*x^2)^(11/4)*(cos(b*x^2)*x
^2*b-sin(b*x^2))*LommelS1(7/4,1/2,b*x^2))-1/8*sin(a)*Pi^(1/2)*2^(1/4)*b^(3/
4)*(12/Pi^(1/2)/x^(3/2)*2^(3/4)/b^(3/4)*(32/81*x^4*b^2+2/3)*sin(b*x^2)+32/3
/Pi^(1/2)/x^(3/2)*2^(3/4)/b^(3/4)*(cos(b*x^2)*x^2*b-sin(b*x^2))-128/27/Pi^(
1/2)*x^(9/2)*b^(9/4)*2^(3/4)/(b*x^2)^(7/4)*sin(b*x^2)*LommelS1(7/4,3/2,b*x^
2)-32/3/Pi^(1/2)*x^(9/2)*b^(9/4)*2^(3/4)/(b*x^2)^(11/4)*(cos(b*x^2)*x^2*b-s
in(b*x^2))*LommelS1(3/4,1/2,b*x^2))

Fricas [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.72

$$\int \frac{\cos(a+bx^2)}{x^{5/2}}\,dx = \frac{(x^2\cos(a)-i\,x^2\sin(a))(i\,b)^{\frac{3}{4}}\,\Gamma\left(\frac{1}{4},i\,bx^2\right)+(x^2\cos(a)+i\,x^2\sin(a))(-i\,b)^{\frac{3}{4}}\,\Gamma\left(\frac{1}{4},-i\,bx^2\cdots\right.}{3\,x^2}$$

[In] integrate(cos(b*x^2+a)/x^(5/2),x, algorithm="fricas")

[Out] 1/3*((x^2*cos(a) - I*x^2*sin(a))*(I*b)^(3/4)*gamma(1/4, I*b*x^2) + (x^2*cos
(a) + I*x^2*sin(a))*(-I*b)^(3/4)*gamma(1/4, -I*b*x^2) - 2*sqrt(x)*cos(b*x^2
 + a))/x^2

Sympy [F]

$$\int \frac{\cos(a+bx^2)}{x^{5/2}}\,dx = \int \frac{\cos(a+bx^2)}{x^{\frac{5}{2}}}\,dx$$

[In] integrate(cos(b*x**2+a)/x**(5/2),x)

[Out] Integral(cos(a + b*x**2)/x**(5/2), x)

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos(a+bx^2)}{x^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(cos(b*x^2+a)/x^(5/2),x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> Encountered operator mismatch in maxima-t
o-sr translation
```

Giac [F]

$$\int \frac{\cos(a+bx^2)}{x^{5/2}}\,dx = \int \frac{\cos(bx^2+a)}{x^{\frac{5}{2}}}\,dx$$

```
[In] integrate(cos(b*x^2+a)/x^(5/2),x, algorithm="giac")
[Out] integrate(cos(b*x^2 + a)/x^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(a+bx^2)}{x^{5/2}}\,dx = \int \frac{\cos(b\,x^2+a)}{x^{5/2}}\,dx$$

```
[In] int(cos(a + b*x^2)/x^(5/2),x)
[Out] int(cos(a + b*x^2)/x^(5/2), x)
```

3.29 $\int x^{5/2}\cos^2\left(a+bx^2\right)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 132

$$\int x^{5/2}\cos^2\left(a+bx^2\right)\,dx=\frac{x^{7/2}}{7}-\frac{3ie^{2ia}x^{3/2}\Gamma\left(\frac{3}{4},-2ibx^2\right)}{64\ 2^{3/4}b\left(-ibx^2\right)^{3/4}}+\frac{3ie^{-2ia}x^{3/2}\Gamma\left(\frac{3}{4},2ibx^2\right)}{64\ 2^{3/4}b\left(ibx^2\right)^{3/4}}+\frac{x^{3/2}\sin\left(2(a+bx^2)\right)}{8b}$$

```
[Out] 1/7*x^(7/2)-3/128*I*exp(2*I*a)*x^(3/2)*GAMMA(3/4,-2*I*b*x^2)*2^(1/4)/b/(-I*
b*x^2)^(3/4)+3/128*I*x^(3/2)*GAMMA(3/4,2*I*b*x^2)*2^(1/4)/b/exp(2*I*a)/(I*b
*x^2)^(3/4)+1/8*x^(3/2)*sin(2*b*x^2+2*a)/b
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {3483, 3485, 3467, 3470, 2250}

$$\int x^{5/2}\cos^2\left(a+bx^2\right)\,dx=\frac{x^{3/2}\sin\left(2(a+bx^2)\right)}{8b}-\frac{3ie^{2ia}x^{3/2}\Gamma\left(\frac{3}{4},-2ibx^2\right)}{64\ 2^{3/4}b\left(-ibx^2\right)^{3/4}}+\frac{3ie^{-2ia}x^{3/2}\Gamma\left(\frac{3}{4},2ibx^2\right)}{64\ 2^{3/4}b\left(ibx^2\right)^{3/4}}+\frac{x^{7/2}}{7}$$

```
[In] Int[x^(5/2)*Cos[a + b*x^2]^2,x]
```

```
[Out] x^(7/2)/7 - (((3*I)/64)*E^((2*I)*a)*x^(3/2)*Gamma[3/4, (-2*I)*b*x^2])/(2^(3
/4)*b*((-I)*b*x^2)^(3/4)) + (((3*I)/64)*x^(3/2)*Gamma[3/4, (2*I)*b*x^2])/(2
^(3/4)*b*E^((2*I)*a)*(I*b*x^2)^(3/4)) + (x^(3/2)*Sin[2*(a + b*x^2)])/(8*b)
```

Rule 2250

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 3467

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[e^(n
 - 1)*(e*x)^(m - n + 1)*(Sin[c + d*x^n]/(d*n)), x] - Dist[e^n*((m - n + 1)/
(d*n)), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3470

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[I/2,
 Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] - Dist[I/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3483

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*((e_.)*(x_))^(m_), x
_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m + 1) - 1
)*(a + b*Cos[c + d*(x^(k*n)/e^n)])^p, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e}, x] && IntegerQ[p] && IGtQ[n, 0] && FractionQ[m]
```

Rule 3485

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_)*((e_.)*(x_))^(m_.), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Cos[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 2\text{Subst}\left(\int x^6 \cos^2\left(a+bx^4\right)\,dx, x, \sqrt{x}\right)\\
&= 2\text{Subst}\left(\int \left(\frac{x^6}{2}+\frac{1}{2}x^6\cos\left(2a+2bx^4\right)\right)\,dx, x, \sqrt{x}\right)\\
&= \frac{x^{7/2}}{7}+\text{Subst}\left(\int x^6\cos\left(2a+2bx^4\right)\,dx, x, \sqrt{x}\right)\\
&= \frac{x^{7/2}}{7}+\frac{x^{3/2}\sin\left(2(a+bx^2)\right)}{8b}-\frac{3\text{Subst}\left(\int x^2\sin\left(2a+2bx^4\right)\,dx, x, \sqrt{x}\right)}{8b}
\end{aligned}$$

$$= \frac{x^{7/2}}{7} + \frac{x^{3/2}\sin(2(a+bx^2))}{8b} - \frac{(3i)\text{Subst}\left(\int e^{-2ia-2ibx^4}x^2\,dx, x, \sqrt{x}\right)}{16b} + \frac{(3i)\text{Subst}\left(\int e^{2ia+2ibx^4}x^2\,dx, x, \sqrt{x}\right)}{16b}$$

$$= \frac{x^{7/2}}{7} - \frac{3ie^{2ia}x^{3/2}\Gamma\left(\frac{3}{4}, -2ibx^2\right)}{64\,2^{3/4}b\left(-ibx^2\right)^{3/4}} + \frac{3ie^{-2ia}x^{3/2}\Gamma\left(\frac{3}{4}, 2ibx^2\right)}{64\,2^{3/4}b\left(ibx^2\right)^{3/4}} + \frac{x^{3/2}\sin(2(a+bx^2))}{8b}$$

Mathematica [A] (verified)

Time = 0.74 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.08

$$\int x^{5/2}\cos^2\left(a+bx^2\right)\,dx = \frac{bx^{11/2}\left(21\sqrt[4]{2}(ibx^2)^{3/4}\Gamma\left(\frac{3}{4}, -2ibx^2\right)(-i\cos(2a)+\sin(2a)) + 21\sqrt[4]{2}(-ibx^2)^{3/4}\Gamma\left(\frac{3}{4}, 2ibx^2\right)(i\cos(2a\right.}{896\left(b^2x^4\right)^{7/4}}$$

```
[In] Integrate[x^(5/2)*Cos[a + b*x^2]^2,x]
```

```
[Out] (b*x^(11/2)*(21*2^(1/4)*(I*b*x^2)^(3/4)*Gamma[3/4, (-2*I)*b*x^2]*((-I)*Cos[
2*a] + Sin[2*a]) + 21*2^(1/4)*((-I)*b*x^2)^(3/4)*Gamma[3/4, (2*I)*b*x^2]*(I
*Cos[2*a] + Sin[2*a]) + 16*(b^2*x^4)^(3/4)*(8*b*x^2 + 7*Sin[2*(a + b*x^2)])
))/(896*(b^2*x^4)^(7/4))
```

Maple [F]

$$\int x^{\frac{5}{2}}\left(\cos^2\left(b\,x^2+a\right)\right)dx$$

```
[In] int(x^(5/2)*cos(b*x^2+a)^2,x)
```

```
[Out] int(x^(5/2)*cos(b*x^2+a)^2,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.11 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.70

$$\int x^{5/2}\cos^2\left(a+bx^2\right)\,dx = \frac{21\,(2i\,b)^{\frac{1}{4}}\left(\cos(2\,a) - i\,\sin(2\,a)\right)\Gamma\left(\frac{3}{4}, 2i\,bx^2\right) + 21\,(-2i\,b)^{\frac{1}{4}}\left(\cos(2\,a) + i\,\sin(2\,a)\right)\Gamma\left(\frac{3}{4}, -2i\,bx^2\right) +}{896\,b^2}$$

```
[In] integrate(x^(5/2)*cos(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/896*(21*(2*I*b)^(1/4)*(cos(2*a) - I*sin(2*a))*gamma(3/4, 2*I*b*x^2) + 21*
(-2*I*b)^(1/4)*(cos(2*a) + I*sin(2*a))*gamma(3/4, -2*I*b*x^2) + 32*(4*b^2*x
^3 + 7*b*x*cos(b*x^2 + a)*sin(b*x^2 + a))*sqrt(x))/b^2
```

Sympy [F]

$$\int x^{5/2}\cos^2\left(a+bx^2\right)\,dx=\int x^{\frac{5}{2}}\cos^2\left(a+bx^2\right)dx$$

```
[In] integrate(x**(5/2)*cos(b*x**2+a)**2,x)
```

```
[Out] Integral(x**(5/2)*cos(a + b*x**2)**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int x^{5/2}\cos^2\left(a+bx^2\right)\,dx=\text{Exception raised: RuntimeError}$$

```
[In] integrate(x^(5/2)*cos(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> Encountered operator mismatch in maxima-t
o-sr translation
```

Giac [F]

$$\int x^{5/2}\cos^2\left(a+bx^2\right)\,dx=\int x^{\frac{5}{2}}\cos\left(bx^2+a\right)^2\,dx$$

```
[In] integrate(x^(5/2)*cos(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^(5/2)*cos(b*x^2 + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{5/2}\cos^2\left(a+bx^2\right)\,dx=\int x^{5/2}\cos\left(b\,x^2+a\right)^2\,dx$$

```
[In] int(x^(5/2)*cos(a + b*x^2)^2,x)
```

```
[Out] int(x^(5/2)*cos(a + b*x^2)^2, x)
```

3.30 $\int x^{3/2} \cos^2(a+bx^2)\, dx$

Optimal result

Integrand size = 16, antiderivative size = 132

$$\int x^{3/2} \cos^2\left(a+bx^2\right)\, dx = \frac{x^{5/2}}{5} - \frac{ie^{2ia}\sqrt{x}\Gamma\left(\frac{1}{4},-2ibx^2\right)}{64\sqrt[4]{2}b\sqrt[4]{-ibx^2}} + \frac{ie^{-2ia}\sqrt{x}\Gamma\left(\frac{1}{4},2ibx^2\right)}{64\sqrt[4]{2}b\sqrt[4]{ibx^2}} + \frac{\sqrt{x}\sin\left(2(a+bx^2)\right)}{8b}$$

```
[Out] 1/5*x^(5/2)-1/128*I*exp(2*I*a)*GAMMA(1/4,-2*I*b*x^2)*x^(1/2)*2^(3/4)/b/(-I*
b*x^2)^(1/4)+1/128*I*GAMMA(1/4,2*I*b*x^2)*x^(1/2)*2^(3/4)/b/exp(2*I*a)/(I*b
*x^2)^(1/4)+1/8*sin(2*b*x^2+2*a)*x^(1/2)/b
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {3483, 3485, 3467, 3436, 2239}

$$\int x^{3/2} \cos^2\left(a+bx^2\right)\, dx = \frac{\sqrt{x}\sin\left(2(a+bx^2)\right)}{8b} - \frac{ie^{2ia}\sqrt{x}\Gamma\left(\frac{1}{4},-2ibx^2\right)}{64\sqrt[4]{2}b\sqrt[4]{-ibx^2}} + \frac{ie^{-2ia}\sqrt{x}\Gamma\left(\frac{1}{4},2ibx^2\right)}{64\sqrt[4]{2}b\sqrt[4]{ibx^2}} + \frac{x^{5/2}}{5}$$

```
[In] Int[x^(3/2)*Cos[a + b*x^2]^2,x]
```

```
[Out] x^(5/2)/5 - ((I/64)*E^((2*I)*a)*Sqrt[x]*Gamma[1/4, (-2*I)*b*x^2])/(2^(1/4)*
b*((-I)*b*x^2)^(1/4)) + ((I/64)*Sqrt[x]*Gamma[1/4, (2*I)*b*x^2])/(2^(1/4)*b
*E^((2*I)*a)*(I*b*x^2)^(1/4)) + (Sqrt[x]*Sin[2*(a + b*x^2)])/(8*b)
```

Rule 2239

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> Simp[(-F^a
)*(c + d*x)*(Gamma[1/n, (-b)*(c + d*x)^n*Log[F]]/(d*n*((-b)*(c + d*x)^n*Log
[F])^(1/n))), x] /; FreeQ[{F, a, b, c, d, n}, x] &&  !IntegerQ[2/n]
```

Rule 3436

```
Int[Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Dist[I/2, In
t[E^((-c)*I - d*I*(e + f*x)^n), x], x] - Dist[I/2, Int[E^(c*I + d*I*(e + f*
x)^n), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ[n, 2]
```

Rule 3467

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[e^(n
 - 1)*(e*x)^(m - n + 1)*(Sin[c + d*x^n]/(d*n)), x] - Dist[e^n*((m - n + 1)/
(d*n)), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3483

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*((e_.)*(x_))^(m_), x
_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m + 1) - 1
)*(a + b*Cos[c + d*(x^(k*n)/e^n)])^p, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e}, x] && IntegerQ[p] && IGtQ[n, 0] && FractionQ[m]
```

Rule 3485

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_)*((e_.)*(x_))^(m_.), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Cos[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 2\text{Subst}\left(\int x^4\cos^2\left(a+bx^4\right)\,dx,x,\sqrt{x}\right)\\
&= 2\text{Subst}\left(\int\left(\frac{x^4}{2}+\frac{1}{2}x^4\cos\left(2a+2bx^4\right)\right)\,dx,x,\sqrt{x}\right)\\
&= \frac{x^{5/2}}{5}+\text{Subst}\left(\int x^4\cos\left(2a+2bx^4\right)\,dx,x,\sqrt{x}\right)\\
&= \frac{x^{5/2}}{5}+\frac{\sqrt{x}\sin\left(2(a+bx^2)\right)}{8b}-\frac{\text{Subst}\left(\int\sin\left(2a+2bx^4\right)\,dx,x,\sqrt{x}\right)}{8b}\\
&= \frac{x^{5/2}}{5}+\frac{\sqrt{x}\sin\left(2(a+bx^2)\right)}{8b}-\frac{i\text{Subst}\left(\int e^{-2ia-2ibx^4}\,dx,x,\sqrt{x}\right)}{16b}\\
&\quad+\frac{i\text{Subst}\left(\int e^{2ia+2ibx^4}\,dx,x,\sqrt{x}\right)}{16b}
\end{aligned}$$

$$= \frac{x^{5/2}}{5} - \frac{ie^{2ia}\sqrt{x}\Gamma\left(\frac{1}{4}, -2ibx^2\right)}{64\sqrt[4]{2}b\sqrt[4]{-ibx^2}} + \frac{ie^{-2ia}\sqrt{x}\Gamma\left(\frac{1}{4}, 2ibx^2\right)}{64\sqrt[4]{2}b\sqrt[4]{ibx^2}} + \frac{\sqrt{x}\sin\left(2(a+bx^2)\right)}{8b}$$

Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.08

$$\int x^{3/2}\cos^2\left(a+bx^2\right)\,dx = \frac{bx^{9/2}\Big(5\ 2^{3/4}\sqrt[4]{ibx^2}\Gamma\left(\frac{1}{4}, -2ibx^2\right)\left(-i\cos(2a)+\sin(2a)\right) + 5\ 2^{3/4}\sqrt[4]{-ibx^2}\Gamma\left(\frac{1}{4}, 2ibx^2\right)\left(i\cos(2a)+\mathrm{si}\right.}{640\left(b^2x^4\right)^{5/4}}$$

```
[In] Integrate[x^(3/2)*Cos[a + b*x^2]^2,x]
```

```
[Out] (b*x^(9/2)*(5*2^(3/4)*(I*b*x^2)^(1/4)*Gamma[1/4, (-2*I)*b*x^2]*((-I)*Cos[2*
a] + Sin[2*a]) + 5*2^(3/4)*((-I)*b*x^2)^(1/4)*Gamma[1/4, (2*I)*b*x^2]*(I*Co
s[2*a] + Sin[2*a]) + 16*(b^2*x^4)^(1/4)*(8*b*x^2 + 5*Sin[2*(a + b*x^2)])))/
(640*(b^2*x^4)^(5/4))
```

Maple [F]

$$\int x^{\frac{3}{2}}\left(\cos^2\left(b\,x^2+a\right)\right)dx$$

```
[In] int(x^(3/2)*cos(b*x^2+a)^2,x)
```

```
[Out] int(x^(3/2)*cos(b*x^2+a)^2,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.10 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.69

$$\int x^{3/2}\cos^2\left(a+bx^2\right)\,dx = \frac{5\left(2i\,b\right)^{\frac{3}{4}}\left(\cos\left(2\,a\right)-i\,\sin\left(2\,a\right)\right)\Gamma\left(\frac{1}{4}, 2i\,bx^2\right) + 5\left(-2i\,b\right)^{\frac{3}{4}}\left(\cos\left(2\,a\right)+i\,\sin\left(2\,a\right)\right)\Gamma\left(\frac{1}{4}, -2i\,bx^2\right) + 3\!\!\!}{640\,b^2}$$

```
[In] integrate(x^(3/2)*cos(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/640*(5*(2*I*b)^(3/4)*(cos(2*a) - I*sin(2*a))*gamma(1/4, 2*I*b*x^2) + 5*(-
2*I*b)^(3/4)*(cos(2*a) + I*sin(2*a))*gamma(1/4, -2*I*b*x^2) + 32*(4*b^2*x^2
 + 5*b*cos(b*x^2 + a)*sin(b*x^2 + a))*sqrt(x))/b^2
```

Sympy [F]

$$\int x^{3/2}\cos^2\left(a+bx^2\right)\,dx=\int x^{\frac{3}{2}}\cos^2\left(a+bx^2\right)dx$$

```
[In] integrate(x**(3/2)*cos(b*x**2+a)**2,x)
[Out] Integral(x**(3/2)*cos(a + b*x**2)**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int x^{3/2}\cos^2\left(a+bx^2\right)\,dx=\text{Exception raised: RuntimeError}$$

```
[In] integrate(x^(3/2)*cos(b*x^2+a)^2,x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> Encountered operator mismatch in maxima-t
o-sr translation
```

Giac [F]

$$\int x^{3/2}\cos^2\left(a+bx^2\right)\,dx=\int x^{\frac{3}{2}}\cos\left(bx^2+a\right)^2\,dx$$

```
[In] integrate(x^(3/2)*cos(b*x^2+a)^2,x, algorithm="giac")
[Out] integrate(x^(3/2)*cos(b*x^2 + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{3/2}\cos^2\left(a+bx^2\right)\,dx=\int x^{3/2}\cos\left(b\,x^2+a\right)^2dx$$

```
[In] int(x^(3/2)*cos(a + b*x^2)^2,x)
[Out] int(x^(3/2)*cos(a + b*x^2)^2, x)
```

3.31 $\int \sqrt{x}\cos^2\left(a+bx^2\right)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 100

$$\int \sqrt{x}\cos^2\left(a+bx^2\right)\,dx=\frac{x^{3/2}}{3}-\frac{e^{2ia}x^{3/2}\Gamma\left(\frac{3}{4},-2ibx^2\right)}{8\ 2^{3/4}\left(-ibx^2\right)^{3/4}}-\frac{e^{-2ia}x^{3/2}\Gamma\left(\frac{3}{4},2ibx^2\right)}{8\ 2^{3/4}\left(ibx^2\right)^{3/4}}$$

```
[Out] 1/3*x^(3/2)-1/16*exp(2*I*a)*x^(3/2)*GAMMA(3/4,-2*I*b*x^2)*2^(1/4)/(-I*b*x^2
)^(3/4)-1/16*x^(3/2)*GAMMA(3/4,2*I*b*x^2)*2^(1/4)/exp(2*I*a)/(I*b*x^2)^(3/4
)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3483, 3485, 3471, 2250}

$$\int \sqrt{x}\cos^2\left(a+bx^2\right)\,dx=-\frac{e^{2ia}x^{3/2}\Gamma\left(\frac{3}{4},-2ibx^2\right)}{8\ 2^{3/4}\left(-ibx^2\right)^{3/4}}-\frac{e^{-2ia}x^{3/2}\Gamma\left(\frac{3}{4},2ibx^2\right)}{8\ 2^{3/4}\left(ibx^2\right)^{3/4}}+\frac{x^{3/2}}{3}$$

```
[In] Int[Sqrt[x]*Cos[a + b*x^2]^2,x]
```

```
[Out] x^(3/2)/3 - (E^((2*I)*a)*x^(3/2)*Gamma[3/4, (-2*I)*b*x^2])/(8*2^(3/4)*((-I)
*b*x^2)^(3/4)) - (x^(3/2)*Gamma[3/4, (2*I)*b*x^2])/(8*2^(3/4)*E^((2*I)*a)*(
I*b*x^2)^(3/4))
```

Rule 2250

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 3471

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Dist[1/2,
 Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] + Dist[1/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3483

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*((e_.)*(x_))^(m_), x
_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m + 1) - 1
)*(a + b*Cos[c + d*(x^(k*n)/e^n)])^p, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e}, x] && IntegerQ[p] && IGtQ[n, 0] && FractionQ[m]
```

Rule 3485

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_)*((e_.)*(x_))^(m_.), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Cos[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= 2\text{Subst}\left(\int x^2\cos^2\left(a+bx^4\right)\,dx,x,\sqrt{x}\right)\\ &= 2\text{Subst}\left(\int\left(\frac{x^2}{2}+\frac{1}{2}x^2\cos\left(2a+2bx^4\right)\right)dx,x,\sqrt{x}\right)\\ &= \frac{x^{3/2}}{3}+\text{Subst}\left(\int x^2\cos\left(2a+2bx^4\right)\,dx,x,\sqrt{x}\right)\\ &= \frac{x^{3/2}}{3}+\frac{1}{2}\text{Subst}\left(\int e^{-2ia-2ibx^4}x^2\,dx,x,\sqrt{x}\right)+\frac{1}{2}\text{Subst}\left(\int e^{2ia+2ibx^4}x^2\,dx,x,\sqrt{x}\right)\\ &= \frac{x^{3/2}}{3}-\frac{e^{2ia}x^{3/2}\Gamma\left(\frac{3}{4},-2ibx^2\right)}{8\;2^{3/4}\left(-ibx^2\right)^{3/4}}-\frac{e^{-2ia}x^{3/2}\Gamma\left(\frac{3}{4},2ibx^2\right)}{8\;2^{3/4}\left(ibx^2\right)^{3/4}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.99

$$\int\sqrt{x}\cos^2\left(a+bx^2\right)\,dx=\frac{1}{48}x^{3/2}\left(16-\frac{3\sqrt[4]{2}\Gamma\left(\frac{3}{4},2ibx^2\right)\left(\cos(2a)-i\sin(2a)\right)}{\left(ibx^2\right)^{3/4}}-\frac{3\sqrt[4]{2}\Gamma\left(\frac{3}{4},-2ibx^2\right)\left(\cos(2a)+i\sin(2a)\right)}{\left(-ibx^2\right)^{3/4}}\right)$$

```
[In] Integrate[Sqrt[x]*Cos[a + b*x^2]^2,x]
```

```
[Out] (x^(3/2)*(16 - (3*2^(1/4)*Gamma[3/4, (2*I)*b*x^2]*(Cos[2*a] - I*Sin[2*a]))/
(I*b*x^2)^(3/4) - (3*2^(1/4)*Gamma[3/4, (-2*I)*b*x^2]*(Cos[2*a] + I*Sin[2*a
]))/((-I)*b*x^2)^(3/4)))/48
```

Maple [F]

$$\int \sqrt{x}\left(\cos^2\left(b\,x^2+a\right)\right)dx$$

```
[In] int(x^(1/2)*cos(b*x^2+a)^2,x)
```

```
[Out] int(x^(1/2)*cos(b*x^2+a)^2,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.10 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.68

$$\int \sqrt{x}\cos^2\left(a+bx^2\right)\,dx = \frac{16\,bx^{\frac{3}{2}} - 3\,(2i\,b)^{\frac{1}{4}}\,(-i\,\cos\left(2\,a\right) - \sin\left(2\,a\right))\Gamma\left(\frac{3}{4}, 2i\,bx^2\right) - 3\,(-2i\,b)^{\frac{1}{4}}\,(i\,\cos\left(2\,a\right) - \sin\left(2\,a\right))\Gamma\left(\frac{3}{4}, -2i\,bx^2\right)}{48\,b}$$

```
[In] integrate(x^(1/2)*cos(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/48*(16*b*x^(3/2) - 3*(2*I*b)^(1/4)*(-I*cos(2*a) - sin(2*a))*gamma(3/4, 2*
I*b*x^2) - 3*(-2*I*b)^(1/4)*(I*cos(2*a) - sin(2*a))*gamma(3/4, -2*I*b*x^2))
/b
```

Sympy [F]

$$\int \sqrt{x}\cos^2\left(a+bx^2\right)\,dx = \int \sqrt{x}\cos^2\left(a+bx^2\right)dx$$

```
[In] integrate(x**(1/2)*cos(b*x**2+a)**2,x)
```

```
[Out] Integral(sqrt(x)*cos(a + b*x**2)**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \sqrt{x}\cos^2\left(a+bx^2\right)\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(x^(1/2)*cos(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> Encountered operator mismatch in maxima-t
o-sr translation
```

Giac [F]

$$\int \sqrt{x}\cos^2\left(a+bx^2\right)\,dx = \int \sqrt{x}\cos\left(bx^2+a\right)^2\,dx$$

```
[In] integrate(x^(1/2)*cos(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(x)*cos(b*x^2 + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{x}\cos^2\left(a+bx^2\right)\,dx = \int \sqrt{x}\cos\left(b\,x^2+a\right)^2\,dx$$

```
[In] int(x^(1/2)*cos(a + b*x^2)^2,x)
```

```
[Out] int(x^(1/2)*cos(a + b*x^2)^2, x)
```

3.32 $\int \frac{\cos^2(a+bx^2)}{\sqrt{x}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 96

$$\int \frac{\cos^2(a+bx^2)}{\sqrt{x}}\,dx = \sqrt{x} - \frac{e^{2ia}\sqrt{x}\Gamma\left(\frac{1}{4},-2ibx^2\right)}{8\sqrt[4]{2}\sqrt[4]{-ibx^2}} - \frac{e^{-2ia}\sqrt{x}\Gamma\left(\frac{1}{4},2ibx^2\right)}{8\sqrt[4]{2}\sqrt[4]{ibx^2}}$$

```
[Out] x^(1/2)-1/16*exp(2*I*a)*GAMMA(1/4,-2*I*b*x^2)*x^(1/2)*2^(3/4)/(-I*b*x^2)^(1
/4)-1/16*GAMMA(1/4,2*I*b*x^2)*x^(1/2)*2^(3/4)/exp(2*I*a)/(I*b*x^2)^(1/4)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3483, 3439, 3437, 2239}

$$\int \frac{\cos^2(a+bx^2)}{\sqrt{x}}\,dx = -\frac{e^{2ia}\sqrt{x}\Gamma\left(\frac{1}{4},-2ibx^2\right)}{8\sqrt[4]{2}\sqrt[4]{-ibx^2}} - \frac{e^{-2ia}\sqrt{x}\Gamma\left(\frac{1}{4},2ibx^2\right)}{8\sqrt[4]{2}\sqrt[4]{ibx^2}} + \sqrt{x}$$

```
[In] Int[Cos[a + b*x^2]^2/Sqrt[x],x]
```

```
[Out] Sqrt[x] - (E^((2*I)*a)*Sqrt[x]*Gamma[1/4, (-2*I)*b*x^2])/(8*2^(1/4)*((-I)*b
*x^2)^(1/4)) - (Sqrt[x]*Gamma[1/4, (2*I)*b*x^2])/(8*2^(1/4)*E^((2*I)*a)*(I*
b*x^2)^(1/4))
```

Rule 2239

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> Simp[(-F^a
)*(c + d*x)*(Gamma[1/n, (-b)*(c + d*x)^n*Log[F]]/(d*n*((-b)*(c + d*x)^n*Log
[F])^(1/n))), x] /; FreeQ[{F, a, b, c, d, n}, x] && !IntegerQ[2/n]
```

Rule 3437

```
Int[Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Dist[1/2, In
t[E^((-c)*I - d*I*(e + f*x)^n), x], x] + Dist[1/2, Int[E^(c*I + d*I*(e + f*
x)^n), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ[n, 2]
```

Rule 3439

```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_), x_Sy
mbol] :> Int[ExpandTrigReduce[(a + b*Cos[c + d*(e + f*x)^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 1] && IGtQ[n, 1]
```

Rule 3483

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*((e_.)*(x_))^(m_), x
_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m + 1) - 1
)*(a + b*Cos[c + d*(x^(k*n)/e^n)])^p, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e}, x] && IntegerQ[p] && IGtQ[n, 0] && FractionQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 2\text{Subst}\left(\int \cos^2\left(a+bx^4\right)\, dx, x, \sqrt{x}\right)\\
&= 2\text{Subst}\left(\int \left(\frac{1}{2}+\frac{1}{2}\cos\left(2a+2bx^4\right)\right)\, dx, x, \sqrt{x}\right)\\
&= \sqrt{x}+\text{Subst}\left(\int \cos\left(2a+2bx^4\right)\, dx, x, \sqrt{x}\right)\\
&= \sqrt{x}+\frac{1}{2}\text{Subst}\left(\int e^{-2ia-2ibx^4}\, dx, x, \sqrt{x}\right)+\frac{1}{2}\text{Subst}\left(\int e^{2ia+2ibx^4}\, dx, x, \sqrt{x}\right)\\
&= \sqrt{x}-\frac{e^{2ia}\sqrt{x}\,\Gamma\left(\frac{1}{4},-2ibx^2\right)}{8\sqrt[4]{2}\sqrt[4]{-ibx^2}}-\frac{e^{-2ia}\sqrt{x}\,\Gamma\left(\frac{1}{4},2ibx^2\right)}{8\sqrt[4]{2}\sqrt[4]{ibx^2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.03

$$\int \frac{\cos^2\left(a+bx^2\right)}{\sqrt{x}}\, dx = \frac{1}{16}\sqrt{x}\left(16-\frac{2^{3/4}\Gamma\left(\frac{1}{4},2ibx^2\right)\left(\cos(2a)-i\sin(2a)\right)}{\sqrt[4]{ibx^2}} - \frac{2^{3/4}\Gamma\left(\frac{1}{4},-2ibx^2\right)\left(\cos(2a)+i\sin(2a)\right)}{\sqrt[4]{-ibx^2}}\right)$$

```
[In] Integrate[Cos[a + b*x^2]^2/Sqrt[x],x]
```

```
[Out] (Sqrt[x]*(16 - (2^(3/4)*Gamma[1/4, (2*I)*b*x^2]*(Cos[2*a] - I*Sin[2*a]))/(I
*b*x^2)^(1/4) - (2^(3/4)*Gamma[1/4, (-2*I)*b*x^2]*(Cos[2*a] + I*Sin[2*a]))/
((-I)*b*x^2)^(1/4)))/16
```

Maple [F]

$$\int \frac{\cos^2(b\,x^2+a)}{\sqrt{x}}dx$$

```
[In] int(cos(b*x^2+a)^2/x^(1/2),x)
```

```
[Out] int(cos(b*x^2+a)^2/x^(1/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.11 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.65

$$\int \frac{\cos^2(a+bx^2)}{\sqrt{x}}\,dx = \frac{(2i\,b)^{\frac{3}{4}}\,(i\,\cos(2\,a)+\sin(2\,a))\Gamma\left(\frac{1}{4},2i\,bx^2\right)+(-2i\,b)^{\frac{3}{4}}\,(-i\,\cos(2\,a)+\sin(2\,a))\Gamma\left(\frac{1}{4},-2i\,bx^2\right)+16\,b\sqrt{x}}{16\,b}$$

```
[In] integrate(cos(b*x^2+a)^2/x^(1/2),x, algorithm="fricas")
```

```
[Out] 1/16*((2*I*b)^(3/4)*(I*cos(2*a) + sin(2*a))*gamma(1/4, 2*I*b*x^2) + (-2*I*b
)^(3/4)*(-I*cos(2*a) + sin(2*a))*gamma(1/4, -2*I*b*x^2) + 16*b*sqrt(x))/b
```

Sympy [F]

$$\int \frac{\cos^2(a+bx^2)}{\sqrt{x}}\,dx = \int \frac{\cos^2(a+bx^2)}{\sqrt{x}}\,dx$$

```
[In] integrate(cos(b*x**2+a)**2/x**(1/2),x)
```

```
[Out] Integral(cos(a + b*x**2)**2/sqrt(x), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^2(a+bx^2)}{\sqrt{x}}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(cos(b*x^2+a)^2/x^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> Encountered operator mismatch in maxima-t
o-sr translation
```

Giac [F]

$$\int \frac{\cos^2(a+bx^2)}{\sqrt{x}}\,dx = \int \frac{\cos(bx^2+a)^2}{\sqrt{x}}\,dx$$

```
[In] integrate(cos(b*x^2+a)^2/x^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x^2 + a)^2/sqrt(x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(a+bx^2)}{\sqrt{x}}\,dx = \int \frac{\cos(b\,x^2+a)^2}{\sqrt{x}}\,dx$$

```
[In] int(cos(a + b*x^2)^2/x^(1/2),x)
```

```
[Out] int(cos(a + b*x^2)^2/x^(1/2), x)
```

3.33 $\int \frac{\cos^2(a+bx^2)}{x^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 117

$$\int \frac{\cos^2(a+bx^2)}{x^{3/2}}\,dx = -\frac{1}{\sqrt{x}} - \frac{\cos(2(a+bx^2))}{\sqrt{x}} - \frac{ibe^{2ia}x^{3/2}\Gamma\left(\frac{3}{4},-2ibx^2\right)}{2^{3/4}(-ibx^2)^{3/4}} + \frac{ibe^{-2ia}x^{3/2}\Gamma\left(\frac{3}{4},2ibx^2\right)}{2^{3/4}(ibx^2)^{3/4}}$$

```
[Out] -1/2*I*b*exp(2*I*a)*x^(3/2)*GAMMA(3/4,-2*I*b*x^2)*2^(1/4)/(-I*b*x^2)^(3/4)+
1/2*I*b*x^(3/2)*GAMMA(3/4,2*I*b*x^2)*2^(1/4)/exp(2*I*a)/(I*b*x^2)^(3/4)-1/x
^(1/2)-cos(2*b*x^2+2*a)/x^(1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {3483, 3485, 3469, 3470, 2250}

$$\int \frac{\cos^2(a+bx^2)}{x^{3/2}}\,dx = -\frac{\cos(2(a+bx^2))}{\sqrt{x}} - \frac{ie^{2ia}bx^{3/2}\Gamma\left(\frac{3}{4},-2ibx^2\right)}{2^{3/4}(-ibx^2)^{3/4}} + \frac{ie^{-2ia}bx^{3/2}\Gamma\left(\frac{3}{4},2ibx^2\right)}{2^{3/4}(ibx^2)^{3/4}} - \frac{1}{\sqrt{x}}$$

```
[In] Int[Cos[a + b*x^2]^2/x^(3/2),x]
```

```
[Out] -(1/Sqrt[x]) - Cos[2*(a + b*x^2)]/Sqrt[x] - (I*b*E^((2*I)*a)*x^(3/2)*Gamma[
3/4, (-2*I)*b*x^2])/(2^(3/4)*((-I)*b*x^2)^(3/4)) + (I*b*x^(3/2)*Gamma[3/4,
(2*I)*b*x^2])/(2^(3/4)*E^((2*I)*a)*(I*b*x^2)^(3/4))
```

Rule 2250

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 3469

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_), x_Symbol] :> Simp[(e*x)
^(m + 1)*(Cos[c + d*x^n]/(e*(m + 1))), x] + Dist[d*(n/(e^n*(m + 1))), Int[(
e*x)^(m + n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3470

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[I/2,
 Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] - Dist[I/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3483

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*((e_.)*(x_))^(m_), x
_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m + 1) - 1
)*(a + b*Cos[c + d*(x^(k*n)/e^n)])^p, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e}, x] && IntegerQ[p] && IGtQ[n, 0] && FractionQ[m]
```

Rule 3485

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_)*((e_.)*(x_))^(m_.), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Cos[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= 2\text{Subst}\left(\int \frac{\cos^2\left(a+bx^4\right)}{x^2}\,dx,x,\sqrt{x}\right)\\
&= 2\text{Subst}\left(\int \left(\frac{1}{2x^2}+\frac{\cos\left(2a+2bx^4\right)}{2x^2}\right)\,dx,x,\sqrt{x}\right)\\
&= -\frac{1}{\sqrt{x}}+\text{Subst}\left(\int \frac{\cos\left(2a+2bx^4\right)}{x^2}\,dx,x,\sqrt{x}\right)\\
&= -\frac{1}{\sqrt{x}}-\frac{\cos\left(2(a+bx^2)\right)}{\sqrt{x}}-(8b)\text{Subst}\left(\int x^2\sin\left(2a+2bx^4\right)\,dx,x,\sqrt{x}\right)
\end{aligned}
$$

$$= -\frac{1}{\sqrt{x}} - \frac{\cos\left(2(a+bx^2)\right)}{\sqrt{x}} - (4ib)\text{Subst}\left(\int e^{-2ia-2ibx^4}x^2\,dx, x, \sqrt{x}\right) + (4ib)\text{Subst}\left(\int e^{2ia+2ibx^4}x^2\,dx, x, \sqrt{x}\right)$$

$$= -\frac{1}{\sqrt{x}} - \frac{\cos\left(2(a+bx^2)\right)}{\sqrt{x}} - \frac{ibe^{2ia}x^{3/2}\Gamma\left(\frac{3}{4}, -2ibx^2\right)}{2^{3/4}\left(-ibx^2\right)^{3/4}} + \frac{ibe^{-2ia}x^{3/2}\Gamma\left(\frac{3}{4}, 2ibx^2\right)}{2^{3/4}\left(ibx^2\right)^{3/4}}$$

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.17

$$\int \frac{\cos^2\left(a+bx^2\right)}{x^{3/2}}\,dx = \frac{-4(b^2x^4)^{3/4}\cos^2\left(a+bx^2\right) + \sqrt[4]{2}bx^2(ibx^2)^{3/4}\,\Gamma\left(\frac{3}{4}, -2ibx^2\right)\left(-i\cos(2a)+\sin(2a)\right) + i \cdots}{2\sqrt{x}\left(b^2x^4\right)^{3/4}}$$

```
[In] Integrate[Cos[a + b*x^2]^2/x^(3/2),x]
```

```
[Out] (-4*(b^2*x^4)^(3/4)*Cos[a + b*x^2]^2 + 2^(1/4)*b*x^2*(I*b*x^2)^(3/4)*Gamma[
3/4, (-2*I)*b*x^2]*((-I)*Cos[2*a] + Sin[2*a]) + I*2^(1/4)*((-I)*b*x^2)^(7/4
)*Gamma[3/4, (2*I)*b*x^2]*(I*Cos[2*a] + Sin[2*a]))/(2*Sqrt[x]*(b^2*x^4)^(3/
4))
```

Maple [F]

$$\int \frac{\cos^2\left(b\,x^2+a\right)}{x^{\frac{3}{2}}}dx$$

```
[In] int(cos(b*x^2+a)^2/x^(3/2),x)
```

```
[Out] int(cos(b*x^2+a)^2/x^(3/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.11 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.68

$$\int \frac{\cos^2\left(a+bx^2\right)}{x^{3/2}}\,dx = -\frac{4\sqrt{x}\cos\left(bx^2+a\right)^2 - \left(x\cos\left(2a\right) - i\,x\sin\left(2a\right)\right)(2i\,b)^{\frac{1}{4}}\,\Gamma\left(\frac{3}{4}, 2i\,bx^2\right) - \left(x\cos\left(2a\right) + i\,x\sin\left(2a\right)\right)(-2i\,b)^{\frac{1}{4}}\,\Gamma\left(\cdots\right)}{2\,x}$$

```
[In] integrate(cos(b*x^2+a)^2/x^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*(4*sqrt(x)*cos(b*x^2 + a)^2 - (x*cos(2*a) - I*x*sin(2*a))*(2*I*b)^(1/4
)*gamma(3/4, 2*I*b*x^2) - (x*cos(2*a) + I*x*sin(2*a))*(-2*I*b)^(1/4)*gamma(
3/4, -2*I*b*x^2))/x
```

Sympy [F]

$$\int \frac{\cos^2(a+bx^2)}{x^{3/2}}\,dx = \int \frac{\cos^2(a+bx^2)}{x^{\frac{3}{2}}}\,dx$$

```
[In] integrate(cos(b*x**2+a)**2/x**(3/2),x)
[Out] Integral(cos(a + b*x**2)**2/x**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^2(a+bx^2)}{x^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(cos(b*x^2+a)^2/x^(3/2),x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> Encountered operator mismatch in maxima-t
o-sr translation
```

Giac [F]

$$\int \frac{\cos^2(a+bx^2)}{x^{3/2}}\,dx = \int \frac{\cos(bx^2+a)^2}{x^{\frac{3}{2}}}\,dx$$

```
[In] integrate(cos(b*x^2+a)^2/x^(3/2),x, algorithm="giac")
[Out] integrate(cos(b*x^2 + a)^2/x^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(a+bx^2)}{x^{3/2}}\,dx = \int \frac{\cos(b\,x^2+a)^2}{x^{3/2}}\,dx$$

```
[In] int(cos(a + b*x^2)^2/x^(3/2),x)
[Out] int(cos(a + b*x^2)^2/x^(3/2), x)
```

3.34 $\int \frac{\cos^2(a+bx^2)}{x^{5/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 116

$$\int \frac{\cos^2(a+bx^2)}{x^{5/2}}\,dx = -\frac{2\cos^2(a+bx^2)}{3x^{3/2}} - \frac{ibe^{2ia}\sqrt{x}\,\Gamma\left(\frac{1}{4},-2ibx^2\right)}{3\sqrt[4]{2}\sqrt[4]{-ibx^2}} + \frac{ibe^{-2ia}\sqrt{x}\,\Gamma\left(\frac{1}{4},2ibx^2\right)}{3\sqrt[4]{2}\sqrt[4]{ibx^2}}$$

```
[Out] -2/3*cos(b*x^2+a)^2/x^(3/2)-1/6*I*b*exp(2*I*a)*GAMMA(1/4,-2*I*b*x^2)*x^(1/2
)*2^(3/4)/(-I*b*x^2)^(1/4)+1/6*I*b*GAMMA(1/4,2*I*b*x^2)*x^(1/2)*2^(3/4)/exp
(2*I*a)/(I*b*x^2)^(1/4)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3483, 3475, 4669, 3454, 3436, 2239}

$$\int \frac{\cos^2(a+bx^2)}{x^{5/2}}\,dx = -\frac{ie^{2ia}b\sqrt{x}\,\Gamma\left(\frac{1}{4},-2ibx^2\right)}{3\sqrt[4]{2}\sqrt[4]{-ibx^2}} + \frac{ie^{-2ia}b\sqrt{x}\,\Gamma\left(\frac{1}{4},2ibx^2\right)}{3\sqrt[4]{2}\sqrt[4]{ibx^2}} - \frac{2\cos^2(a+bx^2)}{3x^{3/2}}$$

```
[In] Int[Cos[a + b*x^2]^2/x^(5/2),x]
```

```
[Out] (-2*Cos[a + b*x^2]^2)/(3*x^(3/2)) - ((I/3)*b*E^((2*I)*a)*Sqrt[x]*Gamma[1/4,
 (-2*I)*b*x^2])/(2^(1/4)*((-I)*b*x^2)^(1/4)) + ((I/3)*b*Sqrt[x]*Gamma[1/4,
(2*I)*b*x^2])/(2^(1/4)*E^((2*I)*a)*(I*b*x^2)^(1/4))
```

Rule 2239

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> Simp[(-F^a
)*(c + d*x)*(Gamma[1/n, (-b)*(c + d*x)^n*Log[F]]/(d*n*((-b)*(c + d*x)^n*Log
[F])^(1/n))), x] /; FreeQ[{F, a, b, c, d, n}, x] && !IntegerQ[2/n]
```

Rule 3436

```
Int[Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Dist[I/2, In
t[E^((-c)*I - d*I*(e + f*x)^n), x], x] - Dist[I/2, Int[E^(c*I + d*I*(e + f*
x)^n), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ[n, 2]
```

Rule 3454

```
Int[((a_.) + (b_.)*Sin[u_])^(p_.), x_Symbol] :> Int[(a + b*Sin[ExpandToSum[
u, x]])^p, x] /; FreeQ[{a, b, p}, x] && BinomialQ[u, x] &&  !BinomialMatchQ
[u, x]
```

Rule 3475

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_)]^(p_)*(x_)^(m_.), x_Symbol] :> Simp[x^(m +
1)*(Cos[a + b*x^n]^p/(m + 1)), x] + Dist[b*n*(p/(m + 1)), Int[Cos[a + b*x^n
]^(p - 1)*Sin[a + b*x^n], x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 1] && EqQ[m
 + n, 0] && NeQ[n, 1] && IntegerQ[n]
```

Rule 3483

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*((e_.)*(x_))^(m_), x
_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m + 1) - 1
)*(a + b*Cos[c + d*(x^(k*n)/e^n)])^p, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e}, x] && IntegerQ[p] && IGtQ[n, 0] && FractionQ[m]
```

Rule 4669

```
Int[Cos[w_]^(p_.)*(u_.)*Sin[v_]^(p_.), x_Symbol] :> Dist[1/2^p, Int[u*Sin[2
*v]^p, x], x] /; EqQ[w, v] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 2\text{Subst}\left(\int \frac{\cos^2\left(a+bx^4\right)}{x^4}\,dx,x,\sqrt{x}\right)\\
&= -\frac{2\cos^2\left(a+bx^2\right)}{3x^{3/2}}-\frac{1}{3}(16b)\text{Subst}\left(\int \cos\left(a+bx^4\right)\sin\left(a+bx^4\right)\,dx,x,\sqrt{x}\right)\\
&= -\frac{2\cos^2\left(a+bx^2\right)}{3x^{3/2}}-\frac{1}{3}(8b)\text{Subst}\left(\int \sin\left(2\left(a+bx^4\right)\right)\,dx,x,\sqrt{x}\right)\\
&= -\frac{2\cos^2\left(a+bx^2\right)}{3x^{3/2}}-\frac{1}{3}(8b)\text{Subst}\left(\int \sin\left(2a+2bx^4\right)\,dx,x,\sqrt{x}\right)\\
&= -\frac{2\cos^2\left(a+bx^2\right)}{3x^{3/2}}\\
&\quad -\frac{1}{3}(4ib)\text{Subst}\left(\int e^{-2ia-2ibx^4}\,dx,x,\sqrt{x}\right)+\frac{1}{3}(4ib)\text{Subst}\left(\int e^{2ia+2ibx^4}\,dx,x,\sqrt{x}\right)
\end{aligned}$$

$$= -\frac{2\cos^2(a+bx^2)}{3x^{3/2}} - \frac{ibe^{2ia}\sqrt{x}\Gamma\left(\frac{1}{4},-2ibx^2\right)}{3\sqrt[4]{2}\sqrt[4]{-ibx^2}} + \frac{ibe^{-2ia}\sqrt{x}\Gamma\left(\frac{1}{4},2ibx^2\right)}{3\sqrt[4]{2}\sqrt[4]{ibx^2}}$$

Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.18

$$\int \frac{\cos^2(a+bx^2)}{x^{5/2}}\,dx = \frac{-4\sqrt[4]{b^2x^4}\cos^2(a+bx^2) + 2^{3/4}bx^2\sqrt[4]{ibx^2}\Gamma\left(\frac{1}{4},-2ibx^2\right)(-i\cos(2a)+\sin(2a)) + i2^{3/4}(-}{6x^{3/2}\sqrt[4]{b^2x^4}}$$

```
[In] Integrate[Cos[a + b*x^2]^2/x^(5/2),x]
```

```
[Out] (-4*(b^2*x^4)^(1/4)*Cos[a + b*x^2]^2 + 2^(3/4)*b*x^2*(I*b*x^2)^(1/4)*Gamma[
1/4, (-2*I)*b*x^2]*((-I)*Cos[2*a] + Sin[2*a]) + I*2^(3/4)*((-I)*b*x^2)^(5/4
)*Gamma[1/4, (2*I)*b*x^2]*(I*Cos[2*a] + Sin[2*a]))/(6*x^(3/2)*(b^2*x^4)^(1/
4))
```

Maple [F]

$$\int \frac{\cos^2(b\,x^2+a)}{x^{\frac{5}{2}}}dx$$

```
[In] int(cos(b*x^2+a)^2/x^(5/2),x)
```

```
[Out] int(cos(b*x^2+a)^2/x^(5/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.73

$$\int \frac{\cos^2(a+bx^2)}{x^{5/2}}\,dx = \frac{(x^2\cos(2\,a) - i\,x^2\sin(2\,a))(2i\,b)^{\frac{3}{4}}\,\Gamma\left(\frac{1}{4},2i\,bx^2\right) + (x^2\cos(2\,a) + i\,x^2\sin(2\,a))(-2i\,b)^{\frac{3}{4}}\,\Gamma}{6\,x^2}$$

```
[In] integrate(cos(b*x^2+a)^2/x^(5/2),x, algorithm="fricas")
```

```
[Out] 1/6*((x^2*cos(2*a) - I*x^2*sin(2*a))*(2*I*b)^(3/4)*gamma(1/4, 2*I*b*x^2) +
(x^2*cos(2*a) + I*x^2*sin(2*a))*(-2*I*b)^(3/4)*gamma(1/4, -2*I*b*x^2) - 4*s
qrt(x)*cos(b*x^2 + a)^2)/x^2
```

Sympy [F]

$$\int \frac{\cos^2(a+bx^2)}{x^{5/2}}\,dx = \int \frac{\cos^2(a+bx^2)}{x^{\frac{5}{2}}}\,dx$$

```
[In] integrate(cos(b*x**2+a)**2/x**(5/2),x)
[Out] Integral(cos(a + b*x**2)**2/x**(5/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^2(a+bx^2)}{x^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(cos(b*x^2+a)^2/x^(5/2),x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> Encountered operator mismatch in maxima-t
o-sr translation
```

Giac [F]

$$\int \frac{\cos^2(a+bx^2)}{x^{5/2}}\,dx = \int \frac{\cos(bx^2+a)^2}{x^{\frac{5}{2}}}\,dx$$

```
[In] integrate(cos(b*x^2+a)^2/x^(5/2),x, algorithm="giac")
[Out] integrate(cos(b*x^2 + a)^2/x^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(a+bx^2)}{x^{5/2}}\,dx = \int \frac{\cos(b\,x^2+a)^2}{x^{5/2}}\,dx$$

```
[In] int(cos(a + b*x^2)^2/x^(5/2),x)
[Out] int(cos(a + b*x^2)^2/x^(5/2), x)
```

3.35 $\int \cos\left(a+\frac{b}{x}\right)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 31

$$\int \cos\left(a+\frac{b}{x}\right)\,dx = x\cos\left(a+\frac{b}{x}\right)+b\,\text{CosIntegral}\left(\frac{b}{x}\right)\sin(a)+b\cos(a)\text{Si}\left(\frac{b}{x}\right)$$

```
[Out] x*cos(a+b/x)+b*cos(a)*Si(b/x)+b*Ci(b/x)*sin(a)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {3443, 3378, 3384, 3380, 3383}

$$\int \cos\left(a+\frac{b}{x}\right)\,dx = b\sin(a)\,\text{CosIntegral}\left(\frac{b}{x}\right)+b\cos(a)\text{Si}\left(\frac{b}{x}\right)+x\cos\left(a+\frac{b}{x}\right)$$

```
[In] Int[Cos[a + b/x],x]
```

```
[Out] x*Cos[a + b/x] + b*CosIntegral[b/x]*Sin[a] + b*Cos[a]*SinIntegral[b/x]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3443

```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_.), x_S
ymbol] :> Dist[1/(n*f), Subst[Int[x^(1/n - 1)*(a + b*Cos[c + d*x])^p, x], x
, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && Integer
Q[1/n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{\cos(a+bx)}{x^2}\,dx, x, \frac{1}{x}\right)\\
&= x\cos\left(a+\frac{b}{x}\right) + b\text{Subst}\left(\int \frac{\sin(a+bx)}{x}\,dx, x, \frac{1}{x}\right)\\
&= x\cos\left(a+\frac{b}{x}\right) + (b\cos(a))\text{Subst}\left(\int \frac{\sin(bx)}{x}\,dx, x, \frac{1}{x}\right)\\
&\quad + (b\sin(a))\text{Subst}\left(\int \frac{\cos(bx)}{x}\,dx, x, \frac{1}{x}\right)\\
&= x\cos\left(a+\frac{b}{x}\right) + b\,\text{CosIntegral}\left(\frac{b}{x}\right)\sin(a) + b\cos(a)\text{Si}\left(\frac{b}{x}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$
\int \cos\left(a+\frac{b}{x}\right)\,dx = x\cos\left(a+\frac{b}{x}\right) + b\,\text{CosIntegral}\left(\frac{b}{x}\right)\sin(a) + b\cos(a)\text{Si}\left(\frac{b}{x}\right)
$$

```
[In] Integrate[Cos[a + b/x],x]
[Out] x*Cos[a + b/x] + b*CosIntegral[b/x]*Sin[a] + b*Cos[a]*SinIntegral[b/x]
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.26

method	result
derivativedivides	$-b\left(-\frac{\cos\left(a+\frac{b}{x}\right)x}{b}-\operatorname{Si}\left(\frac{b}{x}\right)\cos\left(a\right)-\operatorname{Ci}\left(\frac{b}{x}\right)\sin\left(a\right)\right)$
default	$-b\left(-\frac{\cos\left(a+\frac{b}{x}\right)x}{b}-\operatorname{Si}\left(\frac{b}{x}\right)\cos\left(a\right)-\operatorname{Ci}\left(\frac{b}{x}\right)\sin\left(a\right)\right)$
risch	$\frac{ib\,\operatorname{Ei}_1\left(-\frac{ib}{x}\right)\mathrm{e}^{ia}}{2}-\frac{\pi\,\operatorname{csgn}\left(\frac{b}{x}\right)\mathrm{e}^{-ia}b}{2}+\operatorname{Si}\left(\frac{b}{x}\right)\mathrm{e}^{-ia}b-\frac{i\operatorname{Ei}_1\left(-\frac{ib}{x}\right)\mathrm{e}^{-ia}b}{2}+x\cos\left(\frac{ax+b}{x}\right)$
meijerg	$-\frac{\cos(a)\sqrt{\pi}\,\sqrt{b^2}\left(-\frac{4x\,b^2\cos\left(\frac{\sqrt{b^2}}{x}\right)}{\left(b^2\right)^{\frac{3}{2}}\sqrt{\pi}}-\frac{4\operatorname{Si}\left(\frac{\sqrt{b^2}}{x}\right)}{\sqrt{\pi}}\right)}{4}+\frac{\sqrt{\pi}\,\sin(a)b\Bigg(\frac{4\gamma-4-4\ln(x)+4\ln(b)}{\sqrt{\pi}}+\frac{4}{\sqrt{\pi}}-\frac{4\gamma}{\sqrt{\pi}}-\frac{4\ln(2)}{\sqrt{\pi}}-\frac{4\ln\left(\frac{b}{2x}\right)}{\sqrt{\pi}}-}{4}$

```
[In] int(cos(a+b/x),x,method=_RETURNVERBOSE)
[Out] -b*(-cos(a+b/x)/b*x-Si(b/x)*cos(a)-Ci(b/x)*sin(a))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \cos\left(a+\frac{b}{x}\right)\,dx=b\operatorname{Ci}\left(\frac{b}{x}\right)\sin\left(a\right)+b\cos\left(a\right)\operatorname{Si}\left(\frac{b}{x}\right)+x\cos\left(\frac{ax+b}{x}\right)$$

```
[In] integrate(cos(a+b/x),x, algorithm="fricas")
[Out] b*cos_integral(b/x)*sin(a) + b*cos(a)*sin_integral(b/x) + x*cos((a*x + b)/x
)
```

Sympy [F]

$$\int \cos\left(a+\frac{b}{x}\right)\,dx=\int \cos\left(a+\frac{b}{x}\right)\,dx$$

```
[In] integrate(cos(a+b/x),x)
[Out] Integral(cos(a + b/x), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.84

$$\int \cos\left(a + \frac{b}{x}\right)\,dx$$
$$= \frac{1}{2}\left(\left(-i\,\mathrm{Ei}\left(\frac{i\,b}{x}\right) + i\,\mathrm{Ei}\left(-\frac{i\,b}{x}\right)\right)\cos(a) + \left(\mathrm{Ei}\left(\frac{i\,b}{x}\right) + \mathrm{Ei}\left(-\frac{i\,b}{x}\right)\right)\sin(a)\right)b$$
$$+ x\cos\left(\frac{ax+b}{x}\right)$$

```
[In] integrate(cos(a+b/x),x, algorithm="maxima")
```

```
[Out] 1/2*((-I*Ei(I*b/x) + I*Ei(-I*b/x))*cos(a) + (Ei(I*b/x) + Ei(-I*b/x))*sin(a)
)*b + x*cos((a*x + b)/x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 132 vs. 2(31) = 62.

Time = 0.43 (sec) , antiderivative size = 132, normalized size of antiderivative = 4.26

$$\int \cos\left(a + \frac{b}{x}\right)\,dx$$
$$= \frac{ab^2\,\mathrm{Ci}\left(-a + \frac{ax+b}{x}\right)\sin(a) - ab^2\cos(a)\,\mathrm{Si}\left(a - \frac{ax+b}{x}\right) - \frac{(ax+b)b^2\,\mathrm{Ci}\left(-a+\frac{ax+b}{x}\right)\sin(a)}{x} + \frac{(ax+b)b^2\cos(a)\,\mathrm{Si}\left(a-\frac{ax+b}{x}\right)}{x} -}{\left(a - \frac{ax+b}{x}\right)b}$$

```
[In] integrate(cos(a+b/x),x, algorithm="giac")
```

```
[Out] (a*b^2*cos_integral(-a + (a*x + b)/x)*sin(a) - a*b^2*cos(a)*sin_integral(a
- (a*x + b)/x) - (a*x + b)*b^2*cos_integral(-a + (a*x + b)/x)*sin(a)/x + (a
*x + b)*b^2*cos(a)*sin_integral(a - (a*x + b)/x)/x - b^2*cos((a*x + b)/x))/
((a - (a*x + b)/x)*b)
```

Mupad [F(-1)]

Timed out.

$$\int \cos\left(a + \frac{b}{x}\right)\,dx = \int \cos\left(a + \frac{b}{x}\right)\,dx$$

```
[In] int(cos(a + b/x),x)
```

```
[Out] int(cos(a + b/x), x)
```

3.36 $\int \frac{\cos\left(a+\frac{b}{x}\right)}{x} dx$

Optimal result

Integrand size = 12, antiderivative size = 20

$$\int \frac{\cos\left(a+\frac{b}{x}\right)}{x} dx = -\cos(a)\,\text{CosIntegral}\left(\frac{b}{x}\right)+\sin(a)\text{Si}\left(\frac{b}{x}\right)$$

```
[Out] -Ci(b/x)*cos(a)+Si(b/x)*sin(a)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3459, 3457, 3456}

$$\int \frac{\cos\left(a+\frac{b}{x}\right)}{x} dx = \sin(a)\text{Si}\left(\frac{b}{x}\right)-\cos(a)\,\text{CosIntegral}\left(\frac{b}{x}\right)$$

```
[In] Int[Cos[a + b/x]/x,x]
```

```
[Out] -(Cos[a]*CosIntegral[b/x]) + Sin[a]*SinIntegral[b/x]
```

Rule 3456

```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3457

```
Int[Cos[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[CosIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3459

```
Int[Cos[(c_) + (d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Dist[Cos[c], Int[Cos[d*x
^n]/x, x], x] - Dist[Sin[c], Int[Sin[d*x^n]/x, x], x] /; FreeQ[{c, d, n}, x
]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \cos(a)\int \frac{\cos\left(\frac{b}{x}\right)}{x}\,dx - \sin(a)\int \frac{\sin\left(\frac{b}{x}\right)}{x}\,dx\\ &= -\cos(a)\,\text{CosIntegral}\left(\frac{b}{x}\right) + \sin(a)\text{Si}\left(\frac{b}{x}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(a+\frac{b}{x}\right)}{x}\,dx = -\cos(a)\,\text{CosIntegral}\left(\frac{b}{x}\right) + \sin(a)\text{Si}\left(\frac{b}{x}\right)$$

```
[In] Integrate[Cos[a + b/x]/x,x]
[Out] -(Cos[a]*CosIntegral[b/x]) + Sin[a]*SinIntegral[b/x]
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.05

method	result	size
derivativedivides	$-\operatorname{Ci}\left(\frac{b}{x}\right)\cos\left(a\right)+\operatorname{Si}\left(\frac{b}{x}\right)\sin\left(a\right)$	21
default	$-\operatorname{Ci}\left(\frac{b}{x}\right)\cos\left(a\right)+\operatorname{Si}\left(\frac{b}{x}\right)\sin\left(a\right)$	21
risch	$-\frac{i\pi\,\operatorname{csgn}\left(\frac{b}{x}\right)\mathrm{e}^{-ia}}{2}+i\operatorname{Si}\left(\frac{b}{x}\right)\mathrm{e}^{-ia}+\frac{\mathrm{e}^{-ia}\operatorname{Ei}_{1}\left(-\frac{ib}{x}\right)}{2}+\frac{\mathrm{e}^{ia}\operatorname{Ei}_{1}\left(-\frac{ib}{x}\right)}{2}$	63
meijerg	$-\frac{\sqrt{\pi}\,\cos(a)\left(\frac{2\gamma-2\ln(x)+\ln\left(b^{2}\right)}{\sqrt{\pi}}-\frac{2\gamma}{\sqrt{\pi}}-\frac{2\ln(2)}{\sqrt{\pi}}-\frac{2\ln\left(\frac{b}{2x}\right)}{\sqrt{\pi}}+\frac{2\operatorname{Ci}\left(\frac{b}{x}\right)}{\sqrt{\pi}}\right)}{2}+\operatorname{Si}\left(\frac{b}{x}\right)\sin\left(a\right)$	71

```
[In] int(cos(a+b/x)/x,x,method=_RETURNVERBOSE)
[Out] -Ci(b/x)*cos(a)+Si(b/x)*sin(a)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(a+\frac{b}{x}\right)}{x}\,dx = -\cos(a)\operatorname{Ci}\left(\frac{b}{x}\right) + \sin(a)\operatorname{Si}\left(\frac{b}{x}\right)$$

```
[In] integrate(cos(a+b/x)/x,x, algorithm="fricas")
[Out] -cos(a)*cos_integral(b/x) + sin(a)*sin_integral(b/x)
```

Sympy [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.75

$$\int \frac{\cos\left(a+\frac{b}{x}\right)}{x}\,dx = \sin(a)\operatorname{Si}\left(\frac{b}{x}\right) - \cos(a)\operatorname{Ci}\left(\frac{b}{x}\right)$$

```
[In] integrate(cos(a+b/x)/x,x)
[Out] sin(a)*Si(b/x) - cos(a)*Ci(b/x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 43, normalized size of antiderivative = 2.15

$$\int \frac{\cos\left(a+\frac{b}{x}\right)}{x}\,dx = -\frac{1}{2}\left(\operatorname{Ei}\left(\frac{ib}{x}\right) + \operatorname{Ei}\left(-\frac{ib}{x}\right)\right)\cos(a) - \frac{1}{2}\left(i\operatorname{Ei}\left(\frac{ib}{x}\right) - i\operatorname{Ei}\left(-\frac{ib}{x}\right)\right)\sin(a)$$

```
[In] integrate(cos(a+b/x)/x,x, algorithm="maxima")
[Out] -1/2*(Ei(I*b/x) + Ei(-I*b/x))*cos(a) - 1/2*(I*Ei(I*b/x) - I*Ei(-I*b/x))*sin
(a)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(20) = 40$.

Time = 0.38 (sec) , antiderivative size = 41, normalized size of antiderivative = 2.05

$$\int \frac{\cos\left(a+\frac{b}{x}\right)}{x}\,dx = -\frac{b\cos\left(a\right)\operatorname{Ci}\left(-a+\frac{ax+b}{x}\right)+b\sin\left(a\right)\operatorname{Si}\left(a-\frac{ax+b}{x}\right)}{b}$$

```
[In] integrate(cos(a+b/x)/x,x, algorithm="giac")
```

```
[Out] -(b*cos(a)*cos_integral(-a + (a*x + b)/x) + b*sin(a)*sin_integral(a - (a*x
+ b)/x))/b
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos\left(a+\frac{b}{x}\right)}{x}\,dx = \sin\left(a\right)\,\mathrm{sinint}\left(\frac{b}{x}\right)-\cos\left(a\right)\,\mathrm{cosint}\left(\frac{b}{x}\right)$$

```
[In] int(cos(a + b/x)/x,x)
```

```
[Out] sin(a)*sinint(b/x) - cos(a)*cosint(b/x)
```

3.37 $\int \frac{\cos\left(a+\frac{b}{x}\right)}{x^2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 13

$$\int \frac{\cos\left(a+\frac{b}{x}\right)}{x^2}\,dx = -\frac{\sin\left(a+\frac{b}{x}\right)}{b}$$

```
[Out] -sin(a+b/x)/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3461, 2717}

$$\int \frac{\cos\left(a+\frac{b}{x}\right)}{x^2}\,dx = -\frac{\sin\left(a+\frac{b}{x}\right)}{b}$$

```
[In] Int[Cos[a + b/x]/x^2,x]
```

```
[Out] -(Sin[a + b/x]/b)
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
```

```
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\text{Subst}\left(\int \cos(a+bx)\,dx, x, \frac{1}{x}\right)\\ &= -\frac{\sin\left(a+\frac{b}{x}\right)}{b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(a+\frac{b}{x}\right)}{x^2}\,dx = -\frac{\sin\left(a+\frac{b}{x}\right)}{b}$$

```
[In] Integrate[Cos[a + b/x]/x^2,x]
[Out] -(Sin[a + b/x]/b)
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

method	result	size
derivativedivides	$-\frac{\sin\left(a+\frac{b}{x}\right)}{b}$	14
default	$-\frac{\sin\left(a+\frac{b}{x}\right)}{b}$	14
risch	$-\frac{\sin\left(\frac{ax+b}{x}\right)}{b}$	16
parallelrisch	$-\frac{\sin\left(\frac{ax+b}{x}\right)}{b}$	16
norman	$-\frac{2\tan\left(\frac{a}{2}+\frac{b}{2x}\right)}{b\left(1+\tan^2\left(\frac{a}{2}+\frac{b}{2x}\right)\right)}$	34
meijerg	$-\frac{\cos(a)\sin\left(\frac{b}{x}\right)}{b}+\frac{\sqrt{\pi}\,\sin(a)\left(\frac{1}{\sqrt{\pi}}-\frac{\cos\left(\frac{b}{x}\right)}{\sqrt{\pi}}\right)}{b}$	39

```
[In] int(cos(a+b/x)/x^2,x,method=_RETURNVERBOSE)
[Out] -sin(a+b/x)/b
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

$$\int \frac{\cos\left(a+\frac{b}{x}\right)}{x^2}\,dx = -\frac{\sin\left(\frac{ax+b}{x}\right)}{b}$$

```
[In] integrate(cos(a+b/x)/x^2,x, algorithm="fricas")
[Out] -sin((a*x + b)/x)/b
```

Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

$$\int \frac{\cos\left(a+\frac{b}{x}\right)}{x^2}\,dx = \begin{cases} -\frac{\sin\left(a+\frac{b}{x}\right)}{b} & \text{for } b \neq 0 \\ -\frac{\cos(a)}{x} & \text{otherwise} \end{cases}$$

```
[In] integrate(cos(a+b/x)/x**2,x)
[Out] Piecewise((-sin(a + b/x)/b, Ne(b, 0)), (-cos(a)/x, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(a+\frac{b}{x}\right)}{x^2}\,dx = -\frac{\sin\left(a+\frac{b}{x}\right)}{b}$$

```
[In] integrate(cos(a+b/x)/x^2,x, algorithm="maxima")
[Out] -sin(a + b/x)/b
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

$$\int \frac{\cos\left(a+\frac{b}{x}\right)}{x^2}\,dx = -\frac{\sin\left(\frac{ax+b}{x}\right)}{b}$$

```
[In] integrate(cos(a+b/x)/x^2,x, algorithm="giac")
[Out] -sin((a*x + b)/x)/b
```

Mupad [B] (verification not implemented)

Time = 13.87 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(a+\frac{b}{x}\right)}{x^2}\,dx = -\frac{\sin\left(a+\frac{b}{x}\right)}{b}$$

```
[In] int(cos(a + b/x)/x^2,x)
[Out] -sin(a + b/x)/b
```

3.38 $\int \frac{\cos\left(a+\frac{b}{x}\right)}{x^3}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 30

$$\int \frac{\cos\left(a+\frac{b}{x}\right)}{x^3}\,dx = -\frac{\cos\left(a+\frac{b}{x}\right)}{b^2} - \frac{\sin\left(a+\frac{b}{x}\right)}{bx}$$

```
[Out] -cos(a+b/x)/b^2-sin(a+b/x)/b/x
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3461, 3377, 2718}

$$\int \frac{\cos\left(a+\frac{b}{x}\right)}{x^3}\,dx = -\frac{\cos\left(a+\frac{b}{x}\right)}{b^2} - \frac{\sin\left(a+\frac{b}{x}\right)}{bx}$$

```
[In] Int[Cos[a + b/x]/x^3,x]
```

```
[Out] -(Cos[a + b/x]/b^2) - Sin[a + b/x]/(b*x)
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\text{Subst}\left(\int x\cos(a+bx)\,dx, x, \frac{1}{x}\right)\\ &= -\frac{\sin\left(a+\frac{b}{x}\right)}{bx} + \frac{\text{Subst}\left(\int \sin(a+bx)\,dx, x, \frac{1}{x}\right)}{b}\\ &= -\frac{\cos\left(a+\frac{b}{x}\right)}{b^2} - \frac{\sin\left(a+\frac{b}{x}\right)}{bx}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.97

$$\int \frac{\cos\left(a+\frac{b}{x}\right)}{x^3}\,dx = -\frac{x\cos\left(a+\frac{b}{x}\right) + b\sin\left(a+\frac{b}{x}\right)}{b^2x}$$

```
[In] Integrate[Cos[a + b/x]/x^3,x]
[Out] -((x*Cos[a + b/x] + b*Sin[a + b/x])/(b^2*x))
```

Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.17

method	result	size
risch	$-\frac{\cos\left(\frac{ax+b}{x}\right)}{b^{2}}-\frac{\sin\left(\frac{ax+b}{x}\right)}{bx}$	35
parallelrisch	$\frac{x-x\cos\left(\frac{ax+b}{x}\right)-b\sin\left(\frac{ax+b}{x}\right)}{b^{2}x}$	36
derivativedivides	$-\frac{\cos\left(a+\frac{b}{x}\right)+\left(a+\frac{b}{x}\right)\sin\left(a+\frac{b}{x}\right)-a\sin\left(a+\frac{b}{x}\right)}{b^{2}}$	42
default	$-\frac{\cos\left(a+\frac{b}{x}\right)+\left(a+\frac{b}{x}\right)\sin\left(a+\frac{b}{x}\right)-a\sin\left(a+\frac{b}{x}\right)}{b^{2}}$	42
norman	$\frac{\frac{2x^{2}\left(\tan^{2}\left(\frac{a}{2}+\frac{b}{2x}\right)\right)}{b^{2}}-\frac{2x\tan\left(\frac{a}{2}+\frac{b}{2x}\right)}{b}}{\left(1+\tan^{2}\left(\frac{a}{2}+\frac{b}{2x}\right)\right)x^{2}}$	61
meijerg	$-\frac{2\sqrt{\pi}\,\cos(a)\left(-\frac{1}{2\sqrt{\pi}}+\frac{\cos\left(\frac{b}{x}\right)}{2\sqrt{\pi}}+\frac{b\sin\left(\frac{b}{x}\right)}{2\sqrt{\pi}\,x}\right)}{b^{2}}+\frac{2\sqrt{\pi}\,\sin(a)\left(-\frac{b\cos\left(\frac{b}{x}\right)}{2\sqrt{\pi}\,x}+\frac{\sin\left(\frac{b}{x}\right)}{2\sqrt{\pi}}\right)}{b^{2}}$	81

```
[In] int(cos(a+b/x)/x^3,x,method=_RETURNVERBOSE)
[Out] -1/b^2*cos((a*x+b)/x)-1/b/x*sin((a*x+b)/x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.10

$$\int \frac{\cos\left(a+\frac{b}{x}\right)}{x^{3}}\,dx=-\frac{x\cos\left(\frac{ax+b}{x}\right)+b\sin\left(\frac{ax+b}{x}\right)}{b^{2}x}$$

```
[In] integrate(cos(a+b/x)/x^3,x, algorithm="fricas")
[Out] -(x*cos((a*x + b)/x) + b*sin((a*x + b)/x))/(b^2*x)
```

Sympy [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.03

$$\int \frac{\cos\left(a+\frac{b}{x}\right)}{x^{3}}\,dx=\begin{cases}-\frac{\sin\left(a+\frac{b}{x}\right)}{bx}-\frac{\cos\left(a+\frac{b}{x}\right)}{b^{2}} & \text{for}\ b\neq 0\\ -\frac{\cos\left(a\right)}{2x^{2}} & \text{otherwise}\end{cases}$$

```
[In] integrate(cos(a+b/x)/x**3,x)
[Out] Piecewise((-sin(a + b/x)/(b*x) - cos(a + b/x)/b**2, Ne(b, 0)), (-cos(a)/(2*
x**2), True))
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 0.33 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.70

$$\int \frac{\cos\left(a+\frac{b}{x}\right)}{x^3}\,dx = -\frac{\left(\Gamma\left(2,\frac{ib}{x}\right)+\Gamma\left(2,-\frac{ib}{x}\right)\right)\cos\left(a\right)-\left(i\,\Gamma\left(2,\frac{ib}{x}\right)-i\,\Gamma\left(2,-\frac{ib}{x}\right)\right)\sin\left(a\right)}{2\,b^2}$$

```
[In] integrate(cos(a+b/x)/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*((gamma(2, I*b/x) + gamma(2, -I*b/x))*cos(a) - (I*gamma(2, I*b/x) - I*
gamma(2, -I*b/x))*sin(a))/b^2
```

Giac [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.63

$$\int \frac{\cos\left(a+\frac{b}{x}\right)}{x^3}\,dx = \frac{a\sin\left(\frac{ax+b}{x}\right)-\frac{(ax+b)\sin\left(\frac{ax+b}{x}\right)}{x}-\cos\left(\frac{ax+b}{x}\right)}{b^2}$$

```
[In] integrate(cos(a+b/x)/x^3,x, algorithm="giac")
```

```
[Out] (a*sin((a*x + b)/x) - (a*x + b)*sin((a*x + b)/x)/x - cos((a*x + b)/x))/b^2
```

Mupad [B] (verification not implemented)

Time = 13.49 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(a+\frac{b}{x}\right)}{x^3}\,dx = -\frac{\cos\left(a+\frac{b}{x}\right)}{b^2}-\frac{\sin\left(a+\frac{b}{x}\right)}{b\,x}$$

```
[In] int(cos(a + b/x)/x^3,x)
```

```
[Out] - cos(a + b/x)/b^2 - sin(a + b/x)/(b*x)
```

3.39 $\int \frac{\cos\left(a+\frac{b}{x}\right)}{x^4}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 46

$$\int \frac{\cos\left(a+\frac{b}{x}\right)}{x^4}\,dx=-\frac{2\cos\left(a+\frac{b}{x}\right)}{b^2x}+\frac{2\sin\left(a+\frac{b}{x}\right)}{b^3}-\frac{\sin\left(a+\frac{b}{x}\right)}{bx^2}$$

```
[Out] -2*cos(a+b/x)/b^2/x+2*sin(a+b/x)/b^3-sin(a+b/x)/b/x^2
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3461, 3377, 2717}

$$\int \frac{\cos\left(a+\frac{b}{x}\right)}{x^4}\,dx=\frac{2\sin\left(a+\frac{b}{x}\right)}{b^3}-\frac{2\cos\left(a+\frac{b}{x}\right)}{b^2x}-\frac{\sin\left(a+\frac{b}{x}\right)}{bx^2}$$

```
[In] Int[Cos[a + b/x]/x^4,x]
```

```
[Out] (-2*Cos[a + b/x])/(b^2*x) + (2*Sin[a + b/x])/b^3 - Sin[a + b/x]/(b*x^2)
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
```

```
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\text{Subst}\left(\int x^2\cos(a+bx)\,dx, x, \frac{1}{x}\right)\\ &= -\frac{\sin\left(a+\frac{b}{x}\right)}{bx^2}+\frac{2\text{Subst}\left(\int x\sin(a+bx)\,dx, x, \frac{1}{x}\right)}{b}\\ &= -\frac{2\cos\left(a+\frac{b}{x}\right)}{b^2x}-\frac{\sin\left(a+\frac{b}{x}\right)}{bx^2}+\frac{2\text{Subst}\left(\int\cos(a+bx)\,dx, x, \frac{1}{x}\right)}{b^2}\\ &= -\frac{2\cos\left(a+\frac{b}{x}\right)}{b^2x}+\frac{2\sin\left(a+\frac{b}{x}\right)}{b^3}-\frac{\sin\left(a+\frac{b}{x}\right)}{bx^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int\frac{\cos\left(a+\frac{b}{x}\right)}{x^4}\,dx = -\frac{2\cos\left(a+\frac{b}{x}\right)}{b^2x}+\frac{2\sin\left(a+\frac{b}{x}\right)}{b^3}-\frac{\sin\left(a+\frac{b}{x}\right)}{bx^2}$$

```
[In] Integrate[Cos[a + b/x]/x^4,x]
[Out] (-2*Cos[a + b/x])/(b^2*x) + (2*Sin[a + b/x])/b^3 - Sin[a + b/x]/(b*x^2)
```

Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.02

method	result
risch	$-\frac{2\cos\left(\frac{ax+b}{x}\right)}{b^{2}x}-\frac{\left(b^{2}-2x^{2}\right)\sin\left(\frac{ax+b}{x}\right)}{b^{3}x^{2}}$
parallelrisch	$\frac{2x\left(\tan^{2}\left(\frac{ax+b}{2x}\right)\right)b+4\tan\left(\frac{ax+b}{2x}\right)x^{2}-2\tan\left(\frac{ax+b}{2x}\right)b^{2}-2bx}{x^{2}b^{3}\left(1+\tan^{2}\left(\frac{ax+b}{2x}\right)\right)}$
norman	$\frac{-\frac{2x^{2}}{b^{2}}+\frac{4x^{3}\tan\left(\frac{a}{2}+\frac{b}{2x}\right)}{b^{3}}+\frac{2x^{2}\left(\tan^{2}\left(\frac{a}{2}+\frac{b}{2x}\right)\right)}{b^{2}}-\frac{2x\tan\left(\frac{a}{2}+\frac{b}{2x}\right)}{b}}{\left(1+\tan^{2}\left(\frac{a}{2}+\frac{b}{2x}\right)\right)x^{3}}$
derivativedivides	$-\frac{a^{2}\sin\left(a+\frac{b}{x}\right)-2a\left(\cos\left(a+\frac{b}{x}\right)+\left(a+\frac{b}{x}\right)\sin\left(a+\frac{b}{x}\right)\right)+\left(a+\frac{b}{x}\right)^{2}\sin\left(a+\frac{b}{x}\right)-2\sin\left(a+\frac{b}{x}\right)+2\left(a+\frac{b}{x}\right)\cos\left(a+\frac{b}{x}\right)}{b^{3}}$
default	$-\frac{a^{2}\sin\left(a+\frac{b}{x}\right)-2a\left(\cos\left(a+\frac{b}{x}\right)+\left(a+\frac{b}{x}\right)\sin\left(a+\frac{b}{x}\right)\right)+\left(a+\frac{b}{x}\right)^{2}\sin\left(a+\frac{b}{x}\right)-2\sin\left(a+\frac{b}{x}\right)+2\left(a+\frac{b}{x}\right)\cos\left(a+\frac{b}{x}\right)}{b^{3}}$
meijerg	$-\frac{4\sqrt{\pi}\cos(a)\sqrt{b^{2}}\left(\frac{\left(b^{2}\right)^{\frac{3}{2}}\cos\left(\frac{b}{x}\right)}{2\sqrt{\pi}\,x\,b^{2}}-\frac{\left(b^{2}\right)^{\frac{3}{2}}\left(-\frac{3b^{2}}{2x^{2}}+3\right)\sin\left(\frac{b}{x}\right)}{6\sqrt{\pi}\,b^{3}}\right)}{b^{4}}+\frac{4\sqrt{\pi}\sin(a)\left(-\frac{1}{2\sqrt{\pi}}+\frac{\left(-\frac{b^{2}}{2x^{2}}+1\right)\cos\left(\frac{b}{x}\right)}{2\sqrt{\pi}}+\frac{b\sin\left(\frac{b}{x}\right)}{2\sqrt{\pi}\,x}\right)}{b^{3}}$

```
[In] int(cos(a+b/x)/x^4,x,method=_RETURNVERBOSE)
[Out] -2/b^2/x*cos((a*x+b)/x)-(b^2-2*x^2)/b^3/x^2*sin((a*x+b)/x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.93

$$\int \frac{\cos\left(a+\frac{b}{x}\right)}{x^4}\,dx=-\frac{2\,bx\cos\left(\frac{ax+b}{x}\right)+\left(b^2-2\,x^2\right)\sin\left(\frac{ax+b}{x}\right)}{b^3x^2}$$

```
[In] integrate(cos(a+b/x)/x^4,x, algorithm="fricas")
[Out] -(2*b*x*cos((a*x + b)/x) + (b^2 - 2*x^2)*sin((a*x + b)/x))/(b^3*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(a+\frac{b}{x}\right)}{x^4}\,dx=\begin{cases}-\frac{\sin\left(a+\frac{b}{x}\right)}{bx^{2}}-\frac{2\cos\left(a+\frac{b}{x}\right)}{b^{2}x}+\frac{2\sin\left(a+\frac{b}{x}\right)}{b^{3}} & \text{for}\ b\neq 0\\ -\frac{\cos\left(a\right)}{3x^{3}} & \text{otherwise}\end{cases}$$

```
[In] integrate(cos(a+b/x)/x**4,x)
[Out] Piecewise((-sin(a + b/x)/(b*x**2) - 2*cos(a + b/x)/(b**2*x) + 2*sin(a + b/x
)/b**3, Ne(b, 0)), (-cos(a)/(3*x**3), True))
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 0.32 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.09

$$\int \frac{\cos\left(a+\frac{b}{x}\right)}{x^4}\,dx = \frac{\left(i\,\Gamma\left(3,\frac{i\,b}{x}\right)-i\,\Gamma\left(3,-\frac{i\,b}{x}\right)\right)\cos\left(a\right)+\left(\Gamma\left(3,\frac{i\,b}{x}\right)+\Gamma\left(3,-\frac{i\,b}{x}\right)\right)\sin\left(a\right)}{2\,b^3}$$

```
[In] integrate(cos(a+b/x)/x^4,x, algorithm="maxima")
```

```
[Out] 1/2*((I*gamma(3, I*b/x) - I*gamma(3, -I*b/x))*cos(a) + (gamma(3, I*b/x) + g
amma(3, -I*b/x))*sin(a))/b^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 107 vs. 2(46) = 92.

Time = 0.41 (sec) , antiderivative size = 107, normalized size of antiderivative = 2.33

$$\int \frac{\cos\left(a+\frac{b}{x}\right)}{x^4}\,dx =$$

$$-\frac{a^2\sin\left(\frac{ax+b}{x}\right)-2\,a\cos\left(\frac{ax+b}{x}\right)-\frac{2\,(ax+b)a\sin\left(\frac{ax+b}{x}\right)}{x}+\frac{2\,(ax+b)\cos\left(\frac{ax+b}{x}\right)}{x}+\frac{(ax+b)^2\sin\left(\frac{ax+b}{x}\right)}{x^2}-2\,\sin\left(\frac{ax+b}{x}\right)}{b^3}$$

```
[In] integrate(cos(a+b/x)/x^4,x, algorithm="giac")
```

```
[Out] -(a^2*sin((a*x + b)/x) - 2*a*cos((a*x + b)/x) - 2*(a*x + b)*a*sin((a*x + b)
/x)/x + 2*(a*x + b)*cos((a*x + b)/x)/x + (a*x + b)^2*sin((a*x + b)/x)/x^2 -
 2*sin((a*x + b)/x))/b^3
```

Mupad [B] (verification not implemented)

Time = 13.37 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.02

$$\int \frac{\cos\left(a+\frac{b}{x}\right)}{x^4}\,dx = \frac{2\,\sin\left(a+\frac{b}{x}\right)}{b^3}-\frac{b^2\,\sin\left(a+\frac{b}{x}\right)+2\,b\,x\,\cos\left(a+\frac{b}{x}\right)}{b^3\,x^2}$$

```
[In] int(cos(a + b/x)/x^4,x)
```

```
[Out] (2*sin(a + b/x))/b^3 - (b^2*sin(a + b/x) + 2*b*x*cos(a + b/x))/(b^3*x^2)
```

3.40 $\int \cos\left(a+\frac{b}{x^2}\right) dx$

Optimal result

Integrand size = 8, antiderivative size = 79

$$\int \cos\left(a+\frac{b}{x^2}\right) dx = x\cos\left(a+\frac{b}{x^2}\right)+\sqrt{b}\sqrt{2\pi}\cos(a)\operatorname{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{x}\right)+\sqrt{b}\sqrt{2\pi}\operatorname{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{x}\right)\sin(a)$$

```
[Out] x*cos(a+b/x^2)+cos(a)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)/x)*b^(1/2)*2^(1/2)*
Pi^(1/2)+FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)/x)*sin(a)*b^(1/2)*2^(1/2)*Pi^(1/
2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {3441, 3469, 3434, 3433, 3432}

$$\int \cos\left(a+\frac{b}{x^2}\right) dx = \sqrt{2\pi}\sqrt{b}\sin(a)\operatorname{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{x}\right)+\sqrt{2\pi}\sqrt{b}\cos(a)\operatorname{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{x}\right)+x\cos\left(a+\frac{b}{x^2}\right)$$

```
[In] Int[Cos[a + b/x^2],x]
```

```
[Out] x*Cos[a + b/x^2] + Sqrt[b]*Sqrt[2*Pi]*Cos[a]*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/
x] + Sqrt[b]*Sqrt[2*Pi]*FresnelC[(Sqrt[b]*Sqrt[2/Pi])/x]*Sin[a]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3434

```
Int[Sin[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Sin[c], Int
[Cos[d*(e + f*x)^2], x], x] + Dist[Cos[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3441

```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_.), x_S
ymbol] :> Dist[-f^(-1), Subst[Int[(a + b*Cos[c + d/x^n])^p/x^2, x], x, 1/(e
 + f*x)], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && ILtQ[n, 0] &&
 EqQ[n, -2]
```

Rule 3469

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_), x_Symbol] :> Simp[(e*x)
^(m + 1)*(Cos[c + d*x^n]/(e*(m + 1))), x] + Dist[d*(n/(e^n*(m + 1))), Int[(
e*x)^(m + n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{\cos(a+bx^2)}{x^2}\,dx, x, \frac{1}{x}\right)\\
&= x\cos\left(a+\frac{b}{x^2}\right) + (2b)\text{Subst}\left(\int \sin(a+bx^2)\,dx, x, \frac{1}{x}\right)\\
&= x\cos\left(a+\frac{b}{x^2}\right) + (2b\cos(a))\text{Subst}\left(\int \sin(bx^2)\,dx, x, \frac{1}{x}\right)\\
&\quad + (2b\sin(a))\text{Subst}\left(\int \cos(bx^2)\,dx, x, \frac{1}{x}\right)\\
&= x\cos\left(a+\frac{b}{x^2}\right) + \sqrt{b}\sqrt{2\pi}\cos(a)\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{x}\right) + \sqrt{b}\sqrt{2\pi}\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{x}\right)\sin(a)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.01

$$\int \cos\left(a+\frac{b}{x^2}\right)\,dx = x\cos(a)\cos\left(\frac{b}{x^2}\right)+\sqrt{b}\sqrt{2\pi}\left(\cos(a)\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{x}\right)+\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{x}\right)\sin(a)\right)-x\sin(a)\sin\left(\frac{b}{x^2}\right)$$

```
[In] Integrate[Cos[a + b/x^2],x]
```

```
[Out] x*Cos[a]*Cos[b/x^2] + Sqrt[b]*Sqrt[2*Pi]*(Cos[a]*FresnelS[(Sqrt[b]*Sqrt[2/P
i])/x] + FresnelC[(Sqrt[b]*Sqrt[2/Pi])/x]*Sin[a]) - x*Sin[a]*Sin[b/x^2]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.72

method	result	s
derivativedivides	$x\cos\left(a+\frac{b}{x^2}\right)+\sqrt{b}\,\sqrt{2}\,\sqrt{\pi}\left(\cos(a)\,\mathrm{S}\left(\frac{\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}\,x}\right)+\sin(a)\,\mathrm{C}\left(\frac{\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}\,x}\right)\right)$	5
default	$x\cos\left(a+\frac{b}{x^2}\right)+\sqrt{b}\,\sqrt{2}\,\sqrt{\pi}\left(\cos(a)\,\mathrm{S}\left(\frac{\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}\,x}\right)+\sin(a)\,\mathrm{C}\left(\frac{\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}\,x}\right)\right)$	5
risch	$\frac{i\mathrm{e}^{-ia}b\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{ib}}{x}\right)}{2\sqrt{ib}}-\frac{i\mathrm{e}^{ia}b\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{-ib}}{x}\right)}{2\sqrt{-ib}}+x\cos\left(\frac{a\,x^2+b}{x^2}\right)$	7
meijerg	$-\frac{\cos(a)\sqrt{\pi}\,\sqrt{2}\,(b^2)^{\frac{1}{4}}\left(-\frac{4x\sqrt{2}\,\cos\left(\frac{b}{x^2}\right)}{\sqrt{\pi}\,(b^2)^{\frac{1}{4}}}-\frac{8\sqrt{b}\,\mathrm{S}\left(\frac{\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}\,x}\right)}{(b^2)^{\frac{1}{4}}}\right)}{8}+\frac{\sqrt{\pi}\,\sin(a)\sqrt{2}\,\sqrt{b}\left(-\frac{4\sqrt{2}\,x\sin\left(\frac{b}{x^2}\right)}{\sqrt{b}\,\sqrt{\pi}}+8\,\mathrm{C}\left(\frac{\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}\,x}\right)\right)}{8}$	1

```
[In] int(cos(a+b/x^2),x,method=_RETURNVERBOSE)
```

```
[Out] x*cos(a+b/x^2)+b^(1/2)*2^(1/2)*Pi^(1/2)*(cos(a)*FresnelS(b^(1/2)*2^(1/2)/Pi
^(1/2)/x)+sin(a)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)/x))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.92

$$\int \cos\left(a+\frac{b}{x^2}\right)\,dx = \sqrt{2}\pi\sqrt{\frac{b}{\pi}}\cos(a)\,\mathrm{S}\left(\frac{\sqrt{2}\sqrt{\frac{b}{\pi}}}{x}\right) + \sqrt{2}\pi\sqrt{\frac{b}{\pi}}\,\mathrm{C}\left(\frac{\sqrt{2}\sqrt{\frac{b}{\pi}}}{x}\right)\sin(a) + x\cos\left(\frac{ax^2+b}{x^2}\right)$$

```
[In] integrate(cos(a+b/x^2),x, algorithm="fricas")
```

```
[Out] sqrt(2)*pi*sqrt(b/pi)*cos(a)*fresnel_sin(sqrt(2)*sqrt(b/pi)/x) + sqrt(2)*pi
*sqrt(b/pi)*fresnel_cos(sqrt(2)*sqrt(b/pi)/x)*sin(a) + x*cos((a*x^2 + b)/x^
2)
```

Sympy [F]

$$\int \cos\left(a+\frac{b}{x^2}\right)\,dx = \int \cos\left(a+\frac{b}{x^2}\right)\,dx$$

```
[In] integrate(cos(a+b/x**2),x)
```

```
[Out] Integral(cos(a + b/x**2), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.38 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.61

$$\int \cos\left(a+\frac{b}{x^2}\right)\,dx = \frac{\sqrt{2}\left(2\sqrt{2}bx^2\sqrt{\frac{1}{x^4}}\cos\left(\frac{ax^2+b}{x^2}\right) + \left(\left((i+1)\sqrt{\pi}\left(\operatorname{erf}\left(\sqrt{\frac{ib}{x^2}}\right)-1\right) - (i-1)\sqrt{\pi}\left(\operatorname{erf}\left(\sqrt{-\frac{ib}{x^2}}\right)-1\right)\right)\cos(a\right.\right.}{4\,bx}$$

```
[In] integrate(cos(a+b/x^2),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(2)*(2*sqrt(2)*b*x^2*sqrt(x^(-4))*cos((a*x^2 + b)/x^2) + (((I + 1)*
sqrt(pi)*(erf(sqrt(I*b/x^2)) - 1) - (I - 1)*sqrt(pi)*(erf(sqrt(-I*b/x^2)) -
 1))*cos(a) + (-(I - 1)*sqrt(pi)*(erf(sqrt(I*b/x^2)) - 1) + (I + 1)*sqrt(pi
)*(erf(sqrt(-I*b/x^2)) - 1))*sin(a))*b*(b^2/x^4)^(1/4))*sqrt(x^4)/(b*x)
```

Giac [F]

$$\int \cos\left(a + \frac{b}{x^2}\right) dx = \int \cos\left(a + \frac{b}{x^2}\right) dx$$

```
[In] integrate(cos(a+b/x^2),x, algorithm="giac")
[Out] integrate(cos(a + b/x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos\left(a + \frac{b}{x^2}\right) dx = \int \cos\left(a + \frac{b}{x^2}\right) dx$$

```
[In] int(cos(a + b/x^2),x)
[Out] int(cos(a + b/x^2), x)
```

3.41 $\int \frac{\cos\left(a+\frac{b}{x^2}\right)}{x} \, dx$

Optimal result

Integrand size = 12, antiderivative size = 25

$$\int \frac{\cos\left(a+\frac{b}{x^2}\right)}{x} \, dx = -\frac{1}{2}\cos(a)\,\text{CosIntegral}\left(\frac{b}{x^2}\right)+\frac{1}{2}\sin(a)\text{Si}\left(\frac{b}{x^2}\right)$$

```
[Out] -1/2*Ci(b/x^2)*cos(a)+1/2*Si(b/x^2)*sin(a)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3459, 3457, 3456}

$$\int \frac{\cos\left(a+\frac{b}{x^2}\right)}{x} \, dx = \frac{1}{2}\sin(a)\text{Si}\left(\frac{b}{x^2}\right)-\frac{1}{2}\cos(a)\,\text{CosIntegral}\left(\frac{b}{x^2}\right)$$

```
[In] Int[Cos[a + b/x^2]/x,x]
```

```
[Out] -1/2*(Cos[a]*CosIntegral[b/x^2]) + (Sin[a]*SinIntegral[b/x^2])/2
```

Rule 3456

```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3457

```
Int[Cos[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[CosIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3459

```
Int[Cos[(c_) + (d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Dist[Cos[c], Int[Cos[d*x
^n]/x, x], x] - Dist[Sin[c], Int[Sin[d*x^n]/x, x], x] /; FreeQ[{c, d, n}, x
]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \cos(a)\int \frac{\cos\left(\frac{b}{x^2}\right)}{x}\,dx - \sin(a)\int \frac{\sin\left(\frac{b}{x^2}\right)}{x}\,dx\\ &= -\frac{1}{2}\cos(a)\,\text{CosIntegral}\left(\frac{b}{x^2}\right) + \frac{1}{2}\sin(a)\text{Si}\left(\frac{b}{x^2}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{\cos\left(a+\frac{b}{x^2}\right)}{x}\,dx = \frac{1}{2}\left(-\cos(a)\,\text{CosIntegral}\left(\frac{b}{x^2}\right) + \sin(a)\text{Si}\left(\frac{b}{x^2}\right)\right)$$

```
[In] Integrate[Cos[a + b/x^2]/x,x]
[Out] (-(Cos[a]*CosIntegral[b/x^2]) + Sin[a]*SinIntegral[b/x^2])/2
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$-\frac{\operatorname{Ci}\left(\frac{b}{x^2}\right)\cos(a)}{2}+\frac{\operatorname{Si}\left(\frac{b}{x^2}\right)\sin(a)}{2}$	22
default	$-\frac{\operatorname{Ci}\left(\frac{b}{x^2}\right)\cos(a)}{2}+\frac{\operatorname{Si}\left(\frac{b}{x^2}\right)\sin(a)}{2}$	22
risch	$-\frac{i\mathrm{e}^{-ia}\operatorname{csgn}\left(\frac{b}{x^2}\right)\pi}{4}+\frac{i\mathrm{e}^{-ia}\operatorname{Si}\left(\frac{b}{x^2}\right)}{2}+\frac{\mathrm{e}^{-ia}\operatorname{Ei}_1\left(-\frac{ib}{x^2}\right)}{4}+\frac{\mathrm{e}^{ia}\operatorname{Ei}_1\left(-\frac{ib}{x^2}\right)}{4}$	63
meijerg	$-\frac{\sqrt{\pi}\cos(a)\left(\frac{2\gamma-4\ln(x)+\ln\left(b^2\right)}{\sqrt{\pi}}-\frac{2\gamma}{\sqrt{\pi}}-\frac{2\ln(2)}{\sqrt{\pi}}-\frac{2\ln\left(\frac{b}{2x^2}\right)}{\sqrt{\pi}}+\frac{2\operatorname{Ci}\left(\frac{b}{x^2}\right)}{\sqrt{\pi}}\right)}{4}+\frac{\operatorname{Si}\left(\frac{b}{x^2}\right)\sin(a)}{2}$	72

```
[In] int(cos(a+b/x^2)/x,x,method=_RETURNVERBOSE)
[Out] -1/2*Ci(b/x^2)*cos(a)+1/2*Si(b/x^2)*sin(a)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{\cos\left(a+\frac{b}{x^2}\right)}{x}\,dx=-\frac{1}{2}\cos\left(a\right)\operatorname{Ci}\left(\frac{b}{x^2}\right)+\frac{1}{2}\sin\left(a\right)\operatorname{Si}\left(\frac{b}{x^2}\right)$$

```
[In] integrate(cos(a+b/x^2)/x,x, algorithm="fricas")
[Out] -1/2*cos(a)*cos_integral(b/x^2) + 1/2*sin(a)*sin_integral(b/x^2)
```

Sympy [F]

$$\int \frac{\cos\left(a+\frac{b}{x^2}\right)}{x}\,dx=\int \frac{\cos\left(a+\frac{b}{x^2}\right)}{x}\,dx$$

```
[In] integrate(cos(a+b/x**2)/x,x)
[Out] Integral(cos(a + b/x**2)/x, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.72

$$\begin{aligned}\int \frac{\cos\left(a+\frac{b}{x^2}\right)}{x}\,dx=&-\frac{1}{4}\left(\operatorname{Ei}\left(\frac{i\,b}{x^2}\right)+\operatorname{Ei}\left(-\frac{i\,b}{x^2}\right)\right)\cos\left(a\right)\\&-\frac{1}{4}\left(i\operatorname{Ei}\left(\frac{i\,b}{x^2}\right)-i\operatorname{Ei}\left(-\frac{i\,b}{x^2}\right)\right)\sin\left(a\right)\end{aligned}$$

```
[In] integrate(cos(a+b/x^2)/x,x, algorithm="maxima")
[Out] -1/4*(Ei(I*b/x^2) + Ei(-I*b/x^2))*cos(a) - 1/4*(I*Ei(I*b/x^2) - I*Ei(-I*b/x
^2))*sin(a)
```

Giac [F]

$$\int \frac{\cos\left(a+\frac{b}{x^2}\right)}{x}\,dx=\int \frac{\cos\left(a+\frac{b}{x^2}\right)}{x}\,dx$$

```
[In] integrate(cos(a+b/x^2)/x,x, algorithm="giac")
[Out] integrate(cos(a + b/x^2)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos\left(a+\frac{b}{x^2}\right)}{x}\,dx=\frac{\sin(a)\ \mathrm{sinint}\left(\frac{b}{x^2}\right)}{2}-\frac{\cos(a)\ \mathrm{cosint}\left(\frac{b}{x^2}\right)}{2}$$

```
[In] int(cos(a + b/x^2)/x,x)
[Out] (sin(a)*sinint(b/x^2))/2 - (cos(a)*cosint(b/x^2))/2
```

3.42 $\int \frac{\cos\left(a+\frac{b}{x^2}\right)}{x^2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 74

$$\int \frac{\cos\left(a+\frac{b}{x^2}\right)}{x^2}\,dx = -\frac{\sqrt{\frac{\pi}{2}}\cos(a)\operatorname{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{x}\right)}{\sqrt{b}} + \frac{\sqrt{\frac{\pi}{2}}\operatorname{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{x}\right)\sin(a)}{\sqrt{b}}$$

```
[Out] -1/2*cos(a)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)/x)*2^(1/2)*Pi^(1/2)/b^(1/2)+1
/2*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)/x)*sin(a)*2^(1/2)*Pi^(1/2)/b^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3465, 3435, 3433, 3432}

$$\int \frac{\cos\left(a+\frac{b}{x^2}\right)}{x^2}\,dx = \frac{\sqrt{\frac{\pi}{2}}\sin(a)\operatorname{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{x}\right)}{\sqrt{b}} - \frac{\sqrt{\frac{\pi}{2}}\cos(a)\operatorname{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{x}\right)}{\sqrt{b}}$$

```
[In] Int[Cos[a + b/x^2]/x^2,x]
```

```
[Out] -((Sqrt[Pi/2]*Cos[a]*FresnelC[(Sqrt[b]*Sqrt[2/Pi])/x])/Sqrt[b]) + (Sqrt[Pi/
2]*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/x]*Sin[a])/Sqrt[b]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3435

```
Int[Cos[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Cos[c], Int
[Cos[d*(e + f*x)^2], x], x] - Dist[Sin[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3465

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_)]*(x_)^(m_.), x_Symbol] :> Dist[2/n, Subst[I
nt[Cos[a + b*x^2], x], x, x^(n/2)], x] /; FreeQ[{a, b, m, n}, x] && EqQ[m,
n/2 - 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \cos\left(a+bx^2\right)\,dx, x, \frac{1}{x}\right)\\
&= -\left(\cos(a)\text{Subst}\left(\int \cos\left(bx^2\right)\,dx, x, \frac{1}{x}\right)\right) + \sin(a)\text{Subst}\left(\int \sin\left(bx^2\right)\,dx, x, \frac{1}{x}\right)\\
&= -\frac{\sqrt{\frac{\pi}{2}}\cos(a)\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{x}\right)}{\sqrt{b}} + \frac{\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{x}\right)\sin(a)}{\sqrt{b}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.84

$$\int \frac{\cos\left(a+\frac{b}{x^2}\right)}{x^2}\,dx = -\frac{\sqrt{\frac{\pi}{2}}\left(\cos(a)\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{x}\right) - \text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{x}\right)\sin(a)\right)}{\sqrt{b}}$$

```
[In] Integrate[Cos[a + b/x^2]/x^2,x]
[Out] -((Sqrt[Pi/2]*(Cos[a]*FresnelC[(Sqrt[b]*Sqrt[2/Pi])/x] - FresnelS[(Sqrt[b]*
Sqrt[2/Pi])/x]*Sin[a]))/Sqrt[b])
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.65

method	result	size
derivativedivides	$-\frac{\sqrt{2}\,\sqrt{\pi}\,\left(\cos(a)\operatorname{C}\left(\frac{\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}\,x}\right)-\sin(a)\operatorname{S}\left(\frac{\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}\,x}\right)\right)}{2\sqrt{b}}$	48
default	$-\frac{\sqrt{2}\,\sqrt{\pi}\,\left(\cos(a)\operatorname{C}\left(\frac{\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}\,x}\right)-\sin(a)\operatorname{S}\left(\frac{\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}\,x}\right)\right)}{2\sqrt{b}}$	48
meijerg	$-\frac{\cos(a)\operatorname{C}\left(\frac{\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}\,x}\right)\sqrt{2}\,\sqrt{\pi}}{2\sqrt{b}}+\frac{\operatorname{S}\left(\frac{\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}\,x}\right)\sin(a)\sqrt{2}\,\sqrt{\pi}}{2\sqrt{b}}$	56
risch	$-\frac{\mathrm{e}^{-ia}\sqrt{\pi}\,\operatorname{erf}\left(\frac{\sqrt{ib}}{x}\right)}{4\sqrt{ib}}-\frac{\mathrm{e}^{ia}\sqrt{\pi}\,\operatorname{erf}\left(\frac{\sqrt{-ib}}{x}\right)}{4\sqrt{-ib}}$	56

```
[In] int(cos(a+b/x^2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*2^(1/2)*Pi^(1/2)/b^(1/2)*(cos(a)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)/x)-
sin(a)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)/x))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.88

$$\int \frac{\cos\left(a+\frac{b}{x^2}\right)}{x^2}\,dx=-\frac{\sqrt{2}\pi\sqrt{\frac{b}{\pi}}\cos\left(a\right)\operatorname{C}\left(\frac{\sqrt{2}\sqrt{\frac{b}{\pi}}}{x}\right)-\sqrt{2}\pi\sqrt{\frac{b}{\pi}}\operatorname{S}\left(\frac{\sqrt{2}\sqrt{\frac{b}{\pi}}}{x}\right)\sin\left(a\right)}{2\,b}$$

```
[In] integrate(cos(a+b/x^2)/x^2,x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(2)*pi*sqrt(b/pi)*cos(a)*fresnel_cos(sqrt(2)*sqrt(b/pi)/x) - sqrt
(2)*pi*sqrt(b/pi)*fresnel_sin(sqrt(2)*sqrt(b/pi)/x)*sin(a))/b
```

Sympy [F]

$$\int \frac{\cos\left(a+\frac{b}{x^2}\right)}{x^2}\,dx=\int \frac{\cos\left(a+\frac{b}{x^2}\right)}{x^2}\,dx$$

```
[In] integrate(cos(a+b/x**2)/x**2,x)
```

```
[Out] Integral(cos(a + b/x**2)/x**2, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.36 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.32

$$\int \frac{\cos\left(a+\frac{b}{x^2}\right)}{x^2}\,dx =$$
$$-\frac{\sqrt{2}\sqrt{x^4}\left(\left(-(i-1)\,\sqrt{\pi}\left(\operatorname{erf}\left(\sqrt{\frac{i\,b}{x^2}}\right)-1\right)+(i+1)\,\sqrt{\pi}\left(\operatorname{erf}\left(\sqrt{-\frac{i\,b}{x^2}}\right)-1\right)\right)\cos(a)+\left(-(i+1)\,\sqrt{\pi}\left(\mathrm{er}\cdots\right.\right.\right.}{8\,bx}$$

```
[In] integrate(cos(a+b/x^2)/x^2,x, algorithm="maxima")
```

```
[Out] -1/8*sqrt(2)*((-(I - 1)*sqrt(pi)*(erf(sqrt(I*b/x^2)) - 1) + (I + 1)*sqrt(pi
)*(erf(sqrt(-I*b/x^2)) - 1))*cos(a) + (-(I + 1)*sqrt(pi)*(erf(sqrt(I*b/x^2)
) - 1) + (I - 1)*sqrt(pi)*(erf(sqrt(-I*b/x^2)) - 1))*sin(a))*sqrt(x^4)*(b^2
/x^4)^(1/4)/(b*x)
```

Giac [F]

$$\int \frac{\cos\left(a+\frac{b}{x^2}\right)}{x^2}\,dx = \int \frac{\cos\left(a+\frac{b}{x^2}\right)}{x^2}\,dx$$

```
[In] integrate(cos(a+b/x^2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(cos(a + b/x^2)/x^2, x)
```

Mupad [B] (verification not implemented)

Time = 13.34 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.74

$$\int \frac{\cos\left(a+\frac{b}{x^2}\right)}{x^2}\,dx = \frac{\sqrt{2}\,\sqrt{\pi}\,\mathrm{S}\left(\frac{\sqrt{2}\,\sqrt{b}}{x\,\sqrt{\pi}}\right)\,\sin(a)}{2\,\sqrt{b}} - \frac{\sqrt{2}\,\sqrt{\pi}\,\mathrm{C}\left(\frac{\sqrt{2}\,\sqrt{b}}{x\,\sqrt{\pi}}\right)\,\cos(a)}{2\,\sqrt{b}}$$

```
[In] int(cos(a + b/x^2)/x^2,x)
```

```
[Out] (2^(1/2)*pi^(1/2)*fresnels((2^(1/2)*b^(1/2))/(x*pi^(1/2)))*sin(a))/(2*b^(1/
2)) - (2^(1/2)*pi^(1/2)*fresnelc((2^(1/2)*b^(1/2))/(x*pi^(1/2)))*cos(a))/(2
*b^(1/2))
```

3.43 $\int \frac{\cos\left(a+\frac{b}{x^2}\right)}{x^3}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 15

$$\int \frac{\cos\left(a+\frac{b}{x^2}\right)}{x^3}\,dx = -\frac{\sin\left(a+\frac{b}{x^2}\right)}{2b}$$

```
[Out] -1/2*sin(a+b/x^2)/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3461, 2717}

$$\int \frac{\cos\left(a+\frac{b}{x^2}\right)}{x^3}\,dx = -\frac{\sin\left(a+\frac{b}{x^2}\right)}{2b}$$

```
[In] Int[Cos[a + b/x^2]/x^3,x]
```

```
[Out] -1/2*Sin[a + b/x^2]/b
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
```

```
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\text{integral} = -\left(\frac{1}{2}\text{Subst}\left(\int \cos(a+bx)\,dx, x, \frac{1}{x^2}\right)\right)$$
$$= -\frac{\sin\left(a+\frac{b}{x^2}\right)}{2b}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(a+\frac{b}{x^2}\right)}{x^3}\,dx = -\frac{\sin\left(a+\frac{b}{x^2}\right)}{2b}$$

```
[In] Integrate[Cos[a + b/x^2]/x^3,x]
[Out] -1/2*Sin[a + b/x^2]/b
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$-\frac{\sin\left(a+\frac{b}{x^2}\right)}{2b}$	14
default	$-\frac{\sin\left(a+\frac{b}{x^2}\right)}{2b}$	14
risch	$-\frac{\sin\left(\frac{a\,x^2+b}{x^2}\right)}{2b}$	18
parallelrisch	$-\frac{\sin\left(\frac{a\,x^2+b}{x^2}\right)}{2b}$	18
norman	$-\frac{\tan\left(\frac{a}{2}+\frac{b}{2x^2}\right)}{b\left(1+\tan^2\left(\frac{a}{2}+\frac{b}{2x^2}\right)\right)}$	34
meijerg	$-\frac{\cos(a)\sin\left(\frac{b}{x^2}\right)}{2b}+\frac{\sqrt{\pi}\,\sin(a)\left(\frac{1}{\sqrt{\pi}}-\frac{\cos\left(\frac{b}{x^2}\right)}{\sqrt{\pi}}\right)}{2b}$	40

```
[In] int(cos(a+b/x^2)/x^3,x,method=_RETURNVERBOSE)
[Out] -1/2*sin(a+b/x^2)/b
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\cos\left(a+\frac{b}{x^2}\right)}{x^3}\,dx = -\frac{\sin\left(\frac{ax^2+b}{x^2}\right)}{2\,b}$$

```
[In] integrate(cos(a+b/x^2)/x^3,x, algorithm="fricas")
[Out] -1/2*sin((a*x^2 + b)/x^2)/b
```

Sympy [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.47

$$\int \frac{\cos\left(a+\frac{b}{x^2}\right)}{x^3}\,dx = \begin{cases} -\frac{\sin\left(a+\frac{b}{x^2}\right)}{2b} & \text{for } b\neq 0 \\ -\frac{\cos\left(a\right)}{2x^2} & \text{otherwise} \end{cases}$$

```
[In] integrate(cos(a+b/x**2)/x**3,x)
[Out] Piecewise((-sin(a + b/x**2)/(2*b), Ne(b, 0)), (-cos(a)/(2*x**2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{\cos\left(a+\frac{b}{x^2}\right)}{x^3}\,dx = -\frac{\sin\left(a+\frac{b}{x^2}\right)}{2\,b}$$

```
[In] integrate(cos(a+b/x^2)/x^3,x, algorithm="maxima")
[Out] -1/2*sin(a + b/x^2)/b
```

Giac [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\cos\left(a+\frac{b}{x^2}\right)}{x^3}\,dx = -\frac{\sin\left(\frac{ax^2+b}{x^2}\right)}{2\,b}$$

```
[In] integrate(cos(a+b/x^2)/x^3,x, algorithm="giac")
[Out] -1/2*sin((a*x^2 + b)/x^2)/b
```

Mupad [B] (verification not implemented)

Time = 13.98 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{\cos\left(a+\frac{b}{x^2}\right)}{x^3}\,dx = -\frac{\sin\left(a+\frac{b}{x^2}\right)}{2\,b}$$

```
[In] int(cos(a + b/x^2)/x^3,x)
[Out] -sin(a + b/x^2)/(2*b)
```

3.44 $\int \frac{\cos\left(a+\frac{b}{x^2}\right)}{x^4}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 97

$$\int \frac{\cos\left(a+\frac{b}{x^2}\right)}{x^4}\,dx = \frac{\sqrt{\frac{\pi}{2}}\cos(a)\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{x}\right)}{2b^{3/2}} + \frac{\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{x}\right)\sin(a)}{2b^{3/2}} - \frac{\sin\left(a+\frac{b}{x^2}\right)}{2bx}$$

```
[Out] -1/2*sin(a+b/x^2)/b/x+1/4*cos(a)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)/x)*2^(1/
2)*Pi^(1/2)/b^(3/2)+1/4*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)/x)*sin(a)*2^(1/2)
*Pi^(1/2)/b^(3/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {3491, 3467, 3434, 3433, 3432}

$$\int \frac{\cos\left(a+\frac{b}{x^2}\right)}{x^4}\,dx = \frac{\sqrt{\frac{\pi}{2}}\sin(a)\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{x}\right)}{2b^{3/2}} + \frac{\sqrt{\frac{\pi}{2}}\cos(a)\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{x}\right)}{2b^{3/2}} - \frac{\sin\left(a+\frac{b}{x^2}\right)}{2bx}$$

```
[In] Int[Cos[a + b/x^2]/x^4,x]
```

```
[Out] (Sqrt[Pi/2]*Cos[a]*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/x])/(2*b^(3/2)) + (Sqrt[Pi
/2]*FresnelC[(Sqrt[b]*Sqrt[2/Pi])/x]*Sin[a])/(2*b^(3/2)) - Sin[a + b/x^2]/(
2*b*x)
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3434

```
Int[Sin[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Sin[c], Int
[Cos[d*(e + f*x)^2], x], x] + Dist[Cos[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3467

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[e^(n
 - 1)*(e*x)^(m - n + 1)*(Sin[c + d*x^n]/(d*n)), x] - Dist[e^n*((m - n + 1)/
(d*n)), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3491

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> -Subst[Int[(a + b*Cos[c + d/x^n])^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a
, b, c, d}, x] && IGtQ[p, 0] && ILtQ[n, 0] && IntegerQ[m] && EqQ[n, -2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int x^2\cos\left(a+bx^2\right)\,dx,x,\frac{1}{x}\right)\\
&= -\frac{\sin\left(a+\frac{b}{x^2}\right)}{2bx}+\frac{\text{Subst}\left(\int\sin\left(a+bx^2\right)\,dx,x,\frac{1}{x}\right)}{2b}\\
&= -\frac{\sin\left(a+\frac{b}{x^2}\right)}{2bx}+\frac{\cos(a)\text{Subst}\left(\int\sin\left(bx^2\right)\,dx,x,\frac{1}{x}\right)}{2b}+\frac{\sin(a)\text{Subst}\left(\int\cos\left(bx^2\right)\,dx,x,\frac{1}{x}\right)}{2b}\\
&= \frac{\sqrt{\frac{\pi}{2}}\cos(a)\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{x}\right)}{2b^{3/2}}+\frac{\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{x}\right)\sin(a)}{2b^{3/2}}-\frac{\sin\left(a+\frac{b}{x^2}\right)}{2bx}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.91

$$\int \frac{\cos\left(a+\frac{b}{x^2}\right)}{x^4}\,dx$$
$$=\frac{\sqrt{2\pi}x\cos(a)\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{x}\right)+\sqrt{2\pi}x\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{x}\right)\sin(a)-2\sqrt{b}\sin\left(a+\frac{b}{x^2}\right)}{4b^{3/2}x}$$

```
[In] Integrate[Cos[a + b/x^2]/x^4,x]
```

```
[Out] (Sqrt[2*Pi]*x*Cos[a]*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/x] + Sqrt[2*Pi]*x*Fresne
lC[(Sqrt[b]*Sqrt[2/Pi])/x]*Sin[a] - 2*Sqrt[b]*Sin[a + b/x^2])/(4*b^(3/2)*x)
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.66

method	result
derivativedivides	$-\frac{\sin\left(a+\frac{b}{x^2}\right)}{2bx}+\frac{\sqrt{2}\,\sqrt{\pi}\,\left(\cos(a)\,\mathrm{S}\left(\frac{\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}\,x}\right)+\sin(a)\,\mathrm{C}\left(\frac{\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}\,x}\right)\right)}{4b^{\frac{3}{2}}}$
default	$-\frac{\sin\left(a+\frac{b}{x^2}\right)}{2bx}+\frac{\sqrt{2}\,\sqrt{\pi}\,\left(\cos(a)\,\mathrm{S}\left(\frac{\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}\,x}\right)+\sin(a)\,\mathrm{C}\left(\frac{\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}\,x}\right)\right)}{4b^{\frac{3}{2}}}$
risch	$\frac{i\mathrm{e}^{-ia}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{ib}}{x}\right)}{8b\sqrt{ib}}-\frac{i\mathrm{e}^{ia}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{-ib}}{x}\right)}{8b\sqrt{-ib}}-\frac{\sin\left(\frac{a\,x^2+b}{x^2}\right)}{2bx}$
meijerg	$-\frac{\sqrt{\pi}\,\cos(a)\sqrt{2}\,\left(b^2\right)^{\frac{1}{4}}\left(\frac{\sqrt{2}\,\left(b^2\right)^{\frac{3}{4}}\sin\left(\frac{b}{x^2}\right)}{2\sqrt{\pi}\,xb}-\frac{\left(b^2\right)^{\frac{3}{4}}\,\mathrm{S}\left(\frac{\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}\,x}\right)}{2b^{\frac{3}{2}}}\right)}{2b^2}+\frac{\sqrt{\pi}\,\sin(a)\sqrt{2}\,\left(-\frac{\sqrt{2}\,\sqrt{b}\,\cos\left(\frac{b}{x^2}\right)}{2\sqrt{\pi}\,x}+\frac{\mathrm{C}\left(\frac{\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}\,x}\right)}{2}\right)}{2b^{\frac{3}{2}}}$

```
[In] int(cos(a+b/x^2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*sin(a+b/x^2)/b/x+1/4/b^(3/2)*2^(1/2)*Pi^(1/2)*(cos(a)*FresnelS(b^(1/2)
*2^(1/2)/Pi^(1/2)/x)+sin(a)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)/x))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.87

$$\int \frac{\cos\left(a+\frac{b}{x^2}\right)}{x^4}\,dx = \frac{\sqrt{2}\pi x\sqrt{\frac{b}{\pi}}\cos(a)\,\mathrm{S}\left(\frac{\sqrt{2}\sqrt{\frac{b}{\pi}}}{x}\right)+\sqrt{2}\pi x\sqrt{\frac{b}{\pi}}\,\mathrm{C}\left(\frac{\sqrt{2}\sqrt{\frac{b}{\pi}}}{x}\right)\sin(a)-2\,b\sin\left(\frac{ax^2+b}{x^2}\right)}{4\,b^2x}$$

```
[In] integrate(cos(a+b/x^2)/x^4,x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(2)*pi*x*sqrt(b/pi)*cos(a)*fresnel_sin(sqrt(2)*sqrt(b/pi)/x) + sqr
t(2)*pi*x*sqrt(b/pi)*fresnel_cos(sqrt(2)*sqrt(b/pi)/x)*sin(a) - 2*b*sin((a*
x^2 + b)/x^2))/(b^2*x)
```

Sympy [F]

$$\int \frac{\cos\left(a+\frac{b}{x^2}\right)}{x^4}\,dx = \int \frac{\cos\left(a+\frac{b}{x^2}\right)}{x^4}\,dx$$

```
[In] integrate(cos(a+b/x**2)/x**4,x)
```

```
[Out] Integral(cos(a + b/x**2)/x**4, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.76

$$\int \frac{\cos\left(a+\frac{b}{x^2}\right)}{x^4}\,dx = \frac{\sqrt{2}(x^4)^{\frac{3}{2}}\left(\left(-(i+1)\,\Gamma\left(\frac{3}{2},\frac{i\,b}{x^2}\right)+(i-1)\,\Gamma\left(\frac{3}{2},-\frac{i\,b}{x^2}\right)\right)\cos(a)+\left((i-1)\,\Gamma\left(\frac{3}{2},\frac{i\,b}{x^2}\right)-(i+1)\,\Gamma\left(\frac{3}{2},-\frac{i\,b}{x^2}\right)\right)\sin(a)}{8\,b^3x^3}$$

```
[In] integrate(cos(a+b/x^2)/x^4,x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(2)*(x^4)^(3/2)*((-(I + 1)*gamma(3/2, I*b/x^2) + (I - 1)*gamma(3/2,
 -I*b/x^2))*cos(a) + ((I - 1)*gamma(3/2, I*b/x^2) - (I + 1)*gamma(3/2, -I*b
/x^2))*sin(a))*(b^2/x^4)^(3/4)/(b^3*x^3)
```

Giac [F]

$$\int \frac{\cos\left(a + \frac{b}{x^2}\right)}{x^4}\,dx = \int \frac{\cos\left(a + \frac{b}{x^2}\right)}{x^4}\,dx$$

```
[In] integrate(cos(a+b/x^2)/x^4,x, algorithm="giac")
[Out] integrate(cos(a + b/x^2)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos\left(a + \frac{b}{x^2}\right)}{x^4}\,dx = \int \frac{\cos\left(a + \frac{b}{x^2}\right)}{x^4}\,dx$$

```
[In] int(cos(a + b/x^2)/x^4,x)
[Out] int(cos(a + b/x^2)/x^4, x)
```

3.45 $\int \frac{\cos^2(\sqrt{x})}{\sqrt{x}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 19

$$\int \frac{\cos^2\left(\sqrt{x}\right)}{\sqrt{x}}\,dx=\sqrt{x}+\cos\left(\sqrt{x}\right)\sin\left(\sqrt{x}\right)$$

```
[Out] cos(x^(1/2))*sin(x^(1/2))+x^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3461, 2715, 8}

$$\int \frac{\cos^2\left(\sqrt{x}\right)}{\sqrt{x}}\,dx=\sqrt{x}+\sin\left(\sqrt{x}\right)\cos\left(\sqrt{x}\right)$$

```
[In] Int[Cos[Sqrt[x]]^2/Sqrt[x],x]
```

```
[Out] Sqrt[x] + Cos[Sqrt[x]]*Sin[Sqrt[x]]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}\text{integral} &= 2\text{Subst}\left(\int \cos^2(x)\,dx, x, \sqrt{x}\right)\\ &= \cos\left(\sqrt{x}\right)\sin\left(\sqrt{x}\right) + \text{Subst}\left(\int 1\,dx, x, \sqrt{x}\right)\\ &= \sqrt{x} + \cos\left(\sqrt{x}\right)\sin\left(\sqrt{x}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int \frac{\cos^2\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = \sqrt{x} + \frac{1}{2}\sin\left(2\sqrt{x}\right)$$

```
[In] Integrate[Cos[Sqrt[x]]^2/Sqrt[x],x]
[Out] Sqrt[x] + Sin[2*Sqrt[x]]/2
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

method	result	size
derivativedivides	$\cos\left(\sqrt{x}\right)\sin\left(\sqrt{x}\right)+\sqrt{x}$	14
default	$\cos\left(\sqrt{x}\right)\sin\left(\sqrt{x}\right)+\sqrt{x}$	14

```
[In] int(cos(x^(1/2))^2/x^(1/2),x,method=_RETURNVERBOSE)
[Out] cos(x^(1/2))*sin(x^(1/2))+x^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \frac{\cos^2\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = \cos\left(\sqrt{x}\right)\sin\left(\sqrt{x}\right) + \sqrt{x}$$

```
[In] integrate(cos(x^(1/2))^2/x^(1/2),x, algorithm="fricas")
[Out] cos(sqrt(x))*sin(sqrt(x)) + sqrt(x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. 2(17) = 34.

Time = 0.12 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.05

$$\int \frac{\cos^2\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = \sqrt{x}\sin^2\left(\sqrt{x}\right) + \sqrt{x}\cos^2\left(\sqrt{x}\right) + \sin\left(\sqrt{x}\right)\cos\left(\sqrt{x}\right)$$

```
[In] integrate(cos(x**(1/2))**2/x**(1/2),x)
[Out] sqrt(x)*sin(sqrt(x))**2 + sqrt(x)*cos(sqrt(x))**2 + sin(sqrt(x))*cos(sqrt(x
))
```

Maxima [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.63

$$\int \frac{\cos^2\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = \sqrt{x} + \frac{1}{2}\sin\left(2\sqrt{x}\right)$$

```
[In] integrate(cos(x^(1/2))^2/x^(1/2),x, algorithm="maxima")
[Out] sqrt(x) + 1/2*sin(2*sqrt(x))
```

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.63

$$\int \frac{\cos^2\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = \sqrt{x} + \frac{1}{2}\sin\left(2\sqrt{x}\right)$$

```
[In] integrate(cos(x^(1/2))^2/x^(1/2),x, algorithm="giac")
[Out] sqrt(x) + 1/2*sin(2*sqrt(x))
```

Mupad [B] (verification not implemented)

Time = 13.92 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.63

$$\int \frac{\cos^2\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = \frac{\sin\left(2\sqrt{x}\right)}{2} + \sqrt{x}$$

```
[In] int(cos(x^(1/2))^2/x^(1/2),x)
[Out] sin(2*x^(1/2))/2 + x^(1/2)
```

3.46 $\int \frac{\cos(\sqrt{x})}{\sqrt{x}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 8

$$\int \frac{\cos\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = 2\sin\left(\sqrt{x}\right)$$

```
[Out] 2*sin(x^(1/2))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3461, 2717}

$$\int \frac{\cos\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = 2\sin\left(\sqrt{x}\right)$$

```
[In] Int[Cos[Sqrt[x]]/Sqrt[x],x]
```

```
[Out] 2*Sin[Sqrt[x]]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
```

```
m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}\text{integral} &= 2\text{Subst}\left(\int \cos(x)\,dx, x, \sqrt{x}\right)\\ &= 2\sin\left(\sqrt{x}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = 2\sin\left(\sqrt{x}\right)$$

```
[In] Integrate[Cos[Sqrt[x]]/Sqrt[x],x]
[Out] 2*Sin[Sqrt[x]]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$2\sin\left(\sqrt{x}\right)$	7
default	$2\sin\left(\sqrt{x}\right)$	7
meijerg	$2\sin\left(\sqrt{x}\right)$	7

```
[In] int(cos(x^(1/2))/x^(1/2),x,method=_RETURNVERBOSE)
[Out] 2*sin(x^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{\cos\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = 2\,\sin\left(\sqrt{x}\right)$$

```
[In] integrate(cos(x^(1/2))/x^(1/2),x, algorithm="fricas")
[Out] 2*sin(sqrt(x))
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

$$\int \frac{\cos\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = 2\sin\left(\sqrt{x}\right)$$

```
[In] integrate(cos(x**(1/2))/x**(1/2),x)
[Out] 2*sin(sqrt(x))
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{\cos\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = 2\,\sin\left(\sqrt{x}\right)$$

```
[In] integrate(cos(x^(1/2))/x^(1/2),x, algorithm="maxima")
[Out] 2*sin(sqrt(x))
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{\cos\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = 2\,\sin\left(\sqrt{x}\right)$$

```
[In] integrate(cos(x^(1/2))/x^(1/2),x, algorithm="giac")
[Out] 2*sin(sqrt(x))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{\cos\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = 2\,\sin\left(\sqrt{x}\right)$$

```
[In] int(cos(x^(1/2))/x^(1/2),x)
[Out] 2*sin(x^(1/2))
```

3.47 $\int \cos\left(\sqrt{x}\right)\,dx$

Optimal result

Integrand size = 6, antiderivative size = 22

$$\int \cos\left(\sqrt{x}\right)\,dx = 2\cos\left(\sqrt{x}\right) + 2\sqrt{x}\sin\left(\sqrt{x}\right)$$

```
[Out] 2*cos(x^(1/2))+2*sin(x^(1/2))*x^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3443, 3377, 2718}

$$\int \cos\left(\sqrt{x}\right)\,dx = 2\sqrt{x}\sin\left(\sqrt{x}\right) + 2\cos\left(\sqrt{x}\right)$$

```
[In] Int[Cos[Sqrt[x]],x]
```

```
[Out] 2*Cos[Sqrt[x]] + 2*Sqrt[x]*Sin[Sqrt[x]]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3443

```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_.), x_S
ymbol] :> Dist[1/(n*f), Subst[Int[x^(1/n - 1)*(a + b*Cos[c + d*x])^p, x], x
, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && Integer
Q[1/n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= 2\text{Subst}\left(\int x\cos(x)\,dx, x, \sqrt{x}\right)\\ &= 2\sqrt{x}\sin\left(\sqrt{x}\right) - 2\text{Subst}\left(\int \sin(x)\,dx, x, \sqrt{x}\right)\\ &= 2\cos\left(\sqrt{x}\right) + 2\sqrt{x}\sin\left(\sqrt{x}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \cos\left(\sqrt{x}\right)\,dx = 2\cos\left(\sqrt{x}\right) + 2\sqrt{x}\sin\left(\sqrt{x}\right)$$

```
[In] Integrate[Cos[Sqrt[x]],x]
[Out] 2*Cos[Sqrt[x]] + 2*Sqrt[x]*Sin[Sqrt[x]]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

method	result	size
derivativedivides	$2\cos\left(\sqrt{x}\right) + 2\sin\left(\sqrt{x}\right)\sqrt{x}$	17
default	$2\cos\left(\sqrt{x}\right) + 2\sin\left(\sqrt{x}\right)\sqrt{x}$	17
meijerg	$4\sqrt{\pi}\left(-\frac{1}{2\sqrt{\pi}} + \frac{\cos(\sqrt{x})}{2\sqrt{\pi}} + \frac{\sqrt{x}\sin(\sqrt{x})}{2\sqrt{\pi}}\right)$	33

```
[In] int(cos(x^(1/2)),x,method=_RETURNVERBOSE)
[Out] 2*cos(x^(1/2))+2*sin(x^(1/2))*x^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \cos\left(\sqrt{x}\right)\,dx = 2\,\sqrt{x}\sin\left(\sqrt{x}\right) + 2\,\cos\left(\sqrt{x}\right)$$

```
[In] integrate(cos(x^(1/2)),x, algorithm="fricas")
[Out] 2*sqrt(x)*sin(sqrt(x)) + 2*cos(sqrt(x))
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \cos\left(\sqrt{x}\right)\,dx = 2\sqrt{x}\sin\left(\sqrt{x}\right) + 2\cos\left(\sqrt{x}\right)$$

```
[In] integrate(cos(x**(1/2)),x)
[Out] 2*sqrt(x)*sin(sqrt(x)) + 2*cos(sqrt(x))
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \cos\left(\sqrt{x}\right)\,dx = 2\,\sqrt{x}\sin\left(\sqrt{x}\right) + 2\,\cos\left(\sqrt{x}\right)$$

```
[In] integrate(cos(x^(1/2)),x, algorithm="maxima")
[Out] 2*sqrt(x)*sin(sqrt(x)) + 2*cos(sqrt(x))
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \cos\left(\sqrt{x}\right)\,dx = 2\,\sqrt{x}\sin\left(\sqrt{x}\right) + 2\,\cos\left(\sqrt{x}\right)$$

```
[In] integrate(cos(x^(1/2)),x, algorithm="giac")
[Out] 2*sqrt(x)*sin(sqrt(x)) + 2*cos(sqrt(x))
```

Mupad [B] (verification not implemented)

Time = 13.77 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \cos \left(\sqrt{x}\right) \, dx = 2 \cos \left(\sqrt{x}\right) + 2 \sqrt{x} \sin \left(\sqrt{x}\right)$$

```
[In] int(cos(x^(1/2)),x)
[Out] 2*cos(x^(1/2)) + 2*x^(1/2)*sin(x^(1/2))
```

3.48 $\int \cos^2\left(\sqrt{x}\right)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 36

$$\int \cos^2\left(\sqrt{x}\right)\,dx=\frac{x}{2}+\frac{1}{2}\cos^2\left(\sqrt{x}\right)+\sqrt{x}\cos\left(\sqrt{x}\right)\sin\left(\sqrt{x}\right)$$

```
[Out] 1/2*x+1/2*cos(x^(1/2))^2+cos(x^(1/2))*sin(x^(1/2))*x^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3443, 3391, 30}

$$\int \cos^2\left(\sqrt{x}\right)\,dx=\frac{x}{2}+\frac{1}{2}\cos^2\left(\sqrt{x}\right)+\sqrt{x}\sin\left(\sqrt{x}\right)\cos\left(\sqrt{x}\right)$$

```
[In] Int[Cos[Sqrt[x]]^2,x]
```

```
[Out] x/2 + Cos[Sqrt[x]]^2/2 + Sqrt[x]*Cos[Sqrt[x]]*Sin[Sqrt[x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
```

```
]
```

Rule 3443

```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_.), x_S
ymbol] :> Dist[1/(n*f), Subst[Int[x^(1/n - 1)*(a + b*Cos[c + d*x])^p, x], x
, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && Integer
Q[1/n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= 2\text{Subst}\left(\int x\cos^2(x)\,dx, x, \sqrt{x}\right)\\ &= \frac{1}{2}\cos^2\left(\sqrt{x}\right)+\sqrt{x}\cos\left(\sqrt{x}\right)\sin\left(\sqrt{x}\right)+\text{Subst}\left(\int x\,dx, x, \sqrt{x}\right)\\ &= \frac{x}{2}+\frac{1}{2}\cos^2\left(\sqrt{x}\right)+\sqrt{x}\cos\left(\sqrt{x}\right)\sin\left(\sqrt{x}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.86

$$\int \cos^2\left(\sqrt{x}\right)\,dx = \frac{1}{4}\left(\cos\left(2\sqrt{x}\right)+2\left(x+\sqrt{x}\sin\left(2\sqrt{x}\right)\right)\right)$$

```
[In] Integrate[Cos[Sqrt[x]]^2,x]
[Out] (Cos[2*Sqrt[x]] + 2*(x + Sqrt[x]*Sin[2*Sqrt[x]]))/4
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.94

method	result	size
derivativedivides	$2\sqrt{x}\left(\frac{\cos(\sqrt{x})\sin(\sqrt{x})}{2}+\frac{\sqrt{x}}{2}\right)-\frac{x}{2}-\frac{(\sin^2(\sqrt{x}))}{2}$	34
default	$2\sqrt{x}\left(\frac{\cos(\sqrt{x})\sin(\sqrt{x})}{2}+\frac{\sqrt{x}}{2}\right)-\frac{x}{2}-\frac{(\sin^2(\sqrt{x}))}{2}$	34

```
[In] int(cos(x^(1/2))^2,x,method=_RETURNVERBOSE)
[Out] 2*x^(1/2)*(1/2*cos(x^(1/2))*sin(x^(1/2))+1/2*x^(1/2))-1/2*x-1/2*sin(x^(1/2)
)^2
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

$$\int \cos^2\left(\sqrt{x}\right)\,dx = \sqrt{x}\cos\left(\sqrt{x}\right)\sin\left(\sqrt{x}\right) + \frac{1}{2}\cos\left(\sqrt{x}\right)^2 + \frac{1}{2}x$$

```
[In] integrate(cos(x^(1/2))^2,x, algorithm="fricas")
[Out] sqrt(x)*cos(sqrt(x))*sin(sqrt(x)) + 1/2*cos(sqrt(x))^2 + 1/2*x
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.42

$$\int \cos^2\left(\sqrt{x}\right)\,dx = \sqrt{x}\sin\left(\sqrt{x}\right)\cos\left(\sqrt{x}\right) + \frac{x\sin^2\left(\sqrt{x}\right)}{2} + \frac{x\cos^2\left(\sqrt{x}\right)}{2} - \frac{\sin^2\left(\sqrt{x}\right)}{2}$$

```
[In] integrate(cos(x**(1/2))**2,x)
[Out] sqrt(x)*sin(sqrt(x))*cos(sqrt(x)) + x*sin(sqrt(x))**2/2 + x*cos(sqrt(x))**2
/2 - sin(sqrt(x))**2/2
```

Maxima [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.64

$$\int \cos^2\left(\sqrt{x}\right)\,dx = \frac{1}{2}\sqrt{x}\sin\left(2\sqrt{x}\right) + \frac{1}{2}x + \frac{1}{4}\cos\left(2\sqrt{x}\right)$$

```
[In] integrate(cos(x^(1/2))^2,x, algorithm="maxima")
[Out] 1/2*sqrt(x)*sin(2*sqrt(x)) + 1/2*x + 1/4*cos(2*sqrt(x))
```

Giac [A] (verification not implemented)

none

Time = 0.41 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.64

$$\int \cos^2\left(\sqrt{x}\right)\,dx = \frac{1}{2}\sqrt{x}\sin\left(2\sqrt{x}\right) + \frac{1}{2}x + \frac{1}{4}\cos\left(2\sqrt{x}\right)$$

```
[In] integrate(cos(x^(1/2))^2,x, algorithm="giac")
[Out] 1/2*sqrt(x)*sin(2*sqrt(x)) + 1/2*x + 1/4*cos(2*sqrt(x))
```

Mupad [B] (verification not implemented)

Time = 13.71 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.64

$$\int \cos^2\left(\sqrt{x}\right)\,dx = \frac{x}{2} - \frac{\sin\left(\sqrt{x}\right)^2}{2} + \frac{\sqrt{x}\,\sin\left(2\sqrt{x}\right)}{2}$$

```
[In] int(cos(x^(1/2))^2,x)
[Out] x/2 - sin(x^(1/2))^2/2 + (x^(1/2)*sin(2*x^(1/2)))/2
```

3.49 $\int x^{3/2}\cos\left(a+b\sqrt[3]{x}\right)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 235

$$\begin{aligned}\int x^{3/2}\cos\left(a+b\sqrt[3]{x}\right)\,dx=&\frac{135135\sqrt{x}\cos\left(a+b\sqrt[3]{x}\right)}{32b^6}-\frac{3861x^{7/6}\cos\left(a+b\sqrt[3]{x}\right)}{8b^4}\\&+\frac{39x^{11/6}\cos\left(a+b\sqrt[3]{x}\right)}{2b^2}+\frac{405405\sqrt{\frac{\pi}{2}}\cos(a)\operatorname{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[6]{x}\right)}{64b^{15/2}}\\&+\frac{405405\sqrt{\frac{\pi}{2}}\operatorname{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[6]{x}\right)\sin(a)}{64b^{15/2}}-\frac{405405\sqrt[6]{x}\sin\left(a+b\sqrt[3]{x}\right)}{64b^7}\\&+\frac{27027x^{5/6}\sin\left(a+b\sqrt[3]{x}\right)}{16b^5}-\frac{429x^{3/2}\sin\left(a+b\sqrt[3]{x}\right)}{4b^3}+\frac{3x^{13/6}\sin\left(a+b\sqrt[3]{x}\right)}{b}\end{aligned}$$

```
[Out] -3861/8*x^(7/6)*cos(a+b*x^(1/3))/b^4+39/2*x^(11/6)*cos(a+b*x^(1/3))/b^2-405
405/64*x^(1/6)*sin(a+b*x^(1/3))/b^7+27027/16*x^(5/6)*sin(a+b*x^(1/3))/b^5-4
29/4*x^(3/2)*sin(a+b*x^(1/3))/b^3+3*x^(13/6)*sin(a+b*x^(1/3))/b+405405/128*
cos(a)*FresnelS(x^(1/6)*b^(1/2)*2^(1/2)/Pi^(1/2))*2^(1/2)*Pi^(1/2)/b^(15/2)
+405405/128*FresnelC(x^(1/6)*b^(1/2)*2^(1/2)/Pi^(1/2))*sin(a)*2^(1/2)*Pi^(1
/2)/b^(15/2)+135135/32*cos(a+b*x^(1/3))*x^(1/2)/b^6
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used

$= \{3497, 3377, 3387, 3386, 3432, 3385, 3433\}$

$$\int x^{3/2}\cos\left(a+b\sqrt[3]{x}\right)\,dx = \frac{405405\sqrt{\frac{\pi}{2}}\sin(a)\,\mathrm{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[6]{x}\right)}{64b^{15/2}} + \frac{405405\sqrt{\frac{\pi}{2}}\cos(a)\,\mathrm{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[6]{x}\right)}{64b^{15/2}} - \frac{405405\sqrt[6]{x}\sin\left(a+b\sqrt[3]{x}\right)}{64b^7} + \frac{135135\sqrt{x}\cos\left(a+b\sqrt[3]{x}\right)}{32b^6} + \frac{27027x^{5/6}\sin\left(a+b\sqrt[3]{x}\right)}{16b^5} - \frac{3861x^{7/6}\cos\left(a+b\sqrt[3]{x}\right)}{8b^4} - \frac{429x^{3/2}\sin\left(a+b\sqrt[3]{x}\right)}{4b^3} + \frac{39x^{11/6}\cos\left(a+b\sqrt[3]{x}\right)}{2b^2} + \frac{3x^{13/6}\sin\left(a+b\sqrt[3]{x}\right)}{b}$$

```
[In] Int[x^(3/2)*Cos[a + b*x^(1/3)],x]
```

```
[Out] (135135*Sqrt[x]*Cos[a + b*x^(1/3)])/(32*b^6) - (3861*x^(7/6)*Cos[a + b*x^(1
/3)])/(8*b^4) + (39*x^(11/6)*Cos[a + b*x^(1/3)])/(2*b^2) + (405405*Sqrt[Pi/
2]*Cos[a]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*x^(1/6)])/(64*b^(15/2)) + (405405*Sqr
t[Pi/2]*FresnelC[Sqrt[b]*Sqrt[2/Pi]*x^(1/6)]*Sin[a])/(64*b^(15/2)) - (40540
5*x^(1/6)*Sin[a + b*x^(1/3)])/(64*b^7) + (27027*x^(5/6)*Sin[a + b*x^(1/3)])
/(16*b^5) - (429*x^(3/2)*Sin[a + b*x^(1/3)])/(4*b^3) + (3*x^(13/6)*Sin[a +
b*x^(1/3)])/b
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3497

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Module[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a +
b*Cos[c + d*x^(k*n)])^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, m}, x]
&& IntegerQ[p] && FractionQ[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= 3\text{Subst}\left(\int x^{13/2}\cos(a+bx)\,dx, x, \sqrt[3]{x}\right)\\
&= \frac{3x^{13/6}\sin\left(a+b\sqrt[3]{x}\right)}{b} - \frac{39\text{Subst}\left(\int x^{11/2}\sin(a+bx)\,dx, x, \sqrt[3]{x}\right)}{2b}\\
&= \frac{39x^{11/6}\cos\left(a+b\sqrt[3]{x}\right)}{2b^2} + \frac{3x^{13/6}\sin\left(a+b\sqrt[3]{x}\right)}{b} - \frac{429\text{Subst}\left(\int x^{9/2}\cos(a+bx)\,dx, x, \sqrt[3]{x}\right)}{4b^2}\\
&= \frac{39x^{11/6}\cos\left(a+b\sqrt[3]{x}\right)}{2b^2} - \frac{429x^{3/2}\sin\left(a+b\sqrt[3]{x}\right)}{4b^3}\\
&\quad + \frac{3x^{13/6}\sin\left(a+b\sqrt[3]{x}\right)}{b} + \frac{3861\text{Subst}\left(\int x^{7/2}\sin(a+bx)\,dx, x, \sqrt[3]{x}\right)}{8b^3}\\
&= -\frac{3861x^{7/6}\cos\left(a+b\sqrt[3]{x}\right)}{8b^4} + \frac{39x^{11/6}\cos\left(a+b\sqrt[3]{x}\right)}{2b^2} - \frac{429x^{3/2}\sin\left(a+b\sqrt[3]{x}\right)}{4b^3}\\
&\quad + \frac{3x^{13/6}\sin\left(a+b\sqrt[3]{x}\right)}{b} + \frac{27027\text{Subst}\left(\int x^{5/2}\cos(a+bx)\,dx, x, \sqrt[3]{x}\right)}{16b^4}\\
&= -\frac{3861x^{7/6}\cos\left(a+b\sqrt[3]{x}\right)}{8b^4} + \frac{39x^{11/6}\cos\left(a+b\sqrt[3]{x}\right)}{2b^2}\\
&\quad + \frac{27027x^{5/6}\sin\left(a+b\sqrt[3]{x}\right)}{16b^5} - \frac{429x^{3/2}\sin\left(a+b\sqrt[3]{x}\right)}{4b^3}\\
&\quad + \frac{3x^{13/6}\sin\left(a+b\sqrt[3]{x}\right)}{b} - \frac{135135\text{Subst}\left(\int x^{3/2}\sin(a+bx)\,dx, x, \sqrt[3]{x}\right)}{32b^5}
\end{aligned}
$$

$$= \frac{135135\sqrt{x}\cos\left(a+b\sqrt[3]{x}\right)}{32b^6} - \frac{3861x^{7/6}\cos\left(a+b\sqrt[3]{x}\right)}{8b^4} + \frac{39x^{11/6}\cos\left(a+b\sqrt[3]{x}\right)}{2b^2} + \frac{27027x^{5/6}\sin\left(a+b\sqrt[3]{x}\right)}{16b^5} - \frac{429x^{3/2}\sin\left(a+b\sqrt[3]{x}\right)}{4b^3} + \frac{3x^{13/6}\sin\left(a+b\sqrt[3]{x}\right)}{b} - \frac{405405\text{Subst}\left(\int \sqrt{x}\cos(a+bx)\,dx, x, \sqrt[3]{x}\right)}{64b^6}$$

$$= \frac{135135\sqrt{x}\cos\left(a+b\sqrt[3]{x}\right)}{32b^6} - \frac{3861x^{7/6}\cos\left(a+b\sqrt[3]{x}\right)}{8b^4} + \frac{39x^{11/6}\cos\left(a+b\sqrt[3]{x}\right)}{2b^2} - \frac{405405\sqrt[6]{x}\sin\left(a+b\sqrt[3]{x}\right)}{64b^7} + \frac{27027x^{5/6}\sin\left(a+b\sqrt[3]{x}\right)}{16b^5} - \frac{429x^{3/2}\sin\left(a+b\sqrt[3]{x}\right)}{4b^3} + \frac{3x^{13/6}\sin\left(a+b\sqrt[3]{x}\right)}{b} + \frac{405405\text{Subst}\left(\int \frac{\sin(a+bx)}{\sqrt{x}}\,dx, x, \sqrt[3]{x}\right)}{128b^7}$$

$$= \frac{135135\sqrt{x}\cos\left(a+b\sqrt[3]{x}\right)}{32b^6} - \frac{3861x^{7/6}\cos\left(a+b\sqrt[3]{x}\right)}{8b^4} + \frac{39x^{11/6}\cos\left(a+b\sqrt[3]{x}\right)}{2b^2} - \frac{405405\sqrt[6]{x}\sin\left(a+b\sqrt[3]{x}\right)}{64b^7} + \frac{27027x^{5/6}\sin\left(a+b\sqrt[3]{x}\right)}{16b^5} - \frac{429x^{3/2}\sin\left(a+b\sqrt[3]{x}\right)}{4b^3} + \frac{3x^{13/6}\sin\left(a+b\sqrt[3]{x}\right)}{b} + \frac{(405405\cos(a))\text{Subst}\left(\int \frac{\sin(bx)}{\sqrt{x}}\,dx, x, \sqrt[3]{x}\right)}{128b^7} + \frac{(405405\sin(a))\text{Subst}\left(\int \frac{\cos(bx)}{\sqrt{x}}\,dx, x, \sqrt[3]{x}\right)}{128b^7}$$

$$= \frac{135135\sqrt{x}\cos\left(a+b\sqrt[3]{x}\right)}{32b^6} - \frac{3861x^{7/6}\cos\left(a+b\sqrt[3]{x}\right)}{8b^4} + \frac{39x^{11/6}\cos\left(a+b\sqrt[3]{x}\right)}{2b^2} - \frac{405405\sqrt[6]{x}\sin\left(a+b\sqrt[3]{x}\right)}{64b^7} + \frac{27027x^{5/6}\sin\left(a+b\sqrt[3]{x}\right)}{16b^5} - \frac{429x^{3/2}\sin\left(a+b\sqrt[3]{x}\right)}{4b^3} + \frac{3x^{13/6}\sin\left(a+b\sqrt[3]{x}\right)}{b} + \frac{(405405\cos(a))\text{Subst}\left(\int \sin\left(bx^2\right)\,dx, x, \sqrt[6]{x}\right)}{64b^7} + \frac{(405405\sin(a))\text{Subst}\left(\int \cos\left(bx^2\right)\,dx, x, \sqrt[6]{x}\right)}{64b^7}$$

$$= \frac{135135\sqrt{x}\cos\left(a+b\sqrt[3]{x}\right)}{32b^6} - \frac{3861x^{7/6}\cos\left(a+b\sqrt[3]{x}\right)}{8b^4} + \frac{39x^{11/6}\cos\left(a+b\sqrt[3]{x}\right)}{2b^2} + \frac{405405\sqrt{\frac{\pi}{2}}\cos(a)\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[6]{x}\right)}{64b^{15/2}} + \frac{405405\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[6]{x}\right)\sin(a)}{64b^{15/2}} - \frac{405405\sqrt[6]{x}\sin\left(a+b\sqrt[3]{x}\right)}{64b^7} + \frac{27027x^{5/6}\sin\left(a+b\sqrt[3]{x}\right)}{16b^5} - \frac{429x^{3/2}\sin\left(a+b\sqrt[3]{x}\right)}{4b^3} + \frac{3x^{13/6}\sin\left(a+b\sqrt[3]{x}\right)}{b}$$

Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.70

$$\int x^{3/2}\cos\left(a+b\sqrt[3]{x}\right)\,dx = \frac{405405\sqrt{2\pi}\cos(a)\operatorname{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[6]{x}\right)+405405\sqrt{2\pi}\operatorname{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[6]{x}\right)\sin(a)+6\sqrt{b}\sqrt[6]{x}}{}$$

```
[In] Integrate[x^(3/2)*Cos[a + b*x^(1/3)],x]
```

```
[Out] (405405*Sqrt[2*Pi]*Cos[a]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*x^(1/6)] + 405405*Sqr
t[2*Pi]*FresnelC[Sqrt[b]*Sqrt[2/Pi]*x^(1/6)]*Sin[a] + 6*Sqrt[b]*x^(1/6)*(26
*(3465*b*x^(1/3) - 396*b^3*x + 16*b^5*x^(5/3))*Cos[a + b*x^(1/3)] + (-13513
5 + 36036*b^2*x^(2/3) - 2288*b^4*x^(4/3) + 64*b^6*x^2)*Sin[a + b*x^(1/3)]))
/(128*b^(15/2))
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.83

method	result
derivativedivides	$\frac{3x^{\frac{13}{6}}\sin\left(a+b\,x^{\frac{1}{3}}\right)}{b}-\frac{39\left(-\frac{x^{\frac{11}{6}}\cos\left(a+b\,x^{\frac{1}{3}}\right)}{2b}+\frac{11x^{\frac{3}{2}}\sin\left(a+b\,x^{\frac{1}{3}}\right)}{4b}-\frac{99\left(-\frac{x^{\frac{7}{6}}\cos\left(a+b\,x^{\frac{1}{3}}\right)}{2b}+\frac{7x^{\frac{5}{6}}\sin\left(a+b\,x^{\frac{1}{3}}\right)}{4b}-\frac{35\left(-\frac{\sqrt{}}{}\right.}{}\right.}{}\right.}{b}$
default	$\frac{3x^{\frac{13}{6}}\sin\left(a+b\,x^{\frac{1}{3}}\right)}{b}-\frac{39\left(-\frac{x^{\frac{11}{6}}\cos\left(a+b\,x^{\frac{1}{3}}\right)}{2b}+\frac{11x^{\frac{3}{2}}\sin\left(a+b\,x^{\frac{1}{3}}\right)}{4b}-\frac{99\left(-\frac{x^{\frac{7}{6}}\cos\left(a+b\,x^{\frac{1}{3}}\right)}{2b}+\frac{7x^{\frac{5}{6}}\sin\left(a+b\,x^{\frac{1}{3}}\right)}{4b}-\frac{35\left(-\frac{\sqrt{}}{}\right.}{}\right.}{}\right.}{b}$
meijerg	$\frac{192\sqrt{2}\cos(a)\sqrt{\pi}\left(\frac{\sqrt{x}\,\sqrt{2}\left(b^{2}\right)^{\frac{15}{4}}\left(3120x^{\frac{4}{3}}b^{4}-77220x^{\frac{2}{3}}b^{2}+675675\right)\cos\left(b\,x^{\frac{1}{3}}\right)}{61440\sqrt{\pi}\,b^{6}}-\frac{x^{\frac{1}{6}}\sqrt{2}\left(b^{2}\right)^{\frac{15}{4}}\left(-960x^{2}b^{6}+34320x^{\frac{4}{3}}b^{4}-540540\right.}{122880\sqrt{\pi}\,b^{7}}\right.}{\left(b^{2}\right)^{\frac{15}{4}}}$

```
[In] int(x^(3/2)*cos(a+b*x^(1/3)),x,method=_RETURNVERBOSE)
```

```
[Out] 3*x^(13/6)*sin(a+b*x^(1/3))/b-39/b*(-1/2/b*x^(11/6)*cos(a+b*x^(1/3))+11/2/b
*(1/2/b*x^(3/2)*sin(a+b*x^(1/3))-9/2/b*(-1/2/b*x^(7/6)*cos(a+b*x^(1/3))+7/2
/b*(1/2/b*x^(5/6)*sin(a+b*x^(1/3))-5/2/b*(-1/2/b*x^(1/2)*cos(a+b*x^(1/3))+3
/2/b*(1/2*x^(1/6)*sin(a+b*x^(1/3))/b-1/4/b^(3/2)*2^(1/2)*Pi^(1/2)*(cos(a)*F
resnelS(x^(1/6)*b^(1/2)*2^(1/2)/Pi^(1/2))+sin(a)*FresnelC(x^(1/6)*b^(1/2)*2
^(1/2)/Pi^(1/2))))))))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.62

$$\int x^{3/2}\cos\left(a+b\sqrt[3]{x}\right)\,dx=\frac{3\left(135135\,\sqrt{2}\pi\sqrt{\frac{b}{\pi}}\cos\left(a\right)\mathrm{S}\left(\sqrt{2}x^{\frac{1}{6}}\sqrt{\frac{b}{\pi}}\right)+135135\,\sqrt{2}\pi\sqrt{\frac{b}{\pi}}\,\mathrm{C}\left(\sqrt{2}x^{\frac{1}{6}}\sqrt{\frac{b}{\pi}}\right)\sin\left(a\right)+52\left(16\,b^{6}x\right.\right.}{}$$

```
[In] integrate(x^(3/2)*cos(a+b*x^(1/3)),x, algorithm="fricas")
```

```
[Out] 3/128*(135135*sqrt(2)*pi*sqrt(b/pi)*cos(a)*fresnel_sin(sqrt(2)*x^(1/6)*sqrt
(b/pi)) + 135135*sqrt(2)*pi*sqrt(b/pi)*fresnel_cos(sqrt(2)*x^(1/6)*sqrt(b/p
i))*sin(a) + 52*(16*b^6*x^(11/6) - 396*b^4*x^(7/6) + 3465*b^2*sqrt(x))*cos(
b*x^(1/3) + a) - 2*(2288*b^5*x^(3/2) - 36036*b^3*x^(5/6) - (64*b^7*x^2 - 13
5135*b)*x^(1/6))*sin(b*x^(1/3) + a))/b^8
```

Sympy [F]

$$\int x^{3/2}\cos\left(a+b\sqrt[3]{x}\right)\,dx=\int x^{\frac{3}{2}}\cos\left(a+b\sqrt[3]{x}\right)dx$$

```
[In] integrate(x**(3/2)*cos(a+b*x**(1/3)),x)
```

```
[Out] Integral(x**(3/2)*cos(a + b*x**(1/3)), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.46 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.58

$$\int x^{3/2}\cos\left(a+b\sqrt[3]{x}\right)\,dx = \frac{3\left(135135\sqrt{2}\sqrt{\pi}\left(\left((i+1)\cos(a)-(i-1)\sin(a)\right)\operatorname{erf}\left(\sqrt{i\,b}x^{\frac{1}{6}}\right)+\left(-(i-1)\cos(a)+(i+1)\,\mathrm{s}\right.\right.\right.}{}$$

```
[In] integrate(x^(3/2)*cos(a+b*x^(1/3)),x, algorithm="maxima")
```

```
[Out] 3/512*(135135*sqrt(2)*sqrt(pi)*(((I + 1)*cos(a) - (I - 1)*sin(a))*erf(sqrt(
I*b)*x^(1/6)) + (-(I - 1)*cos(a) + (I + 1)*sin(a))*erf(sqrt(-I*b)*x^(1/6)))
*b^(3/2) + 208*(16*b^7*x^(11/6) - 396*b^5*x^(7/6) + 3465*b^3*sqrt(x))*cos(b
*x^(1/3) + a) + 8*(64*b^8*x^(13/6) - 2288*b^6*x^(3/2) + 36036*b^4*x^(5/6) -
 135135*b^2*x^(1/6))*sin(b*x^(1/3) + a))/b^9
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.40 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.03

$$\int x^{3/2}\cos\left(a+b\sqrt[3]{x}\right)\,dx =$$

$$-\frac{3\left(64i\,b^6x^{\frac{13}{6}}-416\,b^5x^{\frac{11}{6}}-2288i\,b^4x^{\frac{3}{2}}+10296\,b^3x^{\frac{7}{6}}+36036i\,b^2x^{\frac{5}{6}}-90090\,b\sqrt{x}-135135i\,x^{\frac{1}{6}}\right)e^{\left(i\,bx^{\frac{1}{3}}+i\,a\right)}}{128\,b^7}$$
$$-\frac{3\left(-64i\,b^6x^{\frac{13}{6}}-416\,b^5x^{\frac{11}{6}}+2288i\,b^4x^{\frac{3}{2}}+10296\,b^3x^{\frac{7}{6}}-36036i\,b^2x^{\frac{5}{6}}-90090\,b\sqrt{x}+135135i\,x^{\frac{1}{6}}\right)e^{\left(-i\,bx^{\frac{1}{3}}-i\right.}}{128\,b^7}$$
$$+\frac{405405\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}i\sqrt{2}x^{\frac{1}{6}}\left(\frac{i\,b}{|b|}+1\right)\sqrt{|b|}\right)e^{(i\,a)}}{256\,b^7\left(\frac{i\,b}{|b|}+1\right)\sqrt{|b|}}$$
$$+\frac{405405\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(\frac{1}{2}i\sqrt{2}x^{\frac{1}{6}}\left(-\frac{i\,b}{|b|}+1\right)\sqrt{|b|}\right)e^{(-i\,a)}}{256\,b^7\left(-\frac{i\,b}{|b|}+1\right)\sqrt{|b|}}$$

```
[In] integrate(x^(3/2)*cos(a+b*x^(1/3)),x, algorithm="giac")
```

```
[Out] -3/128*(64*I*b^6*x^(13/6) - 416*b^5*x^(11/6) - 2288*I*b^4*x^(3/2) + 10296*b
^3*x^(7/6) + 36036*I*b^2*x^(5/6) - 90090*b*sqrt(x) - 135135*I*x^(1/6))*e^(I
*b*x^(1/3) + I*a)/b^7 - 3/128*(-64*I*b^6*x^(13/6) - 416*b^5*x^(11/6) + 2288
*I*b^4*x^(3/2) + 10296*b^3*x^(7/6) - 36036*I*b^2*x^(5/6) - 90090*b*sqrt(x)
```

```
+ 135135*I*x^(1/6))*e^(-I*b*x^(1/3) - I*a)/b^7 + 405405/256*sqrt(2)*sqrt(pi
)*erf(-1/2*I*sqrt(2)*x^(1/6)*(I*b/abs(b) + 1)*sqrt(abs(b)))*e^(I*a)/(b^7*(I
*b/abs(b) + 1)*sqrt(abs(b))) + 405405/256*sqrt(2)*sqrt(pi)*erf(1/2*I*sqrt(2
)*x^(1/6)*(-I*b/abs(b) + 1)*sqrt(abs(b)))*e^(-I*a)/(b^7*(-I*b/abs(b) + 1)*s
qrt(abs(b)))
```

Mupad [F(-1)]

Timed out.

$$\int x^{3/2}\cos\left(a+b\sqrt[3]{x}\right)\,dx=\int x^{3/2}\,\cos\left(a+b\,x^{1/3}\right)\,dx$$

```
[In] int(x^(3/2)*cos(a + b*x^(1/3)),x)
```

```
[Out] int(x^(3/2)*cos(a + b*x^(1/3)), x)
```

3.50 $\int \sqrt{x} \cos \left(a+b\sqrt[3]{x}\right) \, dx$

Optimal result

Integrand size = 16, antiderivative size = 169

$$\int \sqrt{x} \cos \left(a+b\sqrt[3]{x}\right) \, dx = -\frac{315\sqrt[6]{x}\cos\left(a+b\sqrt[3]{x}\right)}{8b^4}+\frac{21x^{5/6}\cos\left(a+b\sqrt[3]{x}\right)}{2b^2} +\frac{315\sqrt{\frac{\pi}{2}}\cos(a)\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[6]{x}\right)}{8b^{9/2}} -\frac{315\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[6]{x}\right)\sin(a)}{8b^{9/2}} -\frac{105\sqrt{x}\sin\left(a+b\sqrt[3]{x}\right)}{4b^3}+\frac{3x^{7/6}\sin\left(a+b\sqrt[3]{x}\right)}{b}$$

```
[Out] -315/8*x^(1/6)*cos(a+b*x^(1/3))/b^4+21/2*x^(5/6)*cos(a+b*x^(1/3))/b^2+3*x^(
7/6)*sin(a+b*x^(1/3))/b+315/16*cos(a)*FresnelC(x^(1/6)*b^(1/2)*2^(1/2)/Pi^(
1/2))*2^(1/2)*Pi^(1/2)/b^(9/2)-315/16*FresnelS(x^(1/6)*b^(1/2)*2^(1/2)/Pi^(
1/2))*sin(a)*2^(1/2)*Pi^(1/2)/b^(9/2)-105/4*sin(a+b*x^(1/3))*x^(1/2)/b^3
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used

= {3497, 3377, 3387, 3386, 3432, 3385, 3433}

$$\int \sqrt{x}\cos\left(a+b\sqrt[3]{x}\right)\,dx = \frac{315\sqrt{\frac{\pi}{2}}\cos(a)\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[6]{x}\right)}{8b^{9/2}} - \frac{315\sqrt{\frac{\pi}{2}}\sin(a)\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[6]{x}\right)}{8b^{9/2}} - \frac{315\sqrt[6]{x}\cos\left(a+b\sqrt[3]{x}\right)}{8b^4} - \frac{105\sqrt{x}\sin\left(a+b\sqrt[3]{x}\right)}{4b^3} + \frac{21x^{5/6}\cos\left(a+b\sqrt[3]{x}\right)}{2b^2} + \frac{3x^{7/6}\sin\left(a+b\sqrt[3]{x}\right)}{b}$$

```
[In] Int[Sqrt[x]*Cos[a + b*x^(1/3)],x]
```

```
[Out] (-315*x^(1/6)*Cos[a + b*x^(1/3)])/(8*b^4) + (21*x^(5/6)*Cos[a + b*x^(1/3)])
/(2*b^2) + (315*Sqrt[Pi/2]*Cos[a]*FresnelC[Sqrt[b]*Sqrt[2/Pi]*x^(1/6)])/(8*
b^(9/2)) - (315*Sqrt[Pi/2]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*x^(1/6)]*Sin[a])/(8*
b^(9/2)) - (105*Sqrt[x]*Sin[a + b*x^(1/3)])/(4*b^3) + (3*x^(7/6)*Sin[a + b*
x^(1/3)])/b
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3497

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Module[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a +
b*Cos[c + d*x^(k*n)])^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, m}, x]
&& IntegerQ[p] && FractionQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 3\text{Subst}\left(\int x^{7/2}\cos(a+bx)\,dx, x, \sqrt[3]{x}\right)\\
&= \frac{3x^{7/6}\sin\left(a+b\sqrt[3]{x}\right)}{b} - \frac{21\text{Subst}\left(\int x^{5/2}\sin(a+bx)\,dx, x, \sqrt[3]{x}\right)}{2b}\\
&= \frac{21x^{5/6}\cos\left(a+b\sqrt[3]{x}\right)}{2b^2} + \frac{3x^{7/6}\sin\left(a+b\sqrt[3]{x}\right)}{b} - \frac{105\text{Subst}\left(\int x^{3/2}\cos(a+bx)\,dx, x, \sqrt[3]{x}\right)}{4b^2}\\
&= \frac{21x^{5/6}\cos\left(a+b\sqrt[3]{x}\right)}{2b^2} - \frac{105\sqrt{x}\sin\left(a+b\sqrt[3]{x}\right)}{4b^3}\\
&\quad + \frac{3x^{7/6}\sin\left(a+b\sqrt[3]{x}\right)}{b} + \frac{315\text{Subst}\left(\int \sqrt{x}\sin(a+bx)\,dx, x, \sqrt[3]{x}\right)}{8b^3}\\
&= -\frac{315\sqrt[6]{x}\cos\left(a+b\sqrt[3]{x}\right)}{8b^4} + \frac{21x^{5/6}\cos\left(a+b\sqrt[3]{x}\right)}{2b^2} - \frac{105\sqrt{x}\sin\left(a+b\sqrt[3]{x}\right)}{4b^3}\\
&\quad + \frac{3x^{7/6}\sin\left(a+b\sqrt[3]{x}\right)}{b} + \frac{315\text{Subst}\left(\int \frac{\cos(a+bx)}{\sqrt{x}}\,dx, x, \sqrt[3]{x}\right)}{16b^4}\\
&= -\frac{315\sqrt[6]{x}\cos\left(a+b\sqrt[3]{x}\right)}{8b^4} + \frac{21x^{5/6}\cos\left(a+b\sqrt[3]{x}\right)}{2b^2}\\
&\quad - \frac{105\sqrt{x}\sin\left(a+b\sqrt[3]{x}\right)}{4b^3} + \frac{3x^{7/6}\sin\left(a+b\sqrt[3]{x}\right)}{b}\\
&\quad + \frac{(315\cos(a))\text{Subst}\left(\int \frac{\cos(bx)}{\sqrt{x}}\,dx, x, \sqrt[3]{x}\right)}{16b^4} - \frac{(315\sin(a))\text{Subst}\left(\int \frac{\sin(bx)}{\sqrt{x}}\,dx, x, \sqrt[3]{x}\right)}{16b^4}
\end{aligned}$$

$$= -\frac{315\sqrt[6]{x}\cos\left(a+b\sqrt[3]{x}\right)}{8b^4} + \frac{21x^{5/6}\cos\left(a+b\sqrt[3]{x}\right)}{2b^2} - \frac{105\sqrt{x}\sin\left(a+b\sqrt[3]{x}\right)}{4b^3} + \frac{3x^{7/6}\sin\left(a+b\sqrt[3]{x}\right)}{b} + \frac{(315\cos(a))\text{Subst}\left(\int\cos\left(bx^2\right)\,dx,x,\sqrt[6]{x}\right)}{8b^4} - \frac{(315\sin(a))\text{Subst}\left(\int\sin\left(bx^2\right)\,dx,x,\sqrt[6]{x}\right)}{8b^4}$$

$$= -\frac{315\sqrt[6]{x}\cos\left(a+b\sqrt[3]{x}\right)}{8b^4} + \frac{21x^{5/6}\cos\left(a+b\sqrt[3]{x}\right)}{2b^2} + \frac{315\sqrt{\frac{\pi}{2}}\cos(a)\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[6]{x}\right)}{8b^{9/2}} - \frac{315\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[6]{x}\right)\sin(a)}{8b^{9/2}} - \frac{105\sqrt{x}\sin\left(a+b\sqrt[3]{x}\right)}{4b^3} + \frac{3x^{7/6}\sin\left(a+b\sqrt[3]{x}\right)}{b}$$

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.83

$$\int \sqrt{x}\cos\left(a+b\sqrt[3]{x}\right)\,dx$$

$$= \frac{315\sqrt{2\pi}\cos(a)\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[6]{x}\right) - 315\sqrt{2\pi}\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[6]{x}\right)\sin(a) + 6\sqrt{b}\sqrt[6]{x}\left(7\left(-15+4b^2x^{2/\cdots}\right.\right.}{16b^{9/2}}$$

```
[In] Integrate[Sqrt[x]*Cos[a + b*x^(1/3)],x]
```

```
[Out] (315*Sqrt[2*Pi]*Cos[a]*FresnelC[Sqrt[b]*Sqrt[2/Pi]*x^(1/6)] - 315*Sqrt[2*Pi
]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*x^(1/6)]*Sin[a] + 6*Sqrt[b]*x^(1/6)*(7*(-15 +
 4*b^2*x^(2/3))*Cos[a + b*x^(1/3)] + 2*b*(-35 + 4*b^2*x^(2/3))*x^(1/3)*Sin[
a + b*x^(1/3)]))/(16*b^(9/2))
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.78

method	result
derivativedivides	$\frac{3x^{\frac{7}{6}}\sin\left(a+b\,x^{\frac{1}{3}}\right)}{b}-\frac{21\left(-\frac{x^{\frac{5}{6}}\cos\left(a+b\,x^{\frac{1}{3}}\right)}{2b}+\frac{5\sqrt{x}\,\sin\left(a+b\,x^{\frac{1}{3}}\right)}{4b}-\frac{15\left(-\frac{x^{\frac{1}{6}}\cos\left(a+b\,x^{\frac{1}{3}}\right)}{2b}+\frac{\sqrt{2}\,\sqrt{\pi}\,\left(\cos(a)\,\mathrm{C}\left(\frac{x^{\frac{1}{6}}\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}}\right)-\right.}{4b^{\frac{3}{2}}}\right.}{4b}\right.}{b}$
default	$\frac{3x^{\frac{7}{6}}\sin\left(a+b\,x^{\frac{1}{3}}\right)}{b}-\frac{21\left(-\frac{x^{\frac{5}{6}}\cos\left(a+b\,x^{\frac{1}{3}}\right)}{2b}+\frac{5\sqrt{x}\,\sin\left(a+b\,x^{\frac{1}{3}}\right)}{4b}-\frac{15\left(-\frac{x^{\frac{1}{6}}\cos\left(a+b\,x^{\frac{1}{3}}\right)}{2b}+\frac{\sqrt{2}\,\sqrt{\pi}\,\left(\cos(a)\,\mathrm{C}\left(\frac{x^{\frac{1}{6}}\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}}\right)-\right.}{4b^{\frac{3}{2}}}\right.}{4b}\right.}{b}$
meijerg	$\frac{24\sqrt{2}\,\cos(a)\sqrt{\pi}\left(-\frac{x^{\frac{1}{6}}\sqrt{2}\,\left(b^{2}\right)^{\frac{9}{4}}\left(-252x^{\frac{2}{3}}b^{2}+945\right)\cos\left(b\,x^{\frac{1}{3}}\right)}{1152\sqrt{\pi}\,b^{4}}-\frac{\sqrt{x}\,\sqrt{2}\,\left(b^{2}\right)^{\frac{9}{4}}\left(-36x^{\frac{2}{3}}b^{2}+315\right)\sin\left(b\,x^{\frac{1}{3}}\right)}{576\sqrt{\pi}\,b^{3}}+\frac{105\left(b^{2}\right)^{\frac{9}{4}}\mathrm{C}\left(\frac{x^{\frac{1}{6}}}{}\right.}{128b^{\frac{9}{2}}}\right.}{\left(b^{2}\right)^{\frac{9}{4}}}$

```
[In] int(x^(1/2)*cos(a+b*x^(1/3)),x,method=_RETURNVERBOSE)
```

```
[Out] 3*x^(7/6)*sin(a+b*x^(1/3))/b-21/b*(-1/2/b*x^(5/6)*cos(a+b*x^(1/3))+5/2/b*(1
/2/b*x^(1/2)*sin(a+b*x^(1/3))-3/2/b*(-1/2/b*x^(1/6)*cos(a+b*x^(1/3))+1/4/b^
(3/2)*2^(1/2)*Pi^(1/2)*(cos(a)*FresnelC(x^(1/6)*b^(1/2)*2^(1/2)/Pi^(1/2))-s
in(a)*FresnelS(x^(1/6)*b^(1/2)*2^(1/2)/Pi^(1/2)))))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.70

$$\int\sqrt{x}\cos\left(a+b\sqrt[3]{x}\right)\,dx$$
$$=\frac{3\left(105\sqrt{2}\pi\sqrt{\frac{b}{\pi}}\cos\left(a\right)\mathrm{C}\left(\sqrt{2}x^{\frac{1}{6}}\sqrt{\frac{b}{\pi}}\right)-105\sqrt{2}\pi\sqrt{\frac{b}{\pi}}\,\mathrm{S}\left(\sqrt{2}x^{\frac{1}{6}}\sqrt{\frac{b}{\pi}}\right)\sin\left(a\right)+14\left(4\,b^{3}x^{\frac{5}{6}}-15\,bx^{\frac{1}{6}}\right)\cos\left(bx\right.}{16\,b^{5}}$$

```
[In] integrate(x^(1/2)*cos(a+b*x^(1/3)),x, algorithm="fricas")
```

```
[Out] 3/16*(105*sqrt(2)*pi*sqrt(b/pi)*cos(a)*fresnel_cos(sqrt(2)*x^(1/6)*sqrt(b/p
i)) - 105*sqrt(2)*pi*sqrt(b/pi)*fresnel_sin(sqrt(2)*x^(1/6)*sqrt(b/pi))*sin
(a) + 14*(4*b^3*x^(5/6) - 15*b*x^(1/6))*cos(b*x^(1/3) + a) + 4*(4*b^4*x^(7/
6) - 35*b^2*sqrt(x))*sin(b*x^(1/3) + a))/b^5
```

Sympy [F]

$$\int \sqrt{x}\cos\left(a+b\sqrt[3]{x}\right)\,dx = \int \sqrt{x}\cos\left(a+b\sqrt[3]{x}\right)dx$$

```
[In] integrate(x**(1/2)*cos(a+b*x**(1/3)),x)
```

```
[Out] Integral(sqrt(x)*cos(a + b*x**(1/3)), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.66

$$\int \sqrt{x}\cos\left(a+b\sqrt[3]{x}\right)\,dx$$
$$= \frac{3\left(105\sqrt{2}\sqrt{\pi}\left(\left(-(i-1)\cos(a)-(i+1)\sin(a)\right)\operatorname{erf}\left(\sqrt{i\,b}x^{\frac{1}{6}}\right)+\left((i+1)\cos(a)+(i-1)\sin(a)\right)\operatorname{erf}\left(\sqrt{\vphantom{x}}\right.\right.\right.}{64\,b^6}$$

```
[In] integrate(x^(1/2)*cos(a+b*x^(1/3)),x, algorithm="maxima")
```

```
[Out] 3/64*(105*sqrt(2)*sqrt(pi)*((-(I - 1)*cos(a) - (I + 1)*sin(a))*erf(sqrt(I*b
)*x^(1/6)) + ((I + 1)*cos(a) + (I - 1)*sin(a))*erf(sqrt(-I*b)*x^(1/6)))*b^(
3/2) + 56*(4*b^4*x^(5/6) - 15*b^2*x^(1/6))*cos(b*x^(1/3) + a) + 16*(4*b^5*x
^(7/6) - 35*b^3*sqrt(x))*sin(b*x^(1/3) + a))/b^6
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.44 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.14

$$\int \sqrt{x}\cos\left(a+b\sqrt[3]{x}\right)\,dx = -\frac{3\left(8i\,b^3x^{\frac{7}{6}}-28\,b^2x^{\frac{5}{6}}-70i\,b\sqrt{x}+105\,x^{\frac{1}{6}}\right)e^{\left(i\,bx^{\frac{1}{3}}+i\,a\right)}}{16\,b^4} - \frac{3\left(-8i\,b^3x^{\frac{7}{6}}-28\,b^2x^{\frac{5}{6}}+70i\,b\sqrt{x}+105\,x^{\frac{1}{6}}\right)e^{\left(-i\,bx^{\frac{1}{3}}-i\,a\right)}}{16\,b^4} + \frac{315i\,\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}i\,\sqrt{2}x^{\frac{1}{6}}\left(\frac{i\,b}{|b|}+1\right)\sqrt{|b|}\right)e^{(i\,a)}}{32\,b^4\left(\frac{i\,b}{|b|}+1\right)\sqrt{|b|}} - \frac{315i\,\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(\frac{1}{2}i\,\sqrt{2}x^{\frac{1}{6}}\left(-\frac{i\,b}{|b|}+1\right)\sqrt{|b|}\right)e^{(-i\,a)}}{32\,b^4\left(-\frac{i\,b}{|b|}+1\right)\sqrt{|b|}}$$

```
[In] integrate(x^(1/2)*cos(a+b*x^(1/3)),x, algorithm="giac")
```

```
[Out] -3/16*(8*I*b^3*x^(7/6) - 28*b^2*x^(5/6) - 70*I*b*sqrt(x) + 105*x^(1/6))*e^(
I*b*x^(1/3) + I*a)/b^4 - 3/16*(-8*I*b^3*x^(7/6) - 28*b^2*x^(5/6) + 70*I*b*s
qrt(x) + 105*x^(1/6))*e^(-I*b*x^(1/3) - I*a)/b^4 + 315/32*I*sqrt(2)*sqrt(pi
)*erf(-1/2*I*sqrt(2)*x^(1/6)*(I*b/abs(b) + 1)*sqrt(abs(b)))*e^(I*a)/(b^4*(I
*b/abs(b) + 1)*sqrt(abs(b))) - 315/32*I*sqrt(2)*sqrt(pi)*erf(1/2*I*sqrt(2)*
x^(1/6)*(-I*b/abs(b) + 1)*sqrt(abs(b)))*e^(-I*a)/(b^4*(-I*b/abs(b) + 1)*sqr
t(abs(b)))
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{x}\cos\left(a+b\sqrt[3]{x}\right)\,dx = \int \sqrt{x}\,\cos\left(a+b\,x^{1/3}\right)\,dx$$

```
[In] int(x^(1/2)*cos(a + b*x^(1/3)),x)
```

```
[Out] int(x^(1/2)*cos(a + b*x^(1/3)), x)
```

3.51 $\int \frac{\cos\left(a+b\sqrt[3]{x}\right)}{\sqrt{x}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 99

$$\int \frac{\cos\left(a+b\sqrt[3]{x}\right)}{\sqrt{x}}\,dx = -\frac{3\sqrt{\frac{\pi}{2}}\cos(a)\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[6]{x}\right)}{b^{3/2}} - \frac{3\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[6]{x}\right)\sin(a)}{b^{3/2}} + \frac{3\sqrt[6]{x}\sin\left(a+b\sqrt[3]{x}\right)}{b}$$

```
[Out] 3*x^(1/6)*sin(a+b*x^(1/3))/b-3/2*cos(a)*FresnelS(x^(1/6)*b^(1/2)*2^(1/2)/Pi
^(1/2))*2^(1/2)*Pi^(1/2)/b^(3/2)-3/2*FresnelC(x^(1/6)*b^(1/2)*2^(1/2)/Pi^(1
/2))*sin(a)*2^(1/2)*Pi^(1/2)/b^(3/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {3497, 3377, 3387, 3386, 3432, 3385, 3433}

$$\int \frac{\cos\left(a+b\sqrt[3]{x}\right)}{\sqrt{x}}\,dx = -\frac{3\sqrt{\frac{\pi}{2}}\sin(a)\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[6]{x}\right)}{b^{3/2}} - \frac{3\sqrt{\frac{\pi}{2}}\cos(a)\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[6]{x}\right)}{b^{3/2}} + \frac{3\sqrt[6]{x}\sin\left(a+b\sqrt[3]{x}\right)}{b}$$

```
[In] Int[Cos[a + b*x^(1/3)]/Sqrt[x],x]
```

```
[Out] (-3*Sqrt[Pi/2]*Cos[a]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*x^(1/6)])/b^(3/2) - (3*Sq
rt[Pi/2]*FresnelC[Sqrt[b]*Sqrt[2/Pi]*x^(1/6)]*Sin[a])/b^(3/2) + (3*x^(1/6)*
Sin[a + b*x^(1/3)])/b
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3497

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Module[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a +
b*Cos[c + d*x^(k*n)])^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, m}, x]
&& IntegerQ[p] && FractionQ[n]
```

Rubi steps

$$\text{integral} = 3\text{Subst}\left(\int \sqrt{x}\cos(a+bx)\,dx, x, \sqrt[3]{x}\right)$$

$$= \frac{3\sqrt[6]{x}\sin\left(a+b\sqrt[3]{x}\right)}{b} - \frac{3\text{Subst}\left(\int \frac{\sin(a+bx)}{\sqrt{x}}\,dx, x, \sqrt[3]{x}\right)}{2b}$$

$$= \frac{3\sqrt[6]{x}\sin\left(a+b\sqrt[3]{x}\right)}{b} - \frac{(3\cos(a))\text{Subst}\left(\int \frac{\sin(bx)}{\sqrt{x}}\,dx, x, \sqrt[3]{x}\right)}{2b} - \frac{(3\sin(a))\text{Subst}\left(\int \frac{\cos(bx)}{\sqrt{x}}\,dx, x, \sqrt[3]{x}\right)}{2b}$$

$$= \frac{3\sqrt[6]{x}\sin\left(a+b\sqrt[3]{x}\right)}{b} - \frac{(3\cos(a))\text{Subst}\left(\int \sin\left(bx^2\right)\,dx, x, \sqrt[6]{x}\right)}{b} - \frac{(3\sin(a))\text{Subst}\left(\int \cos\left(bx^2\right)\,dx, x, \sqrt[6]{x}\right)}{b}$$

$$= -\frac{3\sqrt{\frac{\pi}{2}}\cos(a)\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[6]{x}\right)}{b^{3/2}} - \frac{3\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[6]{x}\right)\sin(a)}{b^{3/2}} + \frac{3\sqrt[6]{x}\sin\left(a+b\sqrt[3]{x}\right)}{b}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.95

$$\int \frac{\cos\left(a+b\sqrt[3]{x}\right)}{\sqrt{x}}\,dx = -\frac{3\left(\sqrt{2\pi}\cos(a)\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[6]{x}\right) + \sqrt{2\pi}\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[6]{x}\right)\sin(a) - 2\sqrt{b}\sqrt[6]{x}\sin\left(a+b\sqrt[3]{x}\right)\right)}{2b^{3/2}}$$

```
[In] Integrate[Cos[a + b*x^(1/3)]/Sqrt[x],x]
[Out] (-3*(Sqrt[2*Pi]*Cos[a]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*x^(1/6)] + Sqrt[2*Pi]*Fr
esnelC[Sqrt[b]*Sqrt[2/Pi]*x^(1/6)]*Sin[a] - 2*Sqrt[b]*x^(1/6)*Sin[a + b*x^(
1/3)]))/(2*b^(3/2))
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.65

method	result
derivativedivides	$\frac{3x^{\frac{1}{6}}\sin\left(a+b\,x^{\frac{1}{3}}\right)}{b}-\frac{3\sqrt{2}\,\sqrt{\pi}\,\left(\cos(a)\,\mathrm{S}\left(\frac{x^{\frac{1}{6}}\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}}\right)+\sin(a)\,\mathrm{C}\left(\frac{x^{\frac{1}{6}}\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}}\right)\right)}{2b^{\frac{3}{2}}}$
default	$\frac{3x^{\frac{1}{6}}\sin\left(a+b\,x^{\frac{1}{3}}\right)}{b}-\frac{3\sqrt{2}\,\sqrt{\pi}\,\left(\cos(a)\,\mathrm{S}\left(\frac{x^{\frac{1}{6}}\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}}\right)+\sin(a)\,\mathrm{C}\left(\frac{x^{\frac{1}{6}}\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}}\right)\right)}{2b^{\frac{3}{2}}}$
meijerg	$\frac{3\cos(a)\sqrt{\pi}\,\sqrt{2}\left(\frac{x^{\frac{1}{6}}\sqrt{2}\,\left(b^{2}\right)^{\frac{3}{4}}\sin\left(b\,x^{\frac{1}{3}}\right)}{2\sqrt{\pi}\,b}-\frac{\left(b^{2}\right)^{\frac{3}{4}}\mathrm{S}\left(\frac{x^{\frac{1}{6}}\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}}\right)}{2b^{\frac{3}{2}}}\right)}{\left(b^{2}\right)^{\frac{3}{4}}}-\frac{3\sin(a)\sqrt{\pi}\,\sqrt{2}\left(-\frac{x^{\frac{1}{6}}\sqrt{2}\,\sqrt{b}\,\cos\left(b\,x^{\frac{1}{3}}\right)}{2\sqrt{\pi}}+\frac{\mathrm{C}\left(\frac{x^{\frac{1}{6}}\sqrt{b}\,\sqrt{\cdots}}{\sqrt{\pi}}\right)}{2}\cdots\right.}{b^{\frac{3}{2}}}$

```
[In] int(cos(a+b*x^(1/3))/x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 3*x^(1/6)*sin(a+b*x^(1/3))/b-3/2/b^(3/2)*2^(1/2)*Pi^(1/2)*(cos(a)*FresnelS(
x^(1/6)*b^(1/2)*2^(1/2)/Pi^(1/2))+sin(a)*FresnelC(x^(1/6)*b^(1/2)*2^(1/2)/P
i^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.79

$$\int \frac{\cos\left(a+b\sqrt[3]{x}\right)}{\sqrt{x}}\,dx = -\frac{3\left(\sqrt{2}\pi\sqrt{\frac{b}{\pi}}\cos\left(a\right)\mathrm{S}\left(\sqrt{2}x^{\frac{1}{6}}\sqrt{\frac{b}{\pi}}\right)+\sqrt{2}\pi\sqrt{\frac{b}{\pi}}\,\mathrm{C}\left(\sqrt{2}x^{\frac{1}{6}}\sqrt{\frac{b}{\pi}}\right)\sin\left(a\right)-2\,bx^{\frac{1}{6}}\sin\left(bx^{\frac{1}{3}}+a\right)\right)}{2\,b^{2}}$$

```
[In] integrate(cos(a+b*x^(1/3))/x^(1/2),x, algorithm="fricas")
```

```
[Out] -3/2*(sqrt(2)*pi*sqrt(b/pi)*cos(a)*fresnel_sin(sqrt(2)*x^(1/6)*sqrt(b/pi))
+ sqrt(2)*pi*sqrt(b/pi)*fresnel_cos(sqrt(2)*x^(1/6)*sqrt(b/pi))*sin(a) - 2*
b*x^(1/6)*sin(b*x^(1/3) + a))/b^2
```

Sympy [F]

$$\int \frac{\cos\left(a+b\sqrt[3]{x}\right)}{\sqrt{x}}\,dx = \int \frac{\cos\left(a+b\sqrt[3]{x}\right)}{\sqrt{x}}\,dx$$

```
[In] integrate(cos(a+b*x**(1/3))/x**(1/2),x)
```

```
[Out] Integral(cos(a + b*x**(1/3))/sqrt(x), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.74

$$\int \frac{\cos\left(a+b\sqrt[3]{x}\right)}{\sqrt{x}}\,dx = \frac{3\left(\sqrt{2}\sqrt{\pi}\left(\left(-(i+1)\cos\left(a\right)+(i-1)\sin\left(a\right)\right)\operatorname{erf}\left(\sqrt{i\,b}x^{\frac{1}{6}}\right)+\left((i-1)\cos\left(a\right)-(i+1)\sin\left(a\right)\right)\operatorname{erf}\left(\sqrt{-i\,b}\cdots\right.\right.\right.}{8\,b^{3}}$$

```
[In] integrate(cos(a+b*x^(1/3))/x^(1/2),x, algorithm="maxima")
```

```
[Out] 3/8*(sqrt(2)*sqrt(pi)*((-(I + 1)*cos(a) + (I - 1)*sin(a))*erf(sqrt(I*b)*x^(
1/6)) + ((I - 1)*cos(a) - (I + 1)*sin(a))*erf(sqrt(-I*b)*x^(1/6)))*b^(3/2)
+ 8*b^2*x^(1/6)*sin(b*x^(1/3) + a))/b^3
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.60 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.44

$$\int \frac{\cos\left(a+b\sqrt[3]{x}\right)}{\sqrt{x}}\,dx = -\frac{3\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}i\sqrt{2}x^{\frac{1}{6}}\left(\frac{i\,b}{|b|}+1\right)\sqrt{|b|}\right)e^{(i\,a)}}{4\,b\left(\frac{i\,b}{|b|}+1\right)\sqrt{|b|}} - \frac{3\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(\frac{1}{2}i\sqrt{2}x^{\frac{1}{6}}\left(-\frac{i\,b}{|b|}+1\right)\sqrt{|b|}\right)e^{(-i\,a)}}{4\,b\left(-\frac{i\,b}{|b|}+1\right)\sqrt{|b|}} - \frac{3i\,x^{\frac{1}{6}}e^{\left(i\,bx^{\frac{1}{3}}+i\,a\right)}}{2\,b} + \frac{3i\,x^{\frac{1}{6}}e^{\left(-i\,bx^{\frac{1}{3}}-i\,a\right)}}{2\,b}$$

```
[In] integrate(cos(a+b*x^(1/3))/x^(1/2),x, algorithm="giac")
```

```
[Out] -3/4*sqrt(2)*sqrt(pi)*erf(-1/2*I*sqrt(2)*x^(1/6)*(I*b/abs(b) + 1)*sqrt(abs(
b)))*e^(I*a)/(b*(I*b/abs(b) + 1)*sqrt(abs(b))) - 3/4*sqrt(2)*sqrt(pi)*erf(1
/2*I*sqrt(2)*x^(1/6)*(-I*b/abs(b) + 1)*sqrt(abs(b)))*e^(-I*a)/(b*(-I*b/abs(
b) + 1)*sqrt(abs(b))) - 3/2*I*x^(1/6)*e^(I*b*x^(1/3) + I*a)/b + 3/2*I*x^(1/
6)*e^(-I*b*x^(1/3) - I*a)/b
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos\left(a+b\sqrt[3]{x}\right)}{\sqrt{x}}\,dx=\int \frac{\cos\left(a+b\,x^{1/3}\right)}{\sqrt{x}}\,dx$$

```
[In] int(cos(a + b*x^(1/3))/x^(1/2),x)
[Out] int(cos(a + b*x^(1/3))/x^(1/2), x)
```

3.52 $\int \frac{\cos\left(a+b\sqrt[3]{x}\right)}{x^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 110

$$\int \frac{\cos\left(a+b\sqrt[3]{x}\right)}{x^{3/2}}\,dx = -\frac{2\cos\left(a+b\sqrt[3]{x}\right)}{\sqrt{x}} - 4b^{3/2}\sqrt{2\pi}\cos(a)\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[6]{x}\right) + 4b^{3/2}\sqrt{2\pi}\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[6]{x}\right)\sin(a) + \frac{4b\sin\left(a+b\sqrt[3]{x}\right)}{\sqrt[6]{x}}$$

```
[Out] 4*b*sin(a+b*x^(1/3))/x^(1/6)-4*b^(3/2)*cos(a)*FresnelC(x^(1/6)*b^(1/2)*2^(1
/2)/Pi^(1/2))*2^(1/2)*Pi^(1/2)+4*b^(3/2)*FresnelS(x^(1/6)*b^(1/2)*2^(1/2)/P
i^(1/2))*sin(a)*2^(1/2)*Pi^(1/2)-2*cos(a+b*x^(1/3))/x^(1/2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {3497, 3378, 3387, 3386, 3432, 3385, 3433}

$$\int \frac{\cos\left(a+b\sqrt[3]{x}\right)}{x^{3/2}}\,dx = -4\sqrt{2\pi}b^{3/2}\cos(a)\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[6]{x}\right) + 4\sqrt{2\pi}b^{3/2}\sin(a)\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[6]{x}\right) + \frac{4b\sin\left(a+b\sqrt[3]{x}\right)}{\sqrt[6]{x}} - \frac{2\cos\left(a+b\sqrt[3]{x}\right)}{\sqrt{x}}$$

```
[In] Int[Cos[a + b*x^(1/3)]/x^(3/2),x]
```

```
[Out] (-2*Cos[a + b*x^(1/3)])/Sqrt[x] - 4*b^(3/2)*Sqrt[2*Pi]*Cos[a]*FresnelC[Sqrt
[b]*Sqrt[2/Pi]*x^(1/6)] + 4*b^(3/2)*Sqrt[2*Pi]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*
x^(1/6)]*Sin[a] + (4*b*Sin[a + b*x^(1/3)])/x^(1/6)
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3497

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Module[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a +
b*Cos[c + d*x^(k*n)])^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, m}, x]
&& IntegerQ[p] && FractionQ[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= 3\text{Subst}\left(\int \frac{\cos(a+bx)}{x^{5/2}}\,dx, x, \sqrt[3]{x}\right) \\
&= -\frac{2\cos\left(a+b\sqrt[3]{x}\right)}{\sqrt{x}} - (2b)\text{Subst}\left(\int \frac{\sin(a+bx)}{x^{3/2}}\,dx, x, \sqrt[3]{x}\right) \\
&= -\frac{2\cos\left(a+b\sqrt[3]{x}\right)}{\sqrt{x}} + \frac{4b\sin\left(a+b\sqrt[3]{x}\right)}{\sqrt[6]{x}} - \left(4b^2\right)\text{Subst}\left(\int \frac{\cos(a+bx)}{\sqrt{x}}\,dx, x, \sqrt[3]{x}\right) \\
&= -\frac{2\cos\left(a+b\sqrt[3]{x}\right)}{\sqrt{x}} + \frac{4b\sin\left(a+b\sqrt[3]{x}\right)}{\sqrt[6]{x}} - \left(4b^2\cos(a)\right)\text{Subst}\left(\int \frac{\cos(bx)}{\sqrt{x}}\,dx, x, \sqrt[3]{x}\right) \\
&\quad + \left(4b^2\sin(a)\right)\text{Subst}\left(\int \frac{\sin(bx)}{\sqrt{x}}\,dx, x, \sqrt[3]{x}\right) \\
&= -\frac{2\cos\left(a+b\sqrt[3]{x}\right)}{\sqrt{x}} + \frac{4b\sin\left(a+b\sqrt[3]{x}\right)}{\sqrt[6]{x}} - \left(8b^2\cos(a)\right)\text{Subst}\left(\int \cos\left(bx^2\right)\,dx, x, \sqrt[6]{x}\right) \\
&\quad + \left(8b^2\sin(a)\right)\text{Subst}\left(\int \sin\left(bx^2\right)\,dx, x, \sqrt[6]{x}\right) \\
&= -\frac{2\cos\left(a+b\sqrt[3]{x}\right)}{\sqrt{x}} - 4b^{3/2}\sqrt{2\pi}\cos(a)\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[6]{x}\right) \\
&\quad + 4b^{3/2}\sqrt{2\pi}\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[6]{x}\right)\sin(a) + \frac{4b\sin\left(a+b\sqrt[3]{x}\right)}{\sqrt[6]{x}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00

$$
\begin{aligned}
\int \frac{\cos\left(a+b\sqrt[3]{x}\right)}{x^{3/2}}\,dx &= -\frac{2\cos\left(a+b\sqrt[3]{x}\right)}{\sqrt{x}} - 4b^{3/2}\sqrt{2\pi}\cos(a)\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[6]{x}\right) \\
&\quad + 4b^{3/2}\sqrt{2\pi}\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[6]{x}\right)\sin(a) + \frac{4b\sin\left(a+b\sqrt[3]{x}\right)}{\sqrt[6]{x}}
\end{aligned}
$$

```
[In] Integrate[Cos[a + b*x^(1/3)]/x^(3/2),x]
```

```
[Out] (-2*Cos[a + b*x^(1/3)])/Sqrt[x] - 4*b^(3/2)*Sqrt[2*Pi]*Cos[a]*FresnelC[Sqrt
[b]*Sqrt[2/Pi]*x^(1/6)] + 4*b^(3/2)*Sqrt[2*Pi]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*
x^(1/6)]*Sin[a] + (4*b*Sin[a + b*x^(1/3)])/x^(1/6)
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.71

method	result
derivativedivides	$-\frac{2\cos\left(a+b\,x^{\frac{1}{3}}\right)}{\sqrt{x}}-4b\left(-\frac{\sin\left(a+b\,x^{\frac{1}{3}}\right)}{x^{\frac{1}{6}}}+\sqrt{b}\,\sqrt{2}\,\sqrt{\pi}\,\left(\cos\left(a\right)\mathrm{C}\left(\frac{x^{\frac{1}{6}}\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}}\right)-\sin\left(a\right)\mathrm{S}\left(\frac{x^{\frac{1}{6}}\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}}\right.\right.\right.$
default	$-\frac{2\cos\left(a+b\,x^{\frac{1}{3}}\right)}{\sqrt{x}}-4b\left(-\frac{\sin\left(a+b\,x^{\frac{1}{3}}\right)}{x^{\frac{1}{6}}}+\sqrt{b}\,\sqrt{2}\,\sqrt{\pi}\,\left(\cos\left(a\right)\mathrm{C}\left(\frac{x^{\frac{1}{6}}\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}}\right)-\sin\left(a\right)\mathrm{S}\left(\frac{x^{\frac{1}{6}}\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}}\right.\right.\right.$
meijerg	$\frac{3\cos(a)\sqrt{\pi}\,\sqrt{2}\,(b^2)^{\frac{3}{4}}\left(-\frac{8\sqrt{2}\,\cos\left(b\,x^{\frac{1}{3}}\right)}{3\sqrt{\pi}\,\sqrt{x}\,\left(b^2\right)^{\frac{3}{4}}}+\frac{16\sqrt{2}\,b\sin\left(b\,x^{\frac{1}{3}}\right)}{3\sqrt{\pi}\,x^{\frac{1}{6}}\left(b^2\right)^{\frac{3}{4}}}-\frac{32b^{\frac{3}{2}}\,\mathrm{C}\left(\frac{x^{\frac{1}{6}}\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}}\right)}{3\left(b^2\right)^{\frac{3}{4}}}\right)}{8}-\frac{3\sin(a)\sqrt{\pi}\,\sqrt{2}\,b^{\frac{3}{2}}\left(-\frac{16\sqrt{2}\,\cos\left(b\,x\right.}{3\sqrt{\pi}\,x^{\frac{1}{6}}\sqrt{}}\right.}{}$

```
[In] int(cos(a+b*x^(1/3))/x^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*cos(a+b*x^(1/3))/x^(1/2)-4*b*(-1/x^(1/6)*sin(a+b*x^(1/3))+b^(1/2)*2^(1/2
)*Pi^(1/2)*(cos(a)*FresnelC(x^(1/6)*b^(1/2)*2^(1/2)/Pi^(1/2))-sin(a)*Fresne
lS(x^(1/6)*b^(1/2)*2^(1/2)/Pi^(1/2))))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.87

$$\int \frac{\cos\left(a+b\sqrt[3]{x}\right)}{x^{3/2}}\,dx =$$

$$-\frac{2\left(2\,\sqrt{2}\pi bx\sqrt{\frac{b}{\pi}}\cos\left(a\right)\mathrm{C}\left(\sqrt{2}x^{\frac{1}{6}}\sqrt{\frac{b}{\pi}}\right)-2\,\sqrt{2}\pi bx\sqrt{\frac{b}{\pi}}\,\mathrm{S}\left(\sqrt{2}x^{\frac{1}{6}}\sqrt{\frac{b}{\pi}}\right)\sin\left(a\right)-2\,bx^{\frac{5}{6}}\sin\left(bx^{\frac{1}{3}}+a\right)+\sqrt{x}\cos\right.}{x}$$

```
[In] integrate(cos(a+b*x^(1/3))/x^(3/2),x, algorithm="fricas")
```

```
[Out] -2*(2*sqrt(2)*pi*b*x*sqrt(b/pi)*cos(a)*fresnel_cos(sqrt(2)*x^(1/6)*sqrt(b/p
i)) - 2*sqrt(2)*pi*b*x*sqrt(b/pi)*fresnel_sin(sqrt(2)*x^(1/6)*sqrt(b/pi))*s
in(a) - 2*b*x^(5/6)*sin(b*x^(1/3) + a) + sqrt(x)*cos(b*x^(1/3) + a))/x
```

Sympy [F]

$$\int \frac{\cos\left(a+b\sqrt[3]{x}\right)}{x^{3/2}}\,dx = \int \frac{\cos\left(a+b\sqrt[3]{x}\right)}{x^{\frac{3}{2}}}\,dx$$

```
[In] integrate(cos(a+b*x**(1/3))/x**(3/2),x)
```

```
[Out] Integral(cos(a + b*x**(1/3))/x**(3/2), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.41 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.67

$$\int \frac{\cos\left(a+b\sqrt[3]{x}\right)}{x^{3/2}}\,dx =$$
$$-\frac{3\left(\left((i-1)\sqrt{2}\Gamma\left(-\frac{3}{2}, i\,bx^{\frac{1}{3}}\right)-(i+1)\sqrt{2}\Gamma\left(-\frac{3}{2}, -i\,bx^{\frac{1}{3}}\right)\right)\cos(a)+\left((i+1)\sqrt{2}\Gamma\left(-\frac{3}{2}, i\,bx^{\frac{1}{3}}\right)-(i-1)\right.}{4\,x^{\frac{1}{6}}}$$

```
[In] integrate(cos(a+b*x^(1/3))/x^(3/2),x, algorithm="maxima")
```

```
[Out] -3/4*(((I - 1)*sqrt(2)*gamma(-3/2, I*b*x^(1/3)) - (I + 1)*sqrt(2)*gamma(-3/
2, -I*b*x^(1/3)))*cos(a) + ((I + 1)*sqrt(2)*gamma(-3/2, I*b*x^(1/3)) - (I -
 1)*sqrt(2)*gamma(-3/2, -I*b*x^(1/3)))*sin(a))*sqrt(b*x^(1/3))*b/x^(1/6)
```

Giac [F]

$$\int \frac{\cos\left(a+b\sqrt[3]{x}\right)}{x^{3/2}}\,dx = \int \frac{\cos\left(bx^{\frac{1}{3}}+a\right)}{x^{\frac{3}{2}}}\,dx$$

```
[In] integrate(cos(a+b*x^(1/3))/x^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x^(1/3) + a)/x^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos \left(a+b\sqrt[3]{x}\right)}{x^{3/2}}\,dx=\int \frac{\cos \left(a+b\,x^{1/3}\right)}{x^{3/2}}\,dx$$

```
[In] int(cos(a + b*x^(1/3))/x^(3/2),x)
[Out] int(cos(a + b*x^(1/3))/x^(3/2), x)
```

3.53 $\int \frac{\cos\left(a+b\sqrt[3]{x}\right)}{x^{5/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 184

$$\int \frac{\cos\left(a+b\sqrt[3]{x}\right)}{x^{5/2}}\,dx=-\frac{2\cos\left(a+b\sqrt[3]{x}\right)}{3x^{3/2}}+\frac{8b^2\cos\left(a+b\sqrt[3]{x}\right)}{105x^{5/6}}-\frac{32b^4\cos\left(a+b\sqrt[3]{x}\right)}{315\sqrt[6]{x}}-\frac{32}{315}b^{9/2}\sqrt{2\pi}\cos(a)\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[6]{x}\right)-\frac{32}{315}b^{9/2}\sqrt{2\pi}\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[6]{x}\right)\sin(a)+\frac{4b\sin\left(a+b\sqrt[3]{x}\right)}{21x^{7/6}}$$

```
[Out] -2/3*cos(a+b*x^(1/3))/x^(3/2)+8/105*b^2*cos(a+b*x^(1/3))/x^(5/6)-32/315*b^4
*cos(a+b*x^(1/3))/x^(1/6)+4/21*b*sin(a+b*x^(1/3))/x^(7/6)-32/315*b^(9/2)*co
s(a)*FresnelS(x^(1/6)*b^(1/2)*2^(1/2)/Pi^(1/2))*2^(1/2)*Pi^(1/2)-32/315*b^(
9/2)*FresnelC(x^(1/6)*b^(1/2)*2^(1/2)/Pi^(1/2))*sin(a)*2^(1/2)*Pi^(1/2)-16/
315*b^3*sin(a+b*x^(1/3))/x^(1/2)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {3497, 3378, 3387, 3386, 3432, 3385, 3433}

$$\int \frac{\cos\left(a+b\sqrt[3]{x}\right)}{x^{5/2}}\,dx=-\frac{32}{315}\sqrt{2\pi}b^{9/2}\sin(a)\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[6]{x}\right)-\frac{32}{315}\sqrt{2\pi}b^{9/2}\cos(a)\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[6]{x}\right)-\frac{32b^4\cos\left(a+b\sqrt[3]{x}\right)}{315\sqrt[6]{x}}-\frac{16b^3\sin\left(a+b\sqrt[3]{x}\right)}{315\sqrt{x}}+\frac{8b^2\cos\left(a+b\sqrt[3]{x}\right)}{105x^{5/6}}$$

```
[In] Int[Cos[a + b*x^(1/3)]/x^(5/2),x]
```

```
[Out] (-2*Cos[a + b*x^(1/3)])/(3*x^(3/2)) + (8*b^2*Cos[a + b*x^(1/3)])/(105*x^(5/
6)) - (32*b^4*Cos[a + b*x^(1/3)])/(315*x^(1/6)) - (32*b^(9/2)*Sqrt[2*Pi]*Co
s[a]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*x^(1/6)])/315 - (32*b^(9/2)*Sqrt[2*Pi]*Fre
snelC[Sqrt[b]*Sqrt[2/Pi]*x^(1/6)]*Sin[a])/315 + (4*b*Sin[a + b*x^(1/3)])/(2
1*x^(7/6)) - (16*b^3*Sin[a + b*x^(1/3)])/(315*Sqrt[x])
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3497

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Module[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a +
b*Cos[c + d*x^(k*n)])^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, m}, x]
```

```
&& IntegerQ[p] && FractionQ[n]
```

Rubi steps

$$\text{integral} = 3\text{Subst}\left(\int \frac{\cos(a+bx)}{x^{11/2}}\,dx, x, \sqrt[3]{x}\right)$$

$$= -\frac{2\cos\left(a+b\sqrt[3]{x}\right)}{3x^{3/2}} - \frac{1}{3}(2b)\text{Subst}\left(\int \frac{\sin(a+bx)}{x^{9/2}}\,dx, x, \sqrt[3]{x}\right)$$

$$= -\frac{2\cos\left(a+b\sqrt[3]{x}\right)}{3x^{3/2}} + \frac{4b\sin\left(a+b\sqrt[3]{x}\right)}{21x^{7/6}} - \frac{1}{21}\left(4b^2\right)\text{Subst}\left(\int \frac{\cos(a+bx)}{x^{7/2}}\,dx, x, \sqrt[3]{x}\right)$$

$$= -\frac{2\cos\left(a+b\sqrt[3]{x}\right)}{3x^{3/2}} + \frac{8b^2\cos\left(a+b\sqrt[3]{x}\right)}{105x^{5/6}} + \frac{4b\sin\left(a+b\sqrt[3]{x}\right)}{21x^{7/6}} + \frac{1}{105}\left(8b^3\right)\text{Subst}\left(\int \frac{\sin(a+bx)}{x^{5/2}}\,dx, x, \sqrt[3]{x}\right)$$

$$= -\frac{2\cos\left(a+b\sqrt[3]{x}\right)}{3x^{3/2}} + \frac{8b^2\cos\left(a+b\sqrt[3]{x}\right)}{105x^{5/6}} + \frac{4b\sin\left(a+b\sqrt[3]{x}\right)}{21x^{7/6}} - \frac{16b^3\sin\left(a+b\sqrt[3]{x}\right)}{315\sqrt{x}} + \frac{1}{315}\left(16b^4\right)\text{Subst}\left(\int \frac{\cos(a+bx)}{x^{3/2}}\,dx, x, \sqrt[3]{x}\right)$$

$$= -\frac{2\cos\left(a+b\sqrt[3]{x}\right)}{3x^{3/2}} + \frac{8b^2\cos\left(a+b\sqrt[3]{x}\right)}{105x^{5/6}} - \frac{32b^4\cos\left(a+b\sqrt[3]{x}\right)}{315\sqrt[6]{x}} + \frac{4b\sin\left(a+b\sqrt[3]{x}\right)}{21x^{7/6}} - \frac{16b^3\sin\left(a+b\sqrt[3]{x}\right)}{315\sqrt{x}} - \frac{1}{315}\left(32b^5\right)\text{Subst}\left(\int \frac{\sin(a+bx)}{\sqrt{x}}\,dx, x, \sqrt[3]{x}\right)$$

$$= -\frac{2\cos\left(a+b\sqrt[3]{x}\right)}{3x^{3/2}} + \frac{8b^2\cos\left(a+b\sqrt[3]{x}\right)}{105x^{5/6}} - \frac{32b^4\cos\left(a+b\sqrt[3]{x}\right)}{315\sqrt[6]{x}} + \frac{4b\sin\left(a+b\sqrt[3]{x}\right)}{21x^{7/6}} - \frac{16b^3\sin\left(a+b\sqrt[3]{x}\right)}{315\sqrt{x}} - \frac{1}{315}\left(32b^5\cos(a)\right)\text{Subst}\left(\int \frac{\sin(bx)}{\sqrt{x}}\,dx, x, \sqrt[3]{x}\right) - \frac{1}{315}\left(32b^5\sin(a)\right)\text{Subst}\left(\int \frac{\cos(bx)}{\sqrt{x}}\,dx, x, \sqrt[3]{x}\right.$$

$$= -\frac{2\cos\left(a+b\sqrt[3]{x}\right)}{3x^{3/2}} + \frac{8b^2\cos\left(a+b\sqrt[3]{x}\right)}{105x^{5/6}} - \frac{32b^4\cos\left(a+b\sqrt[3]{x}\right)}{315\sqrt[6]{x}} + \frac{4b\sin\left(a+b\sqrt[3]{x}\right)}{21x^{7/6}} - \frac{16b^3\sin\left(a+b\sqrt[3]{x}\right)}{315\sqrt{x}} - \frac{1}{315}\left(64b^5\cos(a)\right)\text{Subst}\left(\int \sin\left(bx^2\right)\,dx, x, \sqrt[6]{x}\right) - \frac{1}{315}\left(64b^5\sin(a)\right)\text{Subst}\left(\int \cos\left(bx^2\right)\,dx, x,\right.$$

$$= -\frac{2\cos\left(a+b\sqrt[3]{x}\right)}{3x^{3/2}} + \frac{8b^2\cos\left(a+b\sqrt[3]{x}\right)}{105x^{5/6}} - \frac{32b^4\cos\left(a+b\sqrt[3]{x}\right)}{315\sqrt[6]{x}} - \frac{32}{315}b^{9/2}\sqrt{2\pi}\cos(a)\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[6]{x}\right) - \frac{32}{315}b^{9/2}\sqrt{2\pi}\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[6]{x}\right)\sin(a) + \frac{4b\sin}{}$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.98

$$\int \frac{\cos \left(a+b\sqrt[3]{x}\right)}{x^{5/2}}\,dx =$$

$$-\frac{2\left(105\cos\left(a+b\sqrt[3]{x}\right)-12b^2x^{2/3}\cos\left(a+b\sqrt[3]{x}\right)+16b^4x^{4/3}\cos\left(a+b\sqrt[3]{x}\right)+16b^{9/2}\sqrt{2\pi}x^{3/2}\cos(a)\,\text{FresnelS}\right.}{315}$$

```
[In] Integrate[Cos[a + b*x^(1/3)]/x^(5/2),x]
```

```
[Out] (-2*(105*Cos[a + b*x^(1/3)] - 12*b^2*x^(2/3)*Cos[a + b*x^(1/3)] + 16*b^4*x^
(4/3)*Cos[a + b*x^(1/3)] + 16*b^(9/2)*Sqrt[2*Pi]*x^(3/2)*Cos[a]*FresnelS[Sq
rt[b]*Sqrt[2/Pi]*x^(1/6)] + 16*b^(9/2)*Sqrt[2*Pi]*x^(3/2)*FresnelC[Sqrt[b]*
Sqrt[2/Pi]*x^(1/6)]*Sin[a] - 30*b*x^(1/3)*Sin[a + b*x^(1/3)] + 8*b^3*x*Sin[
a + b*x^(1/3)]))/(315*x^(3/2))
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.70

method	result
derivativedivides	$-\frac{2\cos\left(a+b\,x^{\frac{1}{3}}\right)}{3x^{\frac{3}{2}}}-\frac{4b\left(-\frac{\sin\left(a+b\,x^{\frac{1}{3}}\right)}{7x^{\frac{7}{6}}}+\frac{2b\left(-\frac{\cos\left(a+b\,x^{\frac{1}{3}}\right)}{5x^{\frac{5}{6}}}-\frac{2b\left(-\frac{\sin\left(a+b\,x^{\frac{1}{3}}\right)}{3\sqrt{x}}+\frac{2b\left(-\frac{\cos\left(a+b\,x^{\frac{1}{3}}\right)}{x^{\frac{1}{6}}}-\sqrt{b}\,\sqrt{2}\,\sqrt{\pi}\,\left(\cos(\cdots\right)\cdots}{\cdots}\right)}{5}\cdots\right)}{7}\cdots\right)}{3}\cdots$
default	$-\frac{2\cos\left(a+b\,x^{\frac{1}{3}}\right)}{3x^{\frac{3}{2}}}-\frac{4b\left(-\frac{\sin\left(a+b\,x^{\frac{1}{3}}\right)}{7x^{\frac{7}{6}}}+\frac{2b\left(-\frac{\cos\left(a+b\,x^{\frac{1}{3}}\right)}{5x^{\frac{5}{6}}}-\frac{2b\left(-\frac{\sin\left(a+b\,x^{\frac{1}{3}}\right)}{3\sqrt{x}}+\frac{2b\left(-\frac{\cos\left(a+b\,x^{\frac{1}{3}}\right)}{x^{\frac{1}{6}}}-\sqrt{b}\,\sqrt{2}\,\sqrt{\pi}\,\left(\cos(\cdots\right)\cdots}{\cdots}\right)}{5}\cdots\right)}{7}\cdots\right)}{3}\cdots$
meijerg	$\frac{3\cos(a)\sqrt{\pi}\,\sqrt{2}\,\left(b^{2}\right)^{\frac{9}{4}}\left(-\frac{64\sqrt{2}\,\left(\frac{16x^{\frac{4}{3}}b^{4}}{105}-\frac{4x^{\frac{2}{3}}b^{2}}{35}+1\right)\cos\left(b\,x^{\frac{1}{3}}\right)}{9\sqrt{\pi}\,x^{\frac{3}{2}}\left(b^{2}\right)^{\frac{9}{4}}}+\frac{128\sqrt{2}\,b\left(-4x^{\frac{2}{3}}b^{2}+15\right)\sin\left(b\,x^{\frac{1}{3}}\right)}{945\sqrt{\pi}\,x^{\frac{7}{6}}\left(b^{2}\right)^{\frac{9}{4}}}-\frac{2048b^{\frac{9}{2}}\,\mathrm{S}\left(\frac{x^{\frac{1}{6}}\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}}\right)\cdots}{945\left(b^{2}\right)^{\frac{9}{4}}}\cdots\right)}{64}\cdots$

```
[In] int(cos(a+b*x^(1/3))/x^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*cos(a+b*x^(1/3))/x^(3/2)-4/3*b*(-1/7/x^(7/6)*sin(a+b*x^(1/3))+2/7*b*(-
1/5*cos(a+b*x^(1/3))/x^(5/6)-2/5*b*(-1/3/x^(1/2)*sin(a+b*x^(1/3))+2/3*b*(-1
/x^(1/6)*cos(a+b*x^(1/3))-b^(1/2)*2^(1/2)*Pi^(1/2)*(cos(a)*FresnelS(x^(1/6)
*b^(1/2)*2^(1/2)/Pi^(1/2))+sin(a)*FresnelC(x^(1/6)*b^(1/2)*2^(1/2)/Pi^(1/2)
))))))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.73

$$\int \frac{\cos\left(a+b\sqrt[3]{x}\right)}{x^{5/2}}\,dx = -\frac{2\left(16\sqrt{2}\pi b^4x^2\sqrt{\frac{b}{\pi}}\cos\left(a\right)S\left(\sqrt{2}x^{\frac{1}{6}}\sqrt{\frac{b}{\pi}}\right)+16\sqrt{2}\pi b^4x^2\sqrt{\frac{b}{\pi}}C\left(\sqrt{2}x^{\frac{1}{6}}\sqrt{\frac{b}{\pi}}\right)\sin\left(a\right)+\left(16\,b^4x^{\frac{11}{6}}-12\,b^2x^{\frac{7}{6}}+\right.\right.}{315\,x^2}$$

```
[In] integrate(cos(a+b*x^(1/3))/x^(5/2),x, algorithm="fricas")
```

```
[Out] -2/315*(16*sqrt(2)*pi*b^4*x^2*sqrt(b/pi)*cos(a)*fresnel_sin(sqrt(2)*x^(1/6)
*sqrt(b/pi)) + 16*sqrt(2)*pi*b^4*x^2*sqrt(b/pi)*fresnel_cos(sqrt(2)*x^(1/6)
*sqrt(b/pi))*sin(a) + (16*b^4*x^(11/6) - 12*b^2*x^(7/6) + 105*sqrt(x))*cos(
b*x^(1/3) + a) + 2*(4*b^3*x^(3/2) - 15*b*x^(5/6))*sin(b*x^(1/3) + a))/x^2
```

Sympy [F]

$$\int \frac{\cos\left(a+b\sqrt[3]{x}\right)}{x^{5/2}}\,dx = \int \frac{\cos\left(a+b\sqrt[3]{x}\right)}{x^{\frac{5}{2}}}\,dx$$

```
[In] integrate(cos(a+b*x**(1/3))/x**(5/2),x)
```

```
[Out] Integral(cos(a + b*x**(1/3))/x**(5/2), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.44 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.41

$$\int \frac{\cos\left(a+b\sqrt[3]{x}\right)}{x^{5/2}}\,dx = \frac{3\left(\left(-(i+1)\sqrt{2}\Gamma\left(-\frac{9}{2},i\,bx^{\frac{1}{3}}\right)+(i-1)\sqrt{2}\Gamma\left(-\frac{9}{2},-i\,bx^{\frac{1}{3}}\right)\right)\cos\left(a\right)+\left((i-1)\sqrt{2}\Gamma\left(\right.\right.\right.}{4\,x^{\frac{1}{6}}}$$

```
[In] integrate(cos(a+b*x^(1/3))/x^(5/2),x, algorithm="maxima")
```

```
[Out] 3/4*((-(I + 1)*sqrt(2)*gamma(-9/2, I*b*x^(1/3)) + (I - 1)*sqrt(2)*gamma(-9/
2, -I*b*x^(1/3)))*cos(a) + ((I - 1)*sqrt(2)*gamma(-9/2, I*b*x^(1/3)) - (I +
 1)*sqrt(2)*gamma(-9/2, -I*b*x^(1/3)))*sin(a))*sqrt(b*x^(1/3))*b^4/x^(1/6)
```

Giac [F]

$$\int \frac{\cos\left(a+b\sqrt[3]{x}\right)}{x^{5/2}}\,dx=\int \frac{\cos\left(bx^{\frac{1}{3}}+a\right)}{x^{\frac{5}{2}}}\,dx$$

```
[In] integrate(cos(a+b*x^(1/3))/x^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x^(1/3) + a)/x^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos\left(a+b\sqrt[3]{x}\right)}{x^{5/2}}\,dx=\int \frac{\cos\left(a+b\,x^{1/3}\right)}{x^{5/2}}\,dx$$

```
[In] int(cos(a + b*x^(1/3))/x^(5/2),x)
```

```
[Out] int(cos(a + b*x^(1/3))/x^(5/2), x)
```

3.54 $\int \frac{\cos\left(a+b\sqrt[3]{x}\right)}{x^{7/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 250

$$\int \frac{\cos\left(a+b\sqrt[3]{x}\right)}{x^{7/2}}\,dx = -\frac{2\cos\left(a+b\sqrt[3]{x}\right)}{5x^{5/2}}+\frac{8b^2\cos\left(a+b\sqrt[3]{x}\right)}{715x^{11/6}}-\frac{32b^4\cos\left(a+b\sqrt[3]{x}\right)}{45045x^{7/6}}+\frac{128b^6\cos\left(a+b\sqrt[3]{x}\right)}{675675\sqrt{x}}+\frac{256b^{15/2}\sqrt{2\pi}\cos(a)\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[6]{x}\right)}{675675}-\frac{256b^{15/2}\sqrt{2\pi}\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[6]{x}\right)\sin(a)}{675675}+\frac{4b\sin\left(a+b\sqrt[3]{x}\right)}{65x^{13/6}}-\frac{16b^3\sin\left(a+b\sqrt[3]{x}\right)}{6435x^{3/2}}+\frac{64b^5\sin\left(a+b\sqrt[3]{x}\right)}{225225x^{5/6}}-\frac{256b^7\sin\left(a+b\sqrt[3]{x}\right)}{675675\sqrt[6]{x}}$$

```
[Out] -2/5*cos(a+b*x^(1/3))/x^(5/2)+8/715*b^2*cos(a+b*x^(1/3))/x^(11/6)-32/45045*
b^4*cos(a+b*x^(1/3))/x^(7/6)+4/65*b*sin(a+b*x^(1/3))/x^(13/6)-16/6435*b^3*s
in(a+b*x^(1/3))/x^(3/2)+64/225225*b^5*sin(a+b*x^(1/3))/x^(5/6)-256/675675*b
^7*sin(a+b*x^(1/3))/x^(1/6)+256/675675*b^(15/2)*cos(a)*FresnelC(x^(1/6)*b^(
1/2)*2^(1/2)/Pi^(1/2))*2^(1/2)*Pi^(1/2)-256/675675*b^(15/2)*FresnelS(x^(1/6
)*b^(1/2)*2^(1/2)/Pi^(1/2))*sin(a)*2^(1/2)*Pi^(1/2)+128/675675*b^6*cos(a+b*
x^(1/3))/x^(1/2)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {3497, 3378, 3387, 3386, 3432, 3385, 3433}

$$\int \frac{\cos\left(a+b\sqrt[3]{x}\right)}{x^{7/2}}\,dx = \frac{256\sqrt{2\pi}b^{15/2}\cos(a)\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[6]{x}\right)}{675675} - \frac{256\sqrt{2\pi}b^{15/2}\sin(a)\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[6]{x}\right)}{675675} - \frac{256b^7\sin\left(a+b\sqrt[3]{x}\right)}{675675\sqrt[6]{x}} + \frac{128b^6\cos\left(a+b\sqrt[3]{x}\right)}{675675\sqrt{x}} + \frac{64b^5\sin\left(a+b\sqrt[3]{x}\right)}{225225x^{5/6}} - \frac{32b^4\cos\left(a+b\sqrt[3]{x}\right)}{45045x^{7/6}} - \frac{16b^3\sin\left(a+b\sqrt[3]{x}\right)}{6435x^{3/2}} + \frac{8b^2\cos\left(a+b\sqrt[3]{x}\right)}{715x^{11/6}} + \frac{4b\sin\left(a+b\sqrt[3]{x}\right)}{65x^{13/6}} - \frac{2\cos\left(a+b\sqrt[3]{x}\right)}{5x^{5/2}}$$

```
[In] Int[Cos[a + b*x^(1/3)]/x^(7/2),x]
```

```
[Out] (-2*Cos[a + b*x^(1/3)])/(5*x^(5/2)) + (8*b^2*Cos[a + b*x^(1/3)])/(715*x^(11
/6)) - (32*b^4*Cos[a + b*x^(1/3)])/(45045*x^(7/6)) + (128*b^6*Cos[a + b*x^(
1/3)])/(675675*Sqrt[x]) + (256*b^(15/2)*Sqrt[2*Pi]*Cos[a]*FresnelC[Sqrt[b]*
Sqrt[2/Pi]*x^(1/6)])/675675 - (256*b^(15/2)*Sqrt[2*Pi]*FresnelS[Sqrt[b]*Sqr
t[2/Pi]*x^(1/6)]*Sin[a])/675675 + (4*b*Sin[a + b*x^(1/3)])/(65*x^(13/6)) -
(16*b^3*Sin[a + b*x^(1/3)])/(6435*x^(3/2)) + (64*b^5*Sin[a + b*x^(1/3)])/(2
25225*x^(5/6)) - (256*b^7*Sin[a + b*x^(1/3)])/(675675*x^(1/6))
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3497

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Module[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a +
b*Cos[c + d*x^(k*n)])^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, m}, x]
&& IntegerQ[p] && FractionQ[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= 3\text{Subst}\left(\int \frac{\cos(a+bx)}{x^{17/2}}\,dx, x, \sqrt[3]{x}\right)\\
&= -\frac{2\cos\left(a+b\sqrt[3]{x}\right)}{5x^{5/2}} - \frac{1}{5}(2b)\text{Subst}\left(\int \frac{\sin(a+bx)}{x^{15/2}}\,dx, x, \sqrt[3]{x}\right)\\
&= -\frac{2\cos\left(a+b\sqrt[3]{x}\right)}{5x^{5/2}} + \frac{4b\sin\left(a+b\sqrt[3]{x}\right)}{65x^{13/6}} - \frac{1}{65}\left(4b^2\right)\text{Subst}\left(\int \frac{\cos(a+bx)}{x^{13/2}}\,dx, x, \sqrt[3]{x}\right)\\
&= -\frac{2\cos\left(a+b\sqrt[3]{x}\right)}{5x^{5/2}} + \frac{8b^2\cos\left(a+b\sqrt[3]{x}\right)}{715x^{11/6}} + \frac{4b\sin\left(a+b\sqrt[3]{x}\right)}{65x^{13/6}}\\
&\quad + \frac{1}{715}\left(8b^3\right)\text{Subst}\left(\int \frac{\sin(a+bx)}{x^{11/2}}\,dx, x, \sqrt[3]{x}\right)\\
&= -\frac{2\cos\left(a+b\sqrt[3]{x}\right)}{5x^{5/2}} + \frac{8b^2\cos\left(a+b\sqrt[3]{x}\right)}{715x^{11/6}} + \frac{4b\sin\left(a+b\sqrt[3]{x}\right)}{65x^{13/6}}\\
&\quad - \frac{16b^3\sin\left(a+b\sqrt[3]{x}\right)}{6435x^{3/2}} + \frac{\left(16b^4\right)\text{Subst}\left(\int \frac{\cos(a+bx)}{x^{9/2}}\,dx, x, \sqrt[3]{x}\right)}{6435}\\
&= -\frac{2\cos\left(a+b\sqrt[3]{x}\right)}{5x^{5/2}} + \frac{8b^2\cos\left(a+b\sqrt[3]{x}\right)}{715x^{11/6}} - \frac{32b^4\cos\left(a+b\sqrt[3]{x}\right)}{45045x^{7/6}}\\
&\quad + \frac{4b\sin\left(a+b\sqrt[3]{x}\right)}{65x^{13/6}} - \frac{16b^3\sin\left(a+b\sqrt[3]{x}\right)}{6435x^{3/2}} - \frac{\left(32b^5\right)\text{Subst}\left(\int \frac{\sin(a+bx)}{x^{7/2}}\,dx, x, \sqrt[3]{x}\right)}{45045}
\end{aligned}
$$

$$= -\frac{2\cos\left(a+b\sqrt[3]{x}\right)}{5x^{5/2}} + \frac{8b^2\cos\left(a+b\sqrt[3]{x}\right)}{715x^{11/6}} - \frac{32b^4\cos\left(a+b\sqrt[3]{x}\right)}{45045x^{7/6}} + \frac{4b\sin\left(a+b\sqrt[3]{x}\right)}{65x^{13/6}} - \frac{16b^3\sin\left(a+b\sqrt[3]{x}\right)}{6435x^{3/2}} + \frac{64b^5\sin\left(a+b\sqrt[3]{x}\right)}{225225x^{5/6}} - \frac{(64b^6)\,\text{Subst}\left(\int \frac{\cos(a+bx)}{x^{5/2}}\,dx, x, \sqrt[3]{x}\right)}{225225}$$

$$= -\frac{2\cos\left(a+b\sqrt[3]{x}\right)}{5x^{5/2}} + \frac{8b^2\cos\left(a+b\sqrt[3]{x}\right)}{715x^{11/6}} - \frac{32b^4\cos\left(a+b\sqrt[3]{x}\right)}{45045x^{7/6}} + \frac{128b^6\cos\left(a+b\sqrt[3]{x}\right)}{675675\sqrt{x}} + \frac{4b\sin\left(a+b\sqrt[3]{x}\right)}{65x^{13/6}} - \frac{16b^3\sin\left(a+b\sqrt[3]{x}\right)}{6435x^{3/2}} + \frac{64b^5\sin\left(a+b\sqrt[3]{x}\right)}{225225x^{5/6}} + \frac{(128b^7)\,\text{Subst}\left(\int \frac{\sin(a+bx)}{x^{3/2}}\,dx, x, \sqrt[3]{x}\right)}{675675}$$

$$= -\frac{2\cos\left(a+b\sqrt[3]{x}\right)}{5x^{5/2}} + \frac{8b^2\cos\left(a+b\sqrt[3]{x}\right)}{715x^{11/6}} - \frac{32b^4\cos\left(a+b\sqrt[3]{x}\right)}{45045x^{7/6}} + \frac{128b^6\cos\left(a+b\sqrt[3]{x}\right)}{675675\sqrt{x}} + \frac{4b\sin\left(a+b\sqrt[3]{x}\right)}{65x^{13/6}} - \frac{16b^3\sin\left(a+b\sqrt[3]{x}\right)}{6435x^{3/2}} + \frac{64b^5\sin\left(a+b\sqrt[3]{x}\right)}{225225x^{5/6}} - \frac{256b^7\sin\left(a+b\sqrt[3]{x}\right)}{675675\sqrt[6]{x}} + \frac{(256b^8)\,\text{Subst}\left(\int \frac{\cos(a+bx)}{\sqrt{x}}\,dx, x, \sqrt[3]{x}\right)}{675675}$$

$$= -\frac{2\cos\left(a+b\sqrt[3]{x}\right)}{5x^{5/2}} + \frac{8b^2\cos\left(a+b\sqrt[3]{x}\right)}{715x^{11/6}} - \frac{32b^4\cos\left(a+b\sqrt[3]{x}\right)}{45045x^{7/6}} + \frac{128b^6\cos\left(a+b\sqrt[3]{x}\right)}{675675\sqrt{x}} + \frac{4b\sin\left(a+b\sqrt[3]{x}\right)}{65x^{13/6}} - \frac{16b^3\sin\left(a+b\sqrt[3]{x}\right)}{6435x^{3/2}} + \frac{64b^5\sin\left(a+b\sqrt[3]{x}\right)}{225225x^{5/6}} - \frac{256b^7\sin\left(a+b\sqrt[3]{x}\right)}{675675\sqrt[6]{x}} + \frac{(256b^8\cos(a))\,\text{Subst}\left(\int \frac{\cos(bx)}{\sqrt{x}}\,dx, x, \sqrt[3]{x}\right)}{675675} - \frac{(256b^8\sin(a))\,\text{Subst}\left(\int \frac{\sin(bx)}{\sqrt{x}}\,dx, x, \sqrt[3]{x}\right)}{675675}$$

$$= -\frac{2\cos\left(a+b\sqrt[3]{x}\right)}{5x^{5/2}} + \frac{8b^2\cos\left(a+b\sqrt[3]{x}\right)}{715x^{11/6}} - \frac{32b^4\cos\left(a+b\sqrt[3]{x}\right)}{45045x^{7/6}} + \frac{128b^6\cos\left(a+b\sqrt[3]{x}\right)}{675675\sqrt{x}} + \frac{4b\sin\left(a+b\sqrt[3]{x}\right)}{65x^{13/6}} - \frac{16b^3\sin\left(a+b\sqrt[3]{x}\right)}{6435x^{3/2}} + \frac{64b^5\sin\left(a+b\sqrt[3]{x}\right)}{225225x^{5/6}} - \frac{256b^7\sin\left(a+b\sqrt[3]{x}\right)}{675675\sqrt[6]{x}} + \frac{(512b^8\cos(a))\,\text{Subst}\left(\int \cos\left(bx^2\right)\,dx, x, \sqrt[6]{x}\right)}{675675} - \frac{(512b^8\sin(a))\,\text{Subst}\left(\int \sin\left(bx^2\right)\,dx, x, \sqrt[6]{x}\right)}{675675}$$

$$= -\frac{2\cos\left(a+b\sqrt[3]{x}\right)}{5x^{5/2}} + \frac{8b^2\cos\left(a+b\sqrt[3]{x}\right)}{715x^{11/6}} - \frac{32b^4\cos\left(a+b\sqrt[3]{x}\right)}{45045x^{7/6}} + \frac{128b^6\cos\left(a+b\sqrt[3]{x}\right)}{675675\sqrt{x}} + \frac{256b^{15/2}\sqrt{2\pi}\cos(a)\,\text{FresnelC}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[6]{x}\right)}{675675} - \frac{256b^{15/2}\sqrt{2\pi}\,\text{FresnelS}\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[6]{x}\right)\sin(a)}{675675} + \frac{4b\sin\left(a+b\sqrt[3]{x}\right)}{65x^{13/6}} - \frac{16b^3\sin\left(a+b\sqrt[3]{x}\right)}{6435x^{3/2}} + \frac{64b^5\sin\left(a+b\sqrt[3]{x}\right)}{225225x^{5/6}} - \frac{256b^7\sin\left(a+b\sqrt[3]{x}\right)}{675675\sqrt[6]{x}}$$

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.95

$$\int \frac{\cos\left(a+b\sqrt[3]{x}\right)}{x^{7/2}}\,dx = \frac{2\left(-135135\cos\left(a+b\sqrt[3]{x}\right)+3780b^2x^{2/3}\cos\left(a+b\sqrt[3]{x}\right)-240b^4x^{4/3}\cos\left(a+b\sqrt[3]{x}\right)+6\right.}{}$$

```
[In] Integrate[Cos[a + b*x^(1/3)]/x^(7/2),x]
```

```
[Out] (2*(-135135*Cos[a + b*x^(1/3)] + 3780*b^2*x^(2/3)*Cos[a + b*x^(1/3)] - 240*
b^4*x^(4/3)*Cos[a + b*x^(1/3)] + 64*b^6*x^2*Cos[a + b*x^(1/3)] + 128*b^(15/
2)*Sqrt[2*Pi]*x^(5/2)*Cos[a]*FresnelC[Sqrt[b]*Sqrt[2/Pi]*x^(1/6)] - 128*b^(
15/2)*Sqrt[2*Pi]*x^(5/2)*FresnelS[Sqrt[b]*Sqrt[2/Pi]*x^(1/6)]*Sin[a] + 2079
0*b*x^(1/3)*Sin[a + b*x^(1/3)] - 840*b^3*x*Sin[a + b*x^(1/3)] + 96*b^5*x^(5
/3)*Sin[a + b*x^(1/3)] - 128*b^7*x^(7/3)*Sin[a + b*x^(1/3)]))/(675675*x^(5/
2))
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.72

method	result
	$2b\left(-\frac{\cos\left(a+b\,x^{\frac{1}{3}}\right)}{11x^{\frac{11}{6}}}-\frac{2b\left(-\frac{\sin\left(a+b\,x^{\frac{1}{3}}\right)}{9x^{\frac{3}{2}}}+\frac{2b\left(-\frac{\cos\left(a+b\,x^{\frac{1}{3}}\right)}{7x^{\frac{7}{6}}}-\frac{2b\left(-\frac{\sin\left(a+b\,x\right.}{5x^{\frac{5}{6}}}\right.}{}\right.}{}\right.}{}\right.$

```
[In] int(cos(a+b*x^(1/3))/x^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/5*cos(a+b*x^(1/3))/x^(5/2)-4/5*b*(-1/13/x^(13/6)*sin(a+b*x^(1/3))+2/13*b
*(-1/11*cos(a+b*x^(1/3))/x^(11/6)-2/11*b*(-1/9/x^(3/2)*sin(a+b*x^(1/3))+2/9
*b*(-1/7/x^(7/6)*cos(a+b*x^(1/3))-2/7*b*(-1/5/x^(5/6)*sin(a+b*x^(1/3))+2/5*
b*(-1/3*cos(a+b*x^(1/3))/x^(1/2)-2/3*b*(-1/x^(1/6)*sin(a+b*x^(1/3))+b^(1/2)
*2^(1/2)*Pi^(1/2)*(cos(a)*FresnelC(x^(1/6)*b^(1/2)*2^(1/2)/Pi^(1/2))-sin(a)
*FresnelS(x^(1/6)*b^(1/2)*2^(1/2)/Pi^(1/2)))))))))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.66

$$\int \frac{\cos\left(a+b\sqrt[3]{x}\right)}{x^{7/2}}\,dx = \frac{2\left(128\sqrt{2}\pi b^7x^3\sqrt{\frac{b}{\pi}}\cos\left(a\right)\mathrm{C}\left(\sqrt{2}x^{\frac{1}{6}}\sqrt{\frac{b}{\pi}}\right) - 128\sqrt{2}\pi b^7x^3\sqrt{\frac{b}{\pi}}\,\mathrm{S}\left(\sqrt{2}x^{\frac{1}{6}}\sqrt{\frac{b}{\pi}}\right)\sin\left(a\right) -$$

```
[In] integrate(cos(a+b*x^(1/3))/x^(7/2),x, algorithm="fricas")
```

```
[Out] 2/675675*(128*sqrt(2)*pi*b^7*x^3*sqrt(b/pi)*cos(a)*fresnel_cos(sqrt(2)*x^(1
/6)*sqrt(b/pi)) - 128*sqrt(2)*pi*b^7*x^3*sqrt(b/pi)*fresnel_sin(sqrt(2)*x^(
1/6)*sqrt(b/pi))*sin(a) - (240*b^4*x^(11/6) - 3780*b^2*x^(7/6) - (64*b^6*x^
2 - 135135)*sqrt(x))*cos(b*x^(1/3) + a) + 2*(48*b^5*x^(13/6) - 420*b^3*x^(3
/2) - (64*b^7*x^2 - 10395*b)*x^(5/6))*sin(b*x^(1/3) + a))/x^3
```

Sympy [F]

$$\int \frac{\cos\left(a+b\sqrt[3]{x}\right)}{x^{7/2}}\,dx = \int \frac{\cos\left(a+b\sqrt[3]{x}\right)}{x^{\frac{7}{2}}}\,dx$$

```
[In] integrate(cos(a+b*x**(1/3))/x**(7/2),x)
```

```
[Out] Integral(cos(a + b*x**(1/3))/x**(7/2), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.43 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.30

$$\int \frac{\cos\left(a+b\sqrt[3]{x}\right)}{x^{7/2}}\,dx = \frac{3\left(\left(\left(i-1\right)\sqrt{2}\Gamma\left(-\frac{15}{2}, i\,bx^{\frac{1}{3}}\right) - \left(i+1\right)\sqrt{2}\Gamma\left(-\frac{15}{2}, -i\,bx^{\frac{1}{3}}\right)\right)\cos\left(a\right) + \left(\left(i+1\right)\sqrt{2}\Gamma\left(\right.\right.\right.}{4\,x^{\frac{1}{6}}}$$

```
[In] integrate(cos(a+b*x^(1/3))/x^(7/2),x, algorithm="maxima")
```

```
[Out] 3/4*(((I - 1)*sqrt(2)*gamma(-15/2, I*b*x^(1/3)) - (I + 1)*sqrt(2)*gamma(-15
/2, -I*b*x^(1/3)))*cos(a) + ((I + 1)*sqrt(2)*gamma(-15/2, I*b*x^(1/3)) - (I
 - 1)*sqrt(2)*gamma(-15/2, -I*b*x^(1/3)))*sin(a))*sqrt(b*x^(1/3))*b^7/x^(1/
6)
```

Giac [F]

$$\int \frac{\cos\left(a+b\sqrt[3]{x}\right)}{x^{7/2}}\,dx=\int \frac{\cos\left(bx^{\frac{1}{3}}+a\right)}{x^{\frac{7}{2}}}\,dx$$

```
[In] integrate(cos(a+b*x^(1/3))/x^(7/2),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x^(1/3) + a)/x^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos\left(a+b\sqrt[3]{x}\right)}{x^{7/2}}\,dx=\int \frac{\cos\left(a+b\,x^{1/3}\right)}{x^{7/2}}\,dx$$

```
[In] int(cos(a + b*x^(1/3))/x^(7/2),x)
```

```
[Out] int(cos(a + b*x^(1/3))/x^(7/2), x)
```

3.55 $\int x^{3/2}\cos^2\left(a+b\sqrt[3]{x}\right)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 310

$$\begin{aligned}\int x^{3/2}\cos^2\left(a+b\sqrt[3]{x}\right)\,dx = &-\frac{135135\sqrt{x}}{4096b^6}+\frac{3861x^{7/6}}{256b^4}-\frac{39x^{11/6}}{16b^2}+\frac{x^{5/2}}{5}\\&+\frac{135135\sqrt{x}\cos^2\left(a+b\sqrt[3]{x}\right)}{2048b^6}-\frac{3861x^{7/6}\cos^2\left(a+b\sqrt[3]{x}\right)}{128b^4}+\frac{39x^{11/6}\cos^2\left(a+b\sqrt[3]{x}\right)}{8b^2}\\&+\frac{405405\sqrt{\pi}\cos(2a)\operatorname{FresnelS}\left(\frac{2\sqrt{b}\sqrt[6]{x}}{\sqrt{\pi}}\right)}{32768b^{15/2}}+\frac{405405\sqrt{\pi}\operatorname{FresnelC}\left(\frac{2\sqrt{b}\sqrt[6]{x}}{\sqrt{\pi}}\right)\sin(2a)}{32768b^{15/2}}\\&+\frac{27027x^{5/6}\cos\left(a+b\sqrt[3]{x}\right)\sin\left(a+b\sqrt[3]{x}\right)}{512b^5}-\frac{429x^{3/2}\cos\left(a+b\sqrt[3]{x}\right)\sin\left(a+b\sqrt[3]{x}\right)}{32b^3}\\&+\frac{3x^{13/6}\cos\left(a+b\sqrt[3]{x}\right)\sin\left(a+b\sqrt[3]{x}\right)}{2b}-\frac{405405\sqrt[6]{x}\sin\left(2\left(a+b\sqrt[3]{x}\right)\right)}{16384b^7}\end{aligned}$$

```
[Out] 3861/256*x^(7/6)/b^4-39/16*x^(11/6)/b^2+1/5*x^(5/2)-3861/128*x^(7/6)*cos(a+
b*x^(1/3))^2/b^4+39/8*x^(11/6)*cos(a+b*x^(1/3))^2/b^2+27027/512*x^(5/6)*cos
(a+b*x^(1/3))*sin(a+b*x^(1/3))/b^5-429/32*x^(3/2)*cos(a+b*x^(1/3))*sin(a+b*
x^(1/3))/b^3+3/2*x^(13/6)*cos(a+b*x^(1/3))*sin(a+b*x^(1/3))/b-405405/16384*
x^(1/6)*sin(2*a+2*b*x^(1/3))/b^7+405405/32768*cos(2*a)*FresnelS(2*x^(1/6)*b
^(1/2)/Pi^(1/2))*Pi^(1/2)/b^(15/2)+405405/32768*FresnelC(2*x^(1/6)*b^(1/2)/
Pi^(1/2))*sin(2*a)*Pi^(1/2)/b^(15/2)-135135/4096*x^(1/2)/b^6+135135/2048*co
s(a+b*x^(1/3))^2*x^(1/2)/b^6
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {3497, 3392, 30, 3393, 3377, 3387, 3386, 3432, 3385, 3433}

$$\int x^{3/2}\cos^2\left(a+b\sqrt[3]{x}\right)\,dx=\frac{405405\sqrt{\pi}\sin(2a)\,\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt[6]{x}}{\sqrt{\pi}}\right)}{32768b^{15/2}}+\frac{405405\sqrt{\pi}\cos(2a)\,\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt[6]{x}}{\sqrt{\pi}}\right)}{32768b^{15/2}}-\frac{405405\sqrt[6]{x}\sin\left(2\left(a+b\sqrt[3]{x}\right)\right)}{16384b^7}+\frac{135135\sqrt{x}\cos^2\left(a+b\sqrt[3]{x}\right)}{2048b^6}+\frac{27027x^{5/6}\sin\left(a+b\sqrt[3]{x}\right)\cos\left(a+b\sqrt[3]{x}\right)}{512b^5}-\frac{3861x^{7/6}\cos^2\left(a+b\sqrt[3]{x}\right)}{128b^4}-\frac{429x^{3/2}\sin\left(a+b\sqrt[3]{x}\right)\cos\left(a+b\sqrt[3]{x}\right)}{32b^3}+\frac{39x^{11/6}\cos^2\left(a+b\sqrt[3]{x}\right)}{8b^2}+\frac{3x^{13/6}\sin\left(a+b\sqrt[3]{x}\right)\cos\left(a+b\sqrt[3]{x}\right)}{2b}-\frac{135135\sqrt{x}}{4096b^6}+\frac{3861x^{7/6}}{256b^4}-\frac{39x^{11/6}}{16b^2}+\frac{x^{5/2}}{5}$$

```
[In] Int[x^(3/2)*Cos[a + b*x^(1/3)]^2,x]
```

```
[Out] (-135135*Sqrt[x])/(4096*b^6) + (3861*x^(7/6))/(256*b^4) - (39*x^(11/6))/(16
*b^2) + x^(5/2)/5 + (135135*Sqrt[x]*Cos[a + b*x^(1/3)]^2)/(2048*b^6) - (386
1*x^(7/6)*Cos[a + b*x^(1/3)]^2)/(128*b^4) + (39*x^(11/6)*Cos[a + b*x^(1/3)]
^2)/(8*b^2) + (405405*Sqrt[Pi]*Cos[2*a]*FresnelS[(2*Sqrt[b]*x^(1/6))/Sqrt[P
i]])/(32768*b^(15/2)) + (405405*Sqrt[Pi]*FresnelC[(2*Sqrt[b]*x^(1/6))/Sqrt[
Pi]]*Sin[2*a])/(32768*b^(15/2)) + (27027*x^(5/6)*Cos[a + b*x^(1/3)]*Sin[a +
 b*x^(1/3)])/(512*b^5) - (429*x^(3/2)*Cos[a + b*x^(1/3)]*Sin[a + b*x^(1/3)]
)/(32*b^3) + (3*x^(13/6)*Cos[a + b*x^(1/3)]*Sin[a + b*x^(1/3)])/(2*b) - (40
5405*x^(1/6)*Sin[2*(a + b*x^(1/3))])/(16384*b^7)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3497

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Module[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a +
```

```
b*Cos[c + d*x^(k*n)])^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, m}, x]
&& IntegerQ[p] && FractionQ[n]
```

Rubi steps

$$\text{integral} = 3\text{Subst}\left(\int x^{13/2}\cos^2(a+bx)\,dx, x, \sqrt[3]{x}\right)$$

$$= \frac{39x^{11/6}\cos^2\left(a+b\sqrt[3]{x}\right)}{8b^2} + \frac{3x^{13/6}\cos\left(a+b\sqrt[3]{x}\right)\sin\left(a+b\sqrt[3]{x}\right)}{2b} + \frac{3}{2}\text{Subst}\left(\int x^{13/2}\,dx, x, \sqrt[3]{x}\right) - \frac{429\text{Subst}\left(\int x^{9/2}\cos^2(a+bx)\,dx, x, \sqrt[3]{x}\right)}{16b^2}$$

$$= \frac{x^{5/2}}{5} - \frac{3861x^{7/6}\cos^2\left(a+b\sqrt[3]{x}\right)}{128b^4} + \frac{39x^{11/6}\cos^2\left(a+b\sqrt[3]{x}\right)}{8b^2} - \frac{429x^{3/2}\cos\left(a+b\sqrt[3]{x}\right)\sin\left(a+b\sqrt[3]{x}\right)}{32b^3} + \frac{3x^{13/6}\cos\left(a+b\sqrt[3]{x}\right)\sin\left(a+b\sqrt[3]{x}\right)}{2b} + \frac{27027\text{Subst}\left(\int x^{5/2}\cos^2(a+bx)\,dx, x, \sqrt[3]{x}\right)}{256b^4} - \frac{429\text{Subst}\left(\int x^{9/2}\,dx, x, \sqrt[3]{x}\right)}{32b^2}$$

$$= -\frac{39x^{11/6}}{16b^2} + \frac{x^{5/2}}{5} + \frac{135135\sqrt{x}\cos^2\left(a+b\sqrt[3]{x}\right)}{2048b^6} - \frac{3861x^{7/6}\cos^2\left(a+b\sqrt[3]{x}\right)}{128b^4} + \frac{39x^{11/6}\cos^2\left(a+b\sqrt[3]{x}\right)}{8b^2} + \frac{27027x^{5/6}\cos\left(a+b\sqrt[3]{x}\right)\sin\left(a+b\sqrt[3]{x}\right)}{512b^5} - \frac{429x^{3/2}\cos\left(a+b\sqrt[3]{x}\right)\sin\left(a+b\sqrt[3]{x}\right)}{32b^3} + \frac{3x^{13/6}\cos\left(a+b\sqrt[3]{x}\right)\sin\left(a+b\sqrt[3]{x}\right)}{2b} - \frac{405405\text{Subst}\left(\int \sqrt{x}\cos^2(a+bx)\,dx, x, \sqrt[3]{x}\right)}{4096b^6} + \frac{27027\text{Subst}\left(\int x^{5/2}\,dx, x, \sqrt[3]{x}\right)}{512b^4}$$

$$= \frac{3861x^{7/6}}{256b^4} - \frac{39x^{11/6}}{16b^2} + \frac{x^{5/2}}{5} + \frac{135135\sqrt{x}\cos^2\left(a+b\sqrt[3]{x}\right)}{2048b^6} - \frac{3861x^{7/6}\cos^2\left(a+b\sqrt[3]{x}\right)}{128b^4} + \frac{39x^{11/6}\cos^2\left(a+b\sqrt[3]{x}\right)}{8b^2} + \frac{27027x^{5/6}\cos\left(a+b\sqrt[3]{x}\right)\sin\left(a+b\sqrt[3]{x}\right)}{512b^5} - \frac{429x^{3/2}\cos\left(a+b\sqrt[3]{x}\right)\sin\left(a+b\sqrt[3]{x}\right)}{32b^3} + \frac{3x^{13/6}\cos\left(a+b\sqrt[3]{x}\right)\sin\left(a+b\sqrt[3]{x}\right)}{2b} - \frac{405405\text{Subst}\left(\int\left(\frac{\sqrt{x}}{2}+\frac{1}{2}\sqrt{x}\cos(2a+2bx)\right)dx, x, \sqrt[3]{x}\right)}{4096b^6}$$

$$= -\frac{135135\sqrt{x}}{4096b^6} + \frac{3861x^{7/6}}{256b^4} - \frac{39x^{11/6}}{16b^2} + \frac{x^{5/2}}{5} + \frac{135135\sqrt{x}\cos^2\left(a+b\sqrt[3]{x}\right)}{2048b^6} - \frac{3861x^{7/6}\cos^2\left(a+b\sqrt[3]{x}\right)}{128b^4} + \frac{39x^{11/6}\cos^2\left(a+b\sqrt[3]{x}\right)}{8b^2} + \frac{27027x^{5/6}\cos\left(a+b\sqrt[3]{x}\right)\sin\left(a+b\sqrt[3]{x}\right)}{512b^5} - \frac{429x^{3/2}\cos\left(a+b\sqrt[3]{x}\right)\sin\left(a+b\sqrt[3]{x}\right)}{32b^3} + \frac{3x^{13/6}\cos\left(a+b\sqrt[3]{x}\right)\sin\left(a+b\sqrt[3]{x}\right)}{2b} - \frac{405405\text{Subst}\left(\int\sqrt{x}\cos(2a+2bx)\,dx, x, \sqrt[3]{x}\right)}{8192b^6}$$

$$= -\frac{135135\sqrt{x}}{4096b^6} + \frac{3861x^{7/6}}{256b^4} - \frac{39x^{11/6}}{16b^2} + \frac{x^{5/2}}{5} + \frac{135135\sqrt{x}\cos^2\left(a+b\sqrt[3]{x}\right)}{2048b^6} - \frac{3861x^{7/6}\cos^2\left(a+b\sqrt[3]{x}\right)}{128b^4} + \frac{39x^{11/6}\cos^2\left(a+b\sqrt[3]{x}\right)}{8b^2} + \frac{27027x^{5/6}\cos\left(a+b\sqrt[3]{x}\right)\sin\left(a+b\sqrt[3]{x}\right)}{512b^5} - \frac{429x^{3/2}\cos\left(a+b\sqrt[3]{x}\right)\sin\left(a+b\sqrt[3]{x}\right)}{32b^3} + \frac{3x^{13/6}\cos\left(a+b\sqrt[3]{x}\right)\sin\left(a+b\sqrt[3]{x}\right)}{2b} - \frac{405405\sqrt[6]{x}\sin\left(2\left(a+b\sqrt[3]{x}\right)\right)}{16384b^7} + \frac{405405\text{Subst}\left(\int\frac{\sin(2a+2bx)}{\sqrt{x}}\,dx, x, \sqrt[3]{x}\right)}{32768b^7}$$

$$= -\frac{135135\sqrt{x}}{4096b^6} + \frac{3861x^{7/6}}{256b^4} - \frac{39x^{11/6}}{16b^2} + \frac{x^{5/2}}{5} + \frac{135135\sqrt{x}\cos^2\left(a+b\sqrt[3]{x}\right)}{2048b^6} - \frac{3861x^{7/6}\cos^2\left(a+b\sqrt[3]{x}\right)}{128b^4} + \frac{39x^{11/6}\cos^2\left(a+b\sqrt[3]{x}\right)}{8b^2} + \frac{27027x^{5/6}\cos\left(a+b\sqrt[3]{x}\right)\sin\left(a+b\sqrt[3]{x}\right)}{512b^5} - \frac{429x^{3/2}\cos\left(a+b\sqrt[3]{x}\right)\sin\left(a+b\sqrt[3]{x}\right)}{32b^3} + \frac{3x^{13/6}\cos\left(a+b\sqrt[3]{x}\right)\sin\left(a+b\sqrt[3]{x}\right)}{2b} - \frac{405405\sqrt[6]{x}\sin\left(2\left(a+b\sqrt[3]{x}\right)\right)}{16384b^7} + \frac{(405405\cos(2a))\text{Subst}\left(\int\frac{\sin(2bx)}{\sqrt{x}}\,dx, x, \sqrt[3]{x}\right)}{32768b^7} + \frac{(405405\sin(2a))\text{Subst}\left(\int\frac{\cos(2bx)}{\sqrt{x}}\,dx, x, \sqrt[3]{x}\right)}{32768b^7}$$

$$= -\frac{135135\sqrt{x}}{4096b^6} + \frac{3861x^{7/6}}{256b^4} - \frac{39x^{11/6}}{16b^2} + \frac{x^{5/2}}{5} + \frac{135135\sqrt{x}\cos^2\left(a+b\sqrt[3]{x}\right)}{2048b^6} - \frac{3861x^{7/6}\cos^2\left(a+b\sqrt[3]{x}\right)}{128b^4} + \frac{39x^{11/6}\cos^2\left(a+b\sqrt[3]{x}\right)}{8b^2} + \frac{27027x^{5/6}\cos\left(a+b\sqrt[3]{x}\right)\sin\left(a+b\sqrt[3]{x}\right)}{512b^5} - \frac{429x^{3/2}\cos\left(a+b\sqrt[3]{x}\right)\sin\left(a+b\sqrt[3]{x}\right)}{32b^3} + \frac{3x^{13/6}\cos\left(a+b\sqrt[3]{x}\right)\sin\left(a+b\sqrt[3]{x}\right)}{2b} - \frac{405405\sqrt[6]{x}\sin\left(2\left(a+b\sqrt[3]{x}\right)\right)}{16384b^7} + \frac{(405405\cos(2a))\text{Subst}\left(\int\sin\left(2bx^2\right)\,dx,x,\sqrt[6]{x}\right)}{16384b^7} + \frac{(405405\sin(2a))\text{Subst}\left(\int\cos\left(2bx^2\right)\,dx,x,\sqrt[6]{x}\right)}{16384b^7}$$

$$= -\frac{135135\sqrt{x}}{4096b^6} + \frac{3861x^{7/6}}{256b^4} - \frac{39x^{11/6}}{16b^2} + \frac{x^{5/2}}{5} + \frac{135135\sqrt{x}\cos^2\left(a+b\sqrt[3]{x}\right)}{2048b^6} - \frac{3861x^{7/6}\cos^2\left(a+b\sqrt[3]{x}\right)}{128b^4} + \frac{39x^{11/6}\cos^2\left(a+b\sqrt[3]{x}\right)}{8b^2} + \frac{405405\sqrt{\pi}\cos(2a)\,\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt[6]{x}}{\sqrt{\pi}}\right)}{32768b^{15/2}} + \frac{405405\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt[6]{x}}{\sqrt{\pi}}\right)\sin(2a)}{32768b^{15/2}} + \frac{27027x^{5/6}\cos\left(a+b\sqrt[3]{x}\right)\sin\left(a+b\sqrt[3]{x}\right)}{512b^5} - \frac{429x^{3/2}\cos\left(a+b\sqrt[3]{x}\right)\sin\left(a+b\sqrt[3]{x}\right)}{32b^3} + \frac{3x^{13/6}\cos\left(a+b\sqrt[3]{x}\right)\sin\left(a+b\sqrt[3]{x}\right)}{2b} - \frac{405405\sqrt[6]{x}\sin\left(2\left(a+b\sqrt[3]{x}\right)\right)}{16384b^7}$$

Mathematica [A] (verified)

Time = 0.73 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.56

$$\int x^{3/2}\cos^2\left(a+b\sqrt[3]{x}\right)\,dx = \frac{2027025\sqrt{\pi}\cos(2a)\,\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt[6]{x}}{\sqrt{\pi}}\right) + 2027025\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt[6]{x}}{\sqrt{\pi}}\right)\sin(2a) + 2\sqrt{b}\sqrt[6]{x}(\cdots}{\cdots}$$

```
[In] Integrate[x^(3/2)*Cos[a + b*x^(1/3)]^2,x]
```

```
[Out] (2027025*Sqrt[Pi]*Cos[2*a]*FresnelS[(2*Sqrt[b]*x^(1/6))/Sqrt[Pi]] + 2027025
*Sqrt[Pi]*FresnelC[(2*Sqrt[b]*x^(1/6))/Sqrt[Pi]]*Sin[2*a] + 2*Sqrt[b]*x^(1/
6)*(16384*b^7*x^(7/3) + 780*(3465*b*x^(1/3) - 1584*b^3*x + 256*b^5*x^(5/3))
*Cos[2*(a + b*x^(1/3))] + 15*(-135135 + 144144*b^2*x^(2/3) - 36608*b^4*x^(4
/3) + 4096*b^6*x^2)*Sin[2*(a + b*x^(1/3))]))/(163840*b^(15/2))
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.71

method	result
derivativedivides	$\frac{x^{\frac{5}{2}}}{5}+\frac{3x^{\frac{13}{6}}\sin\left(2a+2b\,x^{\frac{1}{3}}\right)}{4b}-\frac{39\left(-\frac{x^{\frac{11}{6}}\cos\left(2a+2b\,x^{\frac{1}{3}}\right)}{4b}+\frac{11x^{\frac{3}{2}}\sin\left(2a+2b\,x^{\frac{1}{3}}\right)}{16b}-\frac{99\left(-\frac{x^{\frac{7}{6}}\cos\left(2a+2b\,x^{\frac{1}{3}}\right)}{4b}+\frac{7x^{\frac{5}{6}}\sin\left(\right.}{}\right.}{}\right.}{}$
default	$\frac{x^{\frac{5}{2}}}{5}+\frac{3x^{\frac{13}{6}}\sin\left(2a+2b\,x^{\frac{1}{3}}\right)}{4b}-\frac{39\left(-\frac{x^{\frac{11}{6}}\cos\left(2a+2b\,x^{\frac{1}{3}}\right)}{4b}+\frac{11x^{\frac{3}{2}}\sin\left(2a+2b\,x^{\frac{1}{3}}\right)}{16b}-\frac{99\left(-\frac{x^{\frac{7}{6}}\cos\left(2a+2b\,x^{\frac{1}{3}}\right)}{4b}+\frac{7x^{\frac{5}{6}}\sin\left(\right.}{}\right.}{}\right.}{}$

```
[In] int(x^(3/2)*cos(a+b*x^(1/3))^2,x,method=_RETURNVERBOSE)
```

[Out] 1/5*x^(5/2)+3/4/b*x^(13/6)*sin(2*a+2*b*x^(1/3))-39/4/b*(-1/4/b*x^(11/6)*cos(2*a+2*b*x^(1/3))+11/4/b*(1/4/b*x^(3/2)*sin(2*a+2*b*x^(1/3))-9/4/b*(-1/4/b*x^(7/6)*cos(2*a+2*b*x^(1/3))+7/4/b*(1/4/b*x^(5/6)*sin(2*a+2*b*x^(1/3))-5/4/b*(-1/4/b*x^(1/2)*cos(2*a+2*b*x^(1/3))+3/4/b*(1/4*x^(1/6)*sin(2*a+2*b*x^(1/3))/b-1/8/b^(3/2)*Pi^(1/2)*(cos(2*a)*FresnelS(2*x^(1/6)*b^(1/2)/Pi^(1/2))+sin(2*a)*FresnelC(2*x^(1/6)*b^(1/2)/Pi^(1/2))))))))

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.59

$$\int x^{3/2}\cos^2\left(a+b\sqrt[3]{x}\right)\,dx =$$

$$-\frac{399360\,b^6x^{\frac{11}{6}}-2471040\,b^4x^{\frac{7}{6}}-2027025\,\pi\sqrt{\frac{b}{\pi}}\cos\left(2\,a\right)\mathrm{S}\left(2\,x^{\frac{1}{6}}\sqrt{\frac{b}{\pi}}\right)-2027025\,\pi\sqrt{\frac{b}{\pi}}\,\mathrm{C}\left(2\,x^{\frac{1}{6}}\sqrt{\frac{b}{\pi}}\right)\sin\left(2\,a\right)}{}$$

[In] integrate(x^(3/2)*cos(a+b*x^(1/3))^2,x, algorithm="fricas")

[Out] -1/163840*(399360*b^6*x^(11/6) - 2471040*b^4*x^(7/6) - 2027025*pi*sqrt(b/pi)*cos(2*a)*fresnel_sin(2*x^(1/6)*sqrt(b/pi)) - 2027025*pi*sqrt(b/pi)*fresnel_cos(2*x^(1/6)*sqrt(b/pi))*sin(2*a) - 3120*(256*b^6*x^(11/6) - 1584*b^4*x^(7/6) + 3465*b^2*sqrt(x))*cos(b*x^(1/3) + a)^2 + 60*(36608*b^5*x^(3/2) - 144144*b^3*x^(5/6) - (4096*b^7*x^2 - 135135*b)*x^(1/6))*cos(b*x^(1/3) + a)*sin(b*x^(1/3) + a) - 8*(4096*b^8*x^2 - 675675*b^2)*sqrt(x))/b^8

Sympy [F]

$$\int x^{3/2}\cos^2\left(a+b\sqrt[3]{x}\right)\,dx=\int x^{\frac{3}{2}}\cos^2\left(a+b\sqrt[3]{x}\right)dx$$

[In] integrate(x**(3/2)*cos(a+b*x**(1/3))**2,x)

[Out] Integral(x**(3/2)*cos(a + b*x**(1/3))**2, x)

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.42 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.52

$$\int x^{3/2}\cos^2\left(a+b\sqrt[3]{x}\right)\,dx=\frac{262144\,b^9x^{\frac{5}{2}}+2027025\cdot 4^{\frac{1}{4}}\sqrt{2}\sqrt{\pi}\Big(\left((i+1)\,\cos\left(2\,a\right)-(i-1)\,\sin\left(2\,a\right)\right)\mathrm{erf}\left(\sqrt{2i\,b}x^{\frac{1}{6}}\right)+(-(i$$

```
[In] integrate(x^(3/2)*cos(a+b*x^(1/3))^2,x, algorithm="maxima")
```

```
[Out] 1/1310720*(262144*b^9*x^(5/2) + 2027025*4^(1/4)*sqrt(2)*sqrt(pi)*(((I + 1)*
cos(2*a) - (I - 1)*sin(2*a))*erf(sqrt(2*I*b)*x^(1/6)) + (-(I - 1)*cos(2*a)
+ (I + 1)*sin(2*a))*erf(sqrt(-2*I*b)*x^(1/6)))*b^(3/2) + 12480*(256*b^7*x^(
11/6) - 1584*b^5*x^(7/6) + 3465*b^3*sqrt(x))*cos(2*b*x^(1/3) + 2*a) + 240*(
4096*b^8*x^(13/6) - 36608*b^6*x^(3/2) + 144144*b^4*x^(5/6) - 135135*b^2*x^(
1/6))*sin(2*b*x^(1/3) + 2*a))/b^9
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.38 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.72

$$\int x^{3/2}\cos^2\left(a+b\sqrt[3]{x}\right)\,dx=\frac{1}{5}\,x^{\frac{5}{2}}-\frac{3\left(4096i\,b^6x^{\frac{13}{6}}-13312\,b^5x^{\frac{11}{6}}-36608i\,b^4x^{\frac{3}{2}}+82368\,b^3x^{\frac{7}{6}}+144144i\,b^2x^{\frac{5}{6}}-180180\,b\sqrt{x}-135135i\,x^{\frac{1}{6}}\right)e^{(}}{32768\,b^7}-\frac{3\left(-4096i\,b^6x^{\frac{13}{6}}-13312\,b^5x^{\frac{11}{6}}+36608i\,b^4x^{\frac{3}{2}}+82368\,b^3x^{\frac{7}{6}}-144144i\,b^2x^{\frac{5}{6}}-180180\,b\sqrt{x}+135135i\,x^{\frac{1}{6}}\right)}{32768\,b^7}+\frac{405405\,\sqrt{\pi}\,\mathrm{erf}\left(-i\,\sqrt{b}x^{\frac{1}{6}}\left(\frac{i\,b}{|b|}+1\right)\right)e^{(2i\,a)}}{65536\,b^{\frac{15}{2}}\left(\frac{i\,b}{|b|}+1\right)}+\frac{405405\,\sqrt{\pi}\,\mathrm{erf}\left(i\,\sqrt{b}x^{\frac{1}{6}}\left(-\frac{i\,b}{|b|}+1\right)\right)e^{(-2i\,a)}}{65536\,b^{\frac{15}{2}}\left(-\frac{i\,b}{|b|}+1\right)}$$

```
[In] integrate(x^(3/2)*cos(a+b*x^(1/3))^2,x, algorithm="giac")
```

```
[Out] 1/5*x^(5/2) - 3/32768*(4096*I*b^6*x^(13/6) - 13312*b^5*x^(11/6) - 36608*I*b
^4*x^(3/2) + 82368*b^3*x^(7/6) + 144144*I*b^2*x^(5/6) - 180180*b*sqrt(x) -
135135*I*x^(1/6))*e^(2*I*b*x^(1/3) + 2*I*a)/b^7 - 3/32768*(-4096*I*b^6*x^(1
3/6) - 13312*b^5*x^(11/6) + 36608*I*b^4*x^(3/2) + 82368*b^3*x^(7/6) - 14414
4*I*b^2*x^(5/6) - 180180*b*sqrt(x) + 135135*I*x^(1/6))*e^(-2*I*b*x^(1/3) -
2*I*a)/b^7 + 405405/65536*sqrt(pi)*erf(-I*sqrt(b)*x^(1/6)*(I*b/abs(b) + 1))
*e^(2*I*a)/(b^(15/2)*(I*b/abs(b) + 1)) + 405405/65536*sqrt(pi)*erf(I*sqrt(b
)*x^(1/6)*(-I*b/abs(b) + 1))*e^(-2*I*a)/(b^(15/2)*(-I*b/abs(b) + 1))
```

Mupad [F(-1)]

Timed out.

$$\int x^{3/2}\cos^2\left(a+b\sqrt[3]{x}\right)\,dx=\int x^{3/2}\cos\left(a+b\,x^{1/3}\right)^2\,dx$$

```
[In] int(x^(3/2)*cos(a + b*x^(1/3))^2,x)
[Out] int(x^(3/2)*cos(a + b*x^(1/3))^2, x)
```

3.56 $\int \sqrt{x}\cos^2\left(a+b\sqrt[3]{x}\right)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 218

$$\begin{aligned}\int \sqrt{x}\cos^2\left(a+b\sqrt[3]{x}\right)\,dx &= \frac{315\sqrt[6]{x}}{256b^4}-\frac{21x^{5/6}}{16b^2}+\frac{x^{3/2}}{3}-\frac{315\sqrt[6]{x}\cos^2\left(a+b\sqrt[3]{x}\right)}{128b^4}\\ &\quad+\frac{21x^{5/6}\cos^2\left(a+b\sqrt[3]{x}\right)}{8b^2}+\frac{315\sqrt{\pi}\cos(2a)\operatorname{FresnelC}\left(\frac{2\sqrt{b}\sqrt[6]{x}}{\sqrt{\pi}}\right)}{512b^{9/2}}\\ &\quad-\frac{315\sqrt{\pi}\operatorname{FresnelS}\left(\frac{2\sqrt{b}\sqrt[6]{x}}{\sqrt{\pi}}\right)\sin(2a)}{512b^{9/2}}\\ &\quad-\frac{105\sqrt{x}\cos\left(a+b\sqrt[3]{x}\right)\sin\left(a+b\sqrt[3]{x}\right)}{32b^3}\\ &\quad+\frac{3x^{7/6}\cos\left(a+b\sqrt[3]{x}\right)\sin\left(a+b\sqrt[3]{x}\right)}{2b}\end{aligned}$$

```
[Out] 315/256*x^(1/6)/b^4-21/16*x^(5/6)/b^2+1/3*x^(3/2)-315/128*x^(1/6)*cos(a+b*x
^(1/3))^2/b^4+21/8*x^(5/6)*cos(a+b*x^(1/3))^2/b^2+3/2*x^(7/6)*cos(a+b*x^(1/
3))*sin(a+b*x^(1/3))/b+315/512*cos(2*a)*FresnelC(2*x^(1/6)*b^(1/2)/Pi^(1/2)
)*Pi^(1/2)/b^(9/2)-315/512*FresnelS(2*x^(1/6)*b^(1/2)/Pi^(1/2))*sin(2*a)*Pi
^(1/2)/b^(9/2)-105/32*cos(a+b*x^(1/3))*sin(a+b*x^(1/3))*x^(1/2)/b^3
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3497, 3392, 30, 3393, 3387, 3386, 3432, 3385, 3433}

$$\int \sqrt{x}\cos^2\left(a+b\sqrt[3]{x}\right)\,dx=\frac{315\sqrt{\pi}\cos(2a)\operatorname{FresnelC}\left(\frac{2\sqrt{b}\sqrt[6]{x}}{\sqrt{\pi}}\right)}{512b^{9/2}}-\frac{315\sqrt{\pi}\sin(2a)\operatorname{FresnelS}\left(\frac{2\sqrt{b}\sqrt[6]{x}}{\sqrt{\pi}}\right)}{512b^{9/2}}-\frac{315\sqrt[6]{x}\cos^2\left(a+b\sqrt[3]{x}\right)}{128b^4}-\frac{105\sqrt{x}\sin\left(a+b\sqrt[3]{x}\right)\cos\left(a+b\sqrt[3]{x}\right)}{32b^3}+\frac{21x^{5/6}\cos^2\left(a+b\sqrt[3]{x}\right)}{8b^2}+\frac{3x^{7/6}\sin\left(a+b\sqrt[3]{x}\right)\cos\left(a+b\sqrt[3]{x}\right)}{2b}+\frac{315\sqrt[6]{x}}{256b^4}-\frac{21x^{5/6}}{16b^2}+\frac{x^{3/2}}{3}$$

```
[In] Int[Sqrt[x]*Cos[a + b*x^(1/3)]^2,x]
```

```
[Out] (315*x^(1/6))/(256*b^4) - (21*x^(5/6))/(16*b^2) + x^(3/2)/3 - (315*x^(1/6)*
Cos[a + b*x^(1/3)]^2)/(128*b^4) + (21*x^(5/6)*Cos[a + b*x^(1/3)]^2)/(8*b^2)
 + (315*Sqrt[Pi]*Cos[2*a]*FresnelC[(2*Sqrt[b]*x^(1/6))/Sqrt[Pi]])/(512*b^(9
/2)) - (315*Sqrt[Pi]*FresnelS[(2*Sqrt[b]*x^(1/6))/Sqrt[Pi]]*Sin[2*a])/(512*
b^(9/2)) - (105*Sqrt[x]*Cos[a + b*x^(1/3)]*Sin[a + b*x^(1/3)])/(32*b^3) + (
3*x^(7/6)*Cos[a + b*x^(1/3)]*Sin[a + b*x^(1/3)])/(2*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
```

```
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3497

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Module[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a +
b*Cos[c + d*x^(k*n)])^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, m}, x]
&& IntegerQ[p] && FractionQ[n]
```

Rubi steps

$$\text{integral} = 3\text{Subst}\left(\int x^{7/2}\cos^2(a+bx)\,dx, x, \sqrt[3]{x}\right)$$

$$= \frac{21x^{5/6}\cos^2\left(a+b\sqrt[3]{x}\right)}{8b^2} + \frac{3x^{7/6}\cos\left(a+b\sqrt[3]{x}\right)\sin\left(a+b\sqrt[3]{x}\right)}{2b} + \frac{3}{2}\text{Subst}\left(\int x^{7/2}\,dx, x, \sqrt[3]{x}\right) - \frac{105\text{Subst}\left(\int x^{3/2}\cos^2(a+bx)\,dx, x, \sqrt[3]{x}\right)}{16b^2}$$

$$=\frac{x^{3/2}}{3}-\frac{315\sqrt[6]{x}\cos^2\left(a+b\sqrt[3]{x}\right)}{128b^4}+\frac{21x^{5/6}\cos^2\left(a+b\sqrt[3]{x}\right)}{8b^2}-\frac{105\sqrt{x}\cos\left(a+b\sqrt[3]{x}\right)\sin\left(a+b\sqrt[3]{x}\right)}{32b^3}+\frac{3x^{7/6}\cos\left(a+b\sqrt[3]{x}\right)\sin\left(a+b\sqrt[3]{x}\right)}{2b}+\frac{315\text{Subst}\left(\int\frac{\cos^2(a+bx)}{\sqrt{x}}\,dx,x,\sqrt[3]{x}\right)}{256b^4}-\frac{105\text{Subst}\left(\int x^{3/2}\,dx,x,\sqrt[3]{x}\right)}{32b^2}$$

$$=-\frac{21x^{5/6}}{16b^2}+\frac{x^{3/2}}{3}-\frac{315\sqrt[6]{x}\cos^2\left(a+b\sqrt[3]{x}\right)}{128b^4}+\frac{21x^{5/6}\cos^2\left(a+b\sqrt[3]{x}\right)}{8b^2}-\frac{105\sqrt{x}\cos\left(a+b\sqrt[3]{x}\right)\sin\left(a+b\sqrt[3]{x}\right)}{32b^3}+\frac{3x^{7/6}\cos\left(a+b\sqrt[3]{x}\right)\sin\left(a+b\sqrt[3]{x}\right)}{2b}+\frac{315\text{Subst}\left(\int\left(\frac{1}{2\sqrt{x}}+\frac{\cos(2a+2bx)}{2\sqrt{x}}\right)\,dx,x,\sqrt[3]{x}\right)}{256b^4}$$

$$=\frac{315\sqrt[6]{x}}{256b^4}-\frac{21x^{5/6}}{16b^2}+\frac{x^{3/2}}{3}-\frac{315\sqrt[6]{x}\cos^2\left(a+b\sqrt[3]{x}\right)}{128b^4}+\frac{21x^{5/6}\cos^2\left(a+b\sqrt[3]{x}\right)}{8b^2}-\frac{105\sqrt{x}\cos\left(a+b\sqrt[3]{x}\right)\sin\left(a+b\sqrt[3]{x}\right)}{32b^3}+\frac{3x^{7/6}\cos\left(a+b\sqrt[3]{x}\right)\sin\left(a+b\sqrt[3]{x}\right)}{2b}+\frac{315\text{Subst}\left(\int\frac{\cos(2a+2bx)}{\sqrt{x}}\,dx,x,\sqrt[3]{x}\right)}{512b^4}$$

$$=\frac{315\sqrt[6]{x}}{256b^4}-\frac{21x^{5/6}}{16b^2}+\frac{x^{3/2}}{3}-\frac{315\sqrt[6]{x}\cos^2\left(a+b\sqrt[3]{x}\right)}{128b^4}+\frac{21x^{5/6}\cos^2\left(a+b\sqrt[3]{x}\right)}{8b^2}-\frac{105\sqrt{x}\cos\left(a+b\sqrt[3]{x}\right)\sin\left(a+b\sqrt[3]{x}\right)}{32b^3}+\frac{3x^{7/6}\cos\left(a+b\sqrt[3]{x}\right)\sin\left(a+b\sqrt[3]{x}\right)}{2b}+\frac{(315\cos(2a))\text{Subst}\left(\int\frac{\cos(2bx)}{\sqrt{x}}\,dx,x,\sqrt[3]{x}\right)}{512b^4}-\frac{(315\sin(2a))\text{Subst}\left(\int\frac{\sin(2bx)}{\sqrt{x}}\,dx,x,\sqrt[3]{x}\right)}{512b^4}$$

$$=\frac{315\sqrt[6]{x}}{256b^4}-\frac{21x^{5/6}}{16b^2}+\frac{x^{3/2}}{3}-\frac{315\sqrt[6]{x}\cos^2\left(a+b\sqrt[3]{x}\right)}{128b^4}+\frac{21x^{5/6}\cos^2\left(a+b\sqrt[3]{x}\right)}{8b^2}-\frac{105\sqrt{x}\cos\left(a+b\sqrt[3]{x}\right)\sin\left(a+b\sqrt[3]{x}\right)}{32b^3}+\frac{3x^{7/6}\cos\left(a+b\sqrt[3]{x}\right)\sin\left(a+b\sqrt[3]{x}\right)}{2b}+\frac{(315\cos(2a))\text{Subst}\left(\int\cos\left(2bx^2\right)\,dx,x,\sqrt[6]{x}\right)}{256b^4}-\frac{(315\sin(2a))\text{Subst}\left(\int\sin\left(2bx^2\right)\,dx,x,\sqrt[6]{x}\right)}{256b^4}$$

$$=\frac{315\sqrt[6]{x}}{256b^4}-\frac{21x^{5/6}}{16b^2}+\frac{x^{3/2}}{3}-\frac{315\sqrt[6]{x}\cos^2\left(a+b\sqrt[3]{x}\right)}{128b^4}+\frac{21x^{5/6}\cos^2\left(a+b\sqrt[3]{x}\right)}{8b^2}+\frac{315\sqrt{\pi}\cos(2a)\,\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt[6]{x}}{\sqrt{\pi}}\right)}{512b^{9/2}}-\frac{315\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt[6]{x}}{\sqrt{\pi}}\right)\sin(2a)}{512b^{9/2}}-\frac{105\sqrt{x}\cos\left(a+b\sqrt[3]{x}\right)\sin\left(a+b\sqrt[3]{x}\right)}{32b^3}+\frac{3x^{7/6}\cos\left(a+b\sqrt[3]{x}\right)\sin\left(a+b\sqrt[3]{x}\right)}{2b}$$

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.68

$$\int \sqrt{x}\cos^2\left(a+b\sqrt[3]{x}\right)\,dx$$
$$=\frac{945\sqrt{\pi}\cos(2a)\,\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt[6]{x}}{\sqrt{\pi}}\right)-945\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt[6]{x}}{\sqrt{\pi}}\right)\sin(2a)+2\sqrt{b}\sqrt[6]{x}\left(63\left(-15+16b^2x^{2/3}\right)\cdots\right.}{1536b^{9/2}}$$

```
[In] Integrate[Sqrt[x]*Cos[a + b*x^(1/3)]^2,x]
```

```
[Out] (945*Sqrt[Pi]*Cos[2*a]*FresnelC[(2*Sqrt[b]*x^(1/6))/Sqrt[Pi]] - 945*Sqrt[Pi
]*FresnelS[(2*Sqrt[b]*x^(1/6))/Sqrt[Pi]]*Sin[2*a] + 2*Sqrt[b]*x^(1/6)*(63*(
-15 + 16*b^2*x^(2/3))*Cos[2*(a + b*x^(1/3))] + 4*b*x^(1/3)*(64*b^3*x + 9*(-
35 + 16*b^2*x^(2/3))*Sin[2*(a + b*x^(1/3))])))/(1536*b^(9/2))
```

Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.67

method	result
derivativedivides	$\frac{x^{\frac{3}{2}}}{3}+\frac{3x^{\frac{7}{6}}\sin\left(2a+2b\,x^{\frac{1}{3}}\right)}{4b}-\frac{21\left(-\frac{x^{\frac{5}{6}}\cos\left(2a+2b\,x^{\frac{1}{3}}\right)}{4b}+\frac{5\sqrt{x}\,\sin\left(2a+2b\,x^{\frac{1}{3}}\right)}{16b}-\frac{15\left(-\frac{x^{\frac{1}{6}}\cos\left(2a+2b\,x^{\frac{1}{3}}\right)}{4b}+\frac{\sqrt{\pi}\left(\cos(2a)\cdots\right.}{16\cdots}\right)}{b}\cdots\right)}{4b}$
default	$\frac{x^{\frac{3}{2}}}{3}+\frac{3x^{\frac{7}{6}}\sin\left(2a+2b\,x^{\frac{1}{3}}\right)}{4b}-\frac{21\left(-\frac{x^{\frac{5}{6}}\cos\left(2a+2b\,x^{\frac{1}{3}}\right)}{4b}+\frac{5\sqrt{x}\,\sin\left(2a+2b\,x^{\frac{1}{3}}\right)}{16b}-\frac{15\left(-\frac{x^{\frac{1}{6}}\cos\left(2a+2b\,x^{\frac{1}{3}}\right)}{4b}+\frac{\sqrt{\pi}\left(\cos(2a)\cdots\right.}{16\cdots}\right)}{b}\cdots\right)}{4b}$

```
[In] int(x^(1/2)*cos(a+b*x^(1/3))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x^(3/2)+3/4/b*x^(7/6)*sin(2*a+2*b*x^(1/3))-21/4/b*(-1/4/b*x^(5/6)*cos(2
*a+2*b*x^(1/3))+5/4/b*(1/4/b*x^(1/2)*sin(2*a+2*b*x^(1/3))-3/4/b*(-1/4/b*x^(
1/6)*cos(2*a+2*b*x^(1/3))+1/8/b^(3/2)*Pi^(1/2)*(cos(2*a)*FresnelC(2*x^(1/6)
*b^(1/2)/Pi^(1/2))-sin(2*a)*FresnelS(2*x^(1/6)*b^(1/2)/Pi^(1/2))))))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.66

$$\int \sqrt{x}\cos^2\left(a+b\sqrt[3]{x}\right)\,dx$$
$$= \frac{512\,b^5x^{\frac{3}{2}} - 2016\,b^3x^{\frac{5}{6}} + 945\,\pi\sqrt{\frac{b}{\pi}}\cos\left(2\,a\right)\mathrm{C}\left(2\,x^{\frac{1}{6}}\sqrt{\frac{b}{\pi}}\right) - 945\,\pi\sqrt{\frac{b}{\pi}}\,\mathrm{S}\left(2\,x^{\frac{1}{6}}\sqrt{\frac{b}{\pi}}\right)\sin\left(2\,a\right) + 252\left(16\,b^3x^{\frac{5}{6}} -\right.}{1536\,b^5}$$

```
[In] integrate(x^(1/2)*cos(a+b*x^(1/3))^2,x, algorithm="fricas")
```

```
[Out] 1/1536*(512*b^5*x^(3/2) - 2016*b^3*x^(5/6) + 945*pi*sqrt(b/pi)*cos(2*a)*fre
snel_cos(2*x^(1/6)*sqrt(b/pi)) - 945*pi*sqrt(b/pi)*fresnel_sin(2*x^(1/6)*sq
rt(b/pi))*sin(2*a) + 252*(16*b^3*x^(5/6) - 15*b*x^(1/6))*cos(b*x^(1/3) + a)
^2 + 144*(16*b^4*x^(7/6) - 35*b^2*sqrt(x))*cos(b*x^(1/3) + a)*sin(b*x^(1/3)
 + a) + 1890*b*x^(1/6))/b^5
```

Sympy [F]

$$\int \sqrt{x}\cos^2\left(a+b\sqrt[3]{x}\right)\,dx = \int \sqrt{x}\cos^2\left(a+b\sqrt[3]{x}\right)dx$$

```
[In] integrate(x**(1/2)*cos(a+b*x**(1/3))**2,x)
```

```
[Out] Integral(sqrt(x)*cos(a + b*x**(1/3))**2, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.37 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.63

$$\int \sqrt{x}\cos^2\left(a+b\sqrt[3]{x}\right)\,dx$$
$$= \frac{4096\,b^6x^{\frac{3}{2}} + 945\cdot 4^{\frac{1}{4}}\sqrt{2}\sqrt{\pi}\Big(\left(-(i-1)\,\cos\left(2\,a\right) - (i+1)\,\sin\left(2\,a\right)\right)\mathrm{erf}\left(\sqrt{2i\,b}x^{\frac{1}{6}}\right) + \left((i+1)\,\cos\left(2\,a\right) + (i -\right.}{}$$

```
[In] integrate(x^(1/2)*cos(a+b*x^(1/3))^2,x, algorithm="maxima")
```

```
[Out] 1/12288*(4096*b^6*x^(3/2) + 945*4^(1/4)*sqrt(2)*sqrt(pi)*((-(I - 1)*cos(2*a
) - (I + 1)*sin(2*a))*erf(sqrt(2*I*b)*x^(1/6)) + ((I + 1)*cos(2*a) + (I - 1
)*sin(2*a))*erf(sqrt(-2*I*b)*x^(1/6)))*b^(3/2) + 1008*(16*b^4*x^(5/6) - 15*
b^2*x^(1/6))*cos(2*b*x^(1/3) + 2*a) + 576*(16*b^5*x^(7/6) - 35*b^3*sqrt(x))
*sin(2*b*x^(1/3) + 2*a))/b^6
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.36 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.81

$$\int \sqrt{x}\cos^2\left(a+b\sqrt[3]{x}\right)\,dx=\frac{1}{3}\,x^{\frac{3}{2}}-\frac{3\left(64i\,b^3x^{\frac{7}{6}}-112\,b^2x^{\frac{5}{6}}-140i\,b\sqrt{x}+105\,x^{\frac{1}{6}}\right)e^{\left(2i\,bx^{\frac{1}{3}}+2i\,a\right)}}{512\,b^4}-\frac{3\left(-64i\,b^3x^{\frac{7}{6}}-112\,b^2x^{\frac{5}{6}}+140i\,b\sqrt{x}+105\,x^{\frac{1}{6}}\right)e^{\left(-2i\,bx^{\frac{1}{3}}-2i\,a\right)}}{512\,b^4}+\frac{315i\,\sqrt{\pi}\,\mathrm{erf}\left(-i\,\sqrt{b}x^{\frac{1}{6}}\left(\frac{i\,b}{|b|}+1\right)\right)e^{(2i\,a)}}{1024\,b^{\frac{9}{2}}\left(\frac{i\,b}{|b|}+1\right)}-\frac{315i\,\sqrt{\pi}\,\mathrm{erf}\left(i\,\sqrt{b}x^{\frac{1}{6}}\left(-\frac{i\,b}{|b|}+1\right)\right)e^{(-2i\,a)}}{1024\,b^{\frac{9}{2}}\left(-\frac{i\,b}{|b|}+1\right)}$$

```
[In] integrate(x^(1/2)*cos(a+b*x^(1/3))^2,x, algorithm="giac")
```

```
[Out] 1/3*x^(3/2) - 3/512*(64*I*b^3*x^(7/6) - 112*b^2*x^(5/6) - 140*I*b*sqrt(x) +
 105*x^(1/6))*e^(2*I*b*x^(1/3) + 2*I*a)/b^4 - 3/512*(-64*I*b^3*x^(7/6) - 11
2*b^2*x^(5/6) + 140*I*b*sqrt(x) + 105*x^(1/6))*e^(-2*I*b*x^(1/3) - 2*I*a)/b
^4 + 315/1024*I*sqrt(pi)*erf(-I*sqrt(b)*x^(1/6)*(I*b/abs(b) + 1))*e^(2*I*a)
/(b^(9/2)*(I*b/abs(b) + 1)) - 315/1024*I*sqrt(pi)*erf(I*sqrt(b)*x^(1/6)*(-I
*b/abs(b) + 1))*e^(-2*I*a)/(b^(9/2)*(-I*b/abs(b) + 1))
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{x}\cos^2\left(a+b\sqrt[3]{x}\right)\,dx=\int \sqrt{x}\cos\left(a+b\,x^{1/3}\right)^2\,dx$$

```
[In] int(x^(1/2)*cos(a + b*x^(1/3))^2,x)
```

```
[Out] int(x^(1/2)*cos(a + b*x^(1/3))^2, x)
```

3.57 $\int \frac{\cos^2\left(a+b\sqrt[3]{x}\right)}{\sqrt{x}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 102

$$\int \frac{\cos^2\left(a+b\sqrt[3]{x}\right)}{\sqrt{x}}\,dx=\sqrt{x}-\frac{3\sqrt{\pi}\cos(2a)\operatorname{FresnelS}\left(\frac{2\sqrt{b}\sqrt[6]{x}}{\sqrt{\pi}}\right)}{8b^{3/2}}-\frac{3\sqrt{\pi}\operatorname{FresnelC}\left(\frac{2\sqrt{b}\sqrt[6]{x}}{\sqrt{\pi}}\right)\sin(2a)}{8b^{3/2}}+\frac{3\sqrt[6]{x}\sin\left(2\left(a+b\sqrt[3]{x}\right)\right)}{4b}$$

```
[Out] 3/4*x^(1/6)*sin(2*a+2*b*x^(1/3))/b-3/8*cos(2*a)*FresnelS(2*x^(1/6)*b^(1/2)/
Pi^(1/2))*Pi^(1/2)/b^(3/2)-3/8*FresnelC(2*x^(1/6)*b^(1/2)/Pi^(1/2))*sin(2*a
)*Pi^(1/2)/b^(3/2)+x^(1/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {3497, 3393, 3377, 3387, 3386, 3432, 3385, 3433}

$$\int \frac{\cos^2\left(a+b\sqrt[3]{x}\right)}{\sqrt{x}}\,dx=-\frac{3\sqrt{\pi}\sin(2a)\operatorname{FresnelC}\left(\frac{2\sqrt{b}\sqrt[6]{x}}{\sqrt{\pi}}\right)}{8b^{3/2}}-\frac{3\sqrt{\pi}\cos(2a)\operatorname{FresnelS}\left(\frac{2\sqrt{b}\sqrt[6]{x}}{\sqrt{\pi}}\right)}{8b^{3/2}}+\frac{3\sqrt[6]{x}\sin\left(2\left(a+b\sqrt[3]{x}\right)\right)}{4b}+\sqrt{x}$$

```
[In] Int[Cos[a + b*x^(1/3)]^2/Sqrt[x],x]
```

```
[Out] Sqrt[x] - (3*Sqrt[Pi]*Cos[2*a]*FresnelS[(2*Sqrt[b]*x^(1/6))/Sqrt[Pi]])/(8*b
^(3/2)) - (3*Sqrt[Pi]*FresnelC[(2*Sqrt[b]*x^(1/6))/Sqrt[Pi]]*Sin[2*a])/(8*b
^(3/2)) + (3*x^(1/6)*Sin[2*(a + b*x^(1/3))])/(4*b)
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3497

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Module[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a +
b*Cos[c + d*x^(k*n)])^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, m}, x]
&& IntegerQ[p] && FractionQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 3\text{Subst}\left(\int \sqrt{x}\cos^2(a+bx)\,dx, x, \sqrt[3]{x}\right)\\
&= 3\text{Subst}\left(\int \left(\frac{\sqrt{x}}{2}+\frac{1}{2}\sqrt{x}\cos(2a+2bx)\right)\,dx, x, \sqrt[3]{x}\right)\\
&= \sqrt{x}+\frac{3}{2}\text{Subst}\left(\int \sqrt{x}\cos(2a+2bx)\,dx, x, \sqrt[3]{x}\right)\\
&= \sqrt{x}+\frac{3\sqrt[6]{x}\sin\left(2\left(a+b\sqrt[3]{x}\right)\right)}{4b}-\frac{3\text{Subst}\left(\int \frac{\sin(2a+2bx)}{\sqrt{x}}\,dx, x, \sqrt[3]{x}\right)}{8b}\\
&= \sqrt{x}+\frac{3\sqrt[6]{x}\sin\left(2\left(a+b\sqrt[3]{x}\right)\right)}{4b}-\frac{(3\cos(2a))\text{Subst}\left(\int \frac{\sin(2bx)}{\sqrt{x}}\,dx, x, \sqrt[3]{x}\right)}{8b}\\
&\quad-\frac{(3\sin(2a))\text{Subst}\left(\int \frac{\cos(2bx)}{\sqrt{x}}\,dx, x, \sqrt[3]{x}\right)}{8b}\\
&= \sqrt{x}+\frac{3\sqrt[6]{x}\sin\left(2\left(a+b\sqrt[3]{x}\right)\right)}{4b}-\frac{(3\cos(2a))\text{Subst}\left(\int \sin\left(2bx^2\right)\,dx, x, \sqrt[6]{x}\right)}{4b}\\
&\quad-\frac{(3\sin(2a))\text{Subst}\left(\int \cos\left(2bx^2\right)\,dx, x, \sqrt[6]{x}\right)}{4b}\\
&= \sqrt{x}-\frac{3\sqrt{\pi}\cos(2a)\,\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt[6]{x}}{\sqrt{\pi}}\right)}{8b^{3/2}}\\
&\quad-\frac{3\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt[6]{x}}{\sqrt{\pi}}\right)\sin(2a)}{8b^{3/2}}+\frac{3\sqrt[6]{x}\sin\left(2\left(a+b\sqrt[3]{x}\right)\right)}{4b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.01

$$\int \frac{\cos^2\left(a+b\sqrt[3]{x}\right)}{\sqrt{x}}\,dx$$
$$= \frac{-3\sqrt{\pi}\cos(2a)\,\mathrm{FresnelS}\left(\frac{2\sqrt{b}\sqrt[6]{x}}{\sqrt{\pi}}\right)-3\sqrt{\pi}\,\mathrm{FresnelC}\left(\frac{2\sqrt{b}\sqrt[6]{x}}{\sqrt{\pi}}\right)\sin(2a)+2\sqrt{b}\sqrt[6]{x}\left(4b\sqrt[3]{x}+3\sin\left(2\left(a+b\sqrt[3]{\cdots}\right.\right.\right.}{8b^{3/2}}$$

```
[In] Integrate[Cos[a + b*x^(1/3)]^2/Sqrt[x],x]
```

```
[Out] (-3*Sqrt[Pi]*Cos[2*a]*FresnelS[(2*Sqrt[b]*x^(1/6))/Sqrt[Pi]] - 3*Sqrt[Pi]*F
resnelC[(2*Sqrt[b]*x^(1/6))/Sqrt[Pi]]*Sin[2*a] + 2*Sqrt[b]*x^(1/6)*(4*b*x^(
1/3) + 3*Sin[2*(a + b*x^(1/3))]))/(8*b^(3/2))
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.66

method	result	size
derivativedivides	$\sqrt{x}+\frac{3x^{\frac{1}{6}}\sin\left(2a+2b\,x^{\frac{1}{3}}\right)}{4b}-\frac{3\sqrt{\pi}\left(\cos(2a)\,\mathrm{S}\left(\frac{2x^{\frac{1}{6}}\sqrt{b}}{\sqrt{\pi}}\right)+\sin(2a)\,\mathrm{C}\left(\frac{2x^{\frac{1}{6}}\sqrt{b}}{\sqrt{\pi}}\right)\right)}{8b^{\frac{3}{2}}}$	67
default	$\sqrt{x}+\frac{3x^{\frac{1}{6}}\sin\left(2a+2b\,x^{\frac{1}{3}}\right)}{4b}-\frac{3\sqrt{\pi}\left(\cos(2a)\,\mathrm{S}\left(\frac{2x^{\frac{1}{6}}\sqrt{b}}{\sqrt{\pi}}\right)+\sin(2a)\,\mathrm{C}\left(\frac{2x^{\frac{1}{6}}\sqrt{b}}{\sqrt{\pi}}\right)\right)}{8b^{\frac{3}{2}}}$	67

```
[In] int(cos(a+b*x^(1/3))^2/x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] x^(1/2)+3/4*x^(1/6)*sin(2*a+2*b*x^(1/3))/b-3/8/b^(3/2)*Pi^(1/2)*(cos(2*a)*F
resnelS(2*x^(1/6)*b^(1/2)/Pi^(1/2))+sin(2*a)*FresnelC(2*x^(1/6)*b^(1/2)/Pi^
(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.88

$$\int \frac{\cos^2\left(a+b\sqrt[3]{x}\right)}{\sqrt{x}}\,dx =$$
$$-\frac{3\,\pi\sqrt{\frac{b}{\pi}}\cos\left(2\,a\right)\mathrm{S}\left(2\,x^{\frac{1}{6}}\sqrt{\frac{b}{\pi}}\right)+3\,\pi\sqrt{\frac{b}{\pi}}\,\mathrm{C}\left(2\,x^{\frac{1}{6}}\sqrt{\frac{b}{\pi}}\right)\sin\left(2\,a\right)-12\,bx^{\frac{1}{6}}\cos\left(bx^{\frac{1}{3}}+a\right)\sin\left(bx^{\frac{1}{3}}+a\right)-\cdots}{8\,b^{2}}$$

```
[In] integrate(cos(a+b*x^(1/3))^2/x^(1/2),x, algorithm="fricas")
```

```
[Out] -1/8*(3*pi*sqrt(b/pi)*cos(2*a)*fresnel_sin(2*x^(1/6)*sqrt(b/pi)) + 3*pi*sqr
t(b/pi)*fresnel_cos(2*x^(1/6)*sqrt(b/pi))*sin(2*a) - 12*b*x^(1/6)*cos(b*x^(
1/3) + a)*sin(b*x^(1/3) + a) - 8*b^2*sqrt(x))/b^2
```

Sympy [F]

$$\int \frac{\cos^2\left(a+b\sqrt[3]{x}\right)}{\sqrt{x}}\,dx=\int \frac{\cos^2\left(a+b\sqrt[3]{x}\right)}{\sqrt{x}}\,dx$$

```
[In] integrate(cos(a+b*x**(1/3))**2/x**(1/2),x)
```

```
[Out] Integral(cos(a + b*x**(1/3))**2/sqrt(x), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.39 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.94

$$\int \frac{\cos^2\left(a+b\sqrt[3]{x}\right)}{\sqrt{x}}\,dx = -\frac{3\cdot 4^{\frac{1}{4}}\sqrt{2}\sqrt{\pi}\Big(\left(\left(i+1\right)\,\cos\left(2\,a\right)-\left(i-1\right)\,\sin\left(2\,a\right)\right)\operatorname{erf}\left(\sqrt{2i\,b}x^{\frac{1}{6}}\right)+\left(-\left(i-1\right)\,\cos\left(2\,a\right)+\left(i+1\right)\,\sin\left(2\,a\right)\right.}{64\,b^3}$$

```
[In] integrate(cos(a+b*x^(1/3))^2/x^(1/2),x, algorithm="maxima")
```

```
[Out] -1/64*(3*4^(1/4)*sqrt(2)*sqrt(pi)*(((I + 1)*cos(2*a) - (I - 1)*sin(2*a))*er
f(sqrt(2*I*b)*x^(1/6)) + (-(I - 1)*cos(2*a) + (I + 1)*sin(2*a))*erf(sqrt(-2
*I*b)*x^(1/6)))*b^(3/2) - 64*b^3*sqrt(x) - 48*b^2*x^(1/6)*sin(2*b*x^(1/3) +
 2*a))/b^3
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.22

$$\int \frac{\cos^2\left(a+b\sqrt[3]{x}\right)}{\sqrt{x}}\,dx=\sqrt{x}-\frac{3i\,x^{\frac{1}{6}}e^{\left(2i\,bx^{\frac{1}{3}}+2i\,a\right)}}{8\,b}+\frac{3i\,x^{\frac{1}{6}}e^{\left(-2i\,bx^{\frac{1}{3}}-2i\,a\right)}}{8\,b}-\frac{3\,\sqrt{\pi}\operatorname{erf}\left(-i\,\sqrt{b}x^{\frac{1}{6}}\left(\frac{i\,b}{|b|}+1\right)\right)e^{\left(2i\,a\right)}}{16\,b^{\frac{3}{2}}\left(\frac{i\,b}{|b|}+1\right)}-\frac{3\,\sqrt{\pi}\operatorname{erf}\left(i\,\sqrt{b}x^{\frac{1}{6}}\left(-\frac{i\,b}{|b|}+1\right)\right)e^{\left(-2i\,a\right)}}{16\,b^{\frac{3}{2}}\left(-\frac{i\,b}{|b|}+1\right)}$$

```
[In] integrate(cos(a+b*x^(1/3))^2/x^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(x) - 3/8*I*x^(1/6)*e^(2*I*b*x^(1/3) + 2*I*a)/b + 3/8*I*x^(1/6)*e^(-2*I
*b*x^(1/3) - 2*I*a)/b - 3/16*sqrt(pi)*erf(-I*sqrt(b)*x^(1/6)*(I*b/abs(b) +
1))*e^(2*I*a)/(b^(3/2)*(I*b/abs(b) + 1)) - 3/16*sqrt(pi)*erf(I*sqrt(b)*x^(1
/6)*(-I*b/abs(b) + 1))*e^(-2*I*a)/(b^(3/2)*(-I*b/abs(b) + 1))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2\left(a+b\sqrt[3]{x}\right)}{\sqrt{x}}\,dx=\int \frac{\cos\left(a+b\,x^{1/3}\right)^2}{\sqrt{x}}\,dx$$

```
[In] int(cos(a + b*x^(1/3))^2/x^(1/2),x)
```

```
[Out] int(cos(a + b*x^(1/3))^2/x^(1/2), x)
```

3.58 $\int \frac{\cos^2\left(a+b\sqrt[3]{x}\right)}{x^{3/2}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 116

$$\int \frac{\cos^2\left(a+b\sqrt[3]{x}\right)}{x^{3/2}}\,dx=-\frac{2\cos^2\left(a+b\sqrt[3]{x}\right)}{\sqrt{x}}-8b^{3/2}\sqrt{\pi}\cos(2a)\,\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt[6]{x}}{\sqrt{\pi}}\right)+8b^{3/2}\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt[6]{x}}{\sqrt{\pi}}\right)\sin(2a)+\frac{8b\cos\left(a+b\sqrt[3]{x}\right)\sin\left(a+b\sqrt[3]{x}\right)}{\sqrt[6]{x}}$$

```
[Out] 8*b*cos(a+b*x^(1/3))*sin(a+b*x^(1/3))/x^(1/6)-8*b^(3/2)*cos(2*a)*FresnelC(2
*x^(1/6)*b^(1/2)/Pi^(1/2))*Pi^(1/2)+8*b^(3/2)*FresnelS(2*x^(1/6)*b^(1/2)/Pi
^(1/2))*sin(2*a)*Pi^(1/2)-2*cos(a+b*x^(1/3))^2/x^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3497, 3395, 30, 3393, 3387, 3386, 3432, 3385, 3433}

$$\int \frac{\cos^2\left(a+b\sqrt[3]{x}\right)}{x^{3/2}}\,dx=-8\sqrt{\pi}b^{3/2}\cos(2a)\,\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt[6]{x}}{\sqrt{\pi}}\right)+8\sqrt{\pi}b^{3/2}\sin(2a)\,\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt[6]{x}}{\sqrt{\pi}}\right)-\frac{2\cos^2\left(a+b\sqrt[3]{x}\right)}{\sqrt{x}}+\frac{8b\sin\left(a+b\sqrt[3]{x}\right)\cos\left(a+b\sqrt[3]{x}\right)}{\sqrt[6]{x}}$$

```
[In] Int[Cos[a + b*x^(1/3)]^2/x^(3/2),x]
```

```
[Out] (-2*Cos[a + b*x^(1/3)]^2)/Sqrt[x] - 8*b^(3/2)*Sqrt[Pi]*Cos[2*a]*FresnelC[(2
*Sqrt[b]*x^(1/6))/Sqrt[Pi]] + 8*b^(3/2)*Sqrt[Pi]*FresnelS[(2*Sqrt[b]*x^(1/6
))/Sqrt[Pi]]*Sin[2*a] + (8*b*Cos[a + b*x^(1/3)]*Sin[a + b*x^(1/3)])/x^(1/6)
```

<u>Rule 30</u>

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

<u>Rule 3385</u>

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

<u>Rule 3386</u>

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

<u>Rule 3387</u>

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

<u>Rule 3393</u>

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

<u>Rule 3395</u>

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(c + d*x)^(m + 1)*((b*Sin[e + f*x])^n/(d*(m + 1))), x] + (Dist[b
^2*f^2*n*((n - 1)/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[f^2*(n^2/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[b*f*n*(c + d*x)^(m + 2)*Cos[e +
f*x]*((b*Sin[e + f*x])^(n - 1)/(d^2*(m + 1)*(m + 2))), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

<u>Rule 3432</u>

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

<u>Rule 3433</u>

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3497

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Module[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a +
b*Cos[c + d*x^(k*n)])^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, m}, x]
&& IntegerQ[p] && FractionQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 3\text{Subst}\left(\int \frac{\cos^2(a+bx)}{x^{5/2}}\,dx, x, \sqrt[3]{x}\right)\\
&= -\frac{2\cos^2\left(a+b\sqrt[3]{x}\right)}{\sqrt{x}}+\frac{8b\cos\left(a+b\sqrt[3]{x}\right)\sin\left(a+b\sqrt[3]{x}\right)}{\sqrt[6]{x}}\\
&\quad+\left(8b^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}}\,dx, x, \sqrt[3]{x}\right)-\left(16b^2\right)\text{Subst}\left(\int \frac{\cos^2(a+bx)}{\sqrt{x}}\,dx, x, \sqrt[3]{x}\right)\\
&= 16b^2\sqrt[6]{x}-\frac{2\cos^2\left(a+b\sqrt[3]{x}\right)}{\sqrt{x}}+\frac{8b\cos\left(a+b\sqrt[3]{x}\right)\sin\left(a+b\sqrt[3]{x}\right)}{\sqrt[6]{x}}\\
&\quad-\left(16b^2\right)\text{Subst}\left(\int \left(\frac{1}{2\sqrt{x}}+\frac{\cos(2a+2bx)}{2\sqrt{x}}\right)dx, x, \sqrt[3]{x}\right)\\
&= -\frac{2\cos^2\left(a+b\sqrt[3]{x}\right)}{\sqrt{x}}+\frac{8b\cos\left(a+b\sqrt[3]{x}\right)\sin\left(a+b\sqrt[3]{x}\right)}{\sqrt[6]{x}}\\
&\quad-\left(8b^2\right)\text{Subst}\left(\int \frac{\cos(2a+2bx)}{\sqrt{x}}\,dx, x, \sqrt[3]{x}\right)\\
&= -\frac{2\cos^2\left(a+b\sqrt[3]{x}\right)}{\sqrt{x}}+\frac{8b\cos\left(a+b\sqrt[3]{x}\right)\sin\left(a+b\sqrt[3]{x}\right)}{\sqrt[6]{x}}\\
&\quad-\left(8b^2\cos(2a)\right)\text{Subst}\left(\int \frac{\cos(2bx)}{\sqrt{x}}\,dx, x, \sqrt[3]{x}\right)\\
&\quad+\left(8b^2\sin(2a)\right)\text{Subst}\left(\int \frac{\sin(2bx)}{\sqrt{x}}\,dx, x, \sqrt[3]{x}\right)\\
&= -\frac{2\cos^2\left(a+b\sqrt[3]{x}\right)}{\sqrt{x}}+\frac{8b\cos\left(a+b\sqrt[3]{x}\right)\sin\left(a+b\sqrt[3]{x}\right)}{\sqrt[6]{x}}\\
&\quad-\left(16b^2\cos(2a)\right)\text{Subst}\left(\int \cos\left(2bx^2\right)\,dx, x, \sqrt[6]{x}\right)\\
&\quad+\left(16b^2\sin(2a)\right)\text{Subst}\left(\int \sin\left(2bx^2\right)\,dx, x, \sqrt[6]{x}\right)
\end{aligned}$$

$$= -\frac{2\cos^2\left(a+b\sqrt[3]{x}\right)}{\sqrt{x}} - 8b^{3/2}\sqrt{\pi}\cos(2a)\,\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt[6]{x}}{\sqrt{\pi}}\right) + 8b^{3/2}\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt[6]{x}}{\sqrt{\pi}}\right)\sin(2a) + \frac{8b\cos\left(a+b\sqrt[3]{x}\right)\sin\left(a+b\sqrt[3]{x}\right)}{\sqrt[6]{x}}$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00

$$\int \frac{\cos^2\left(a+b\sqrt[3]{x}\right)}{x^{3/2}}\,dx = -8b^{3/2}\sqrt{\pi}\cos(2a)\,\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt[6]{x}}{\sqrt{\pi}}\right) + \frac{-1-\cos\left(2\left(a+b\sqrt[3]{x}\right)\right)+8b^{3/2}\sqrt{\pi}\sqrt{x}\,\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt[6]{x}}{\sqrt{\pi}}\right)\sin(2a)+4b\sqrt[3]{x}\sin\left(2\left(a+b\sqrt[3]{x}\right)\right)}{\sqrt{x}}$$

```
[In] Integrate[Cos[a + b*x^(1/3)]^2/x^(3/2),x]
```

```
[Out] -8*b^(3/2)*Sqrt[Pi]*Cos[2*a]*FresnelC[(2*Sqrt[b]*x^(1/6))/Sqrt[Pi]] + (-1 -
 Cos[2*(a + b*x^(1/3))] + 8*b^(3/2)*Sqrt[Pi]*Sqrt[x]*FresnelS[(2*Sqrt[b]*x^
(1/6))/Sqrt[Pi]]*Sin[2*a] + 4*b*x^(1/3)*Sin[2*(a + b*x^(1/3))])/Sqrt[x]
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.75

method	result
derivativedivides	$-\frac{1}{\sqrt{x}}-\frac{\cos\left(2a+2b\,x^{\frac{1}{3}}\right)}{\sqrt{x}}-4b\left(-\frac{\sin\left(2a+2b\,x^{\frac{1}{3}}\right)}{x^{\frac{1}{6}}}+2\sqrt{b}\,\sqrt{\pi}\,\left(\cos\left(2a\right)\text{C}\left(\frac{2x^{\frac{1}{6}}\sqrt{b}}{\sqrt{\pi}}\right)-\sin\left(2a\right)\text{S}\left(\right.\right.\right.$
default	$-\frac{1}{\sqrt{x}}-\frac{\cos\left(2a+2b\,x^{\frac{1}{3}}\right)}{\sqrt{x}}-4b\left(-\frac{\sin\left(2a+2b\,x^{\frac{1}{3}}\right)}{x^{\frac{1}{6}}}+2\sqrt{b}\,\sqrt{\pi}\,\left(\cos\left(2a\right)\text{C}\left(\frac{2x^{\frac{1}{6}}\sqrt{b}}{\sqrt{\pi}}\right)-\sin\left(2a\right)\text{S}\left(\right.\right.\right.$

```
[In] int(cos(a+b*x^(1/3))^2/x^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/x^(1/2)-1/x^(1/2)*cos(2*a+2*b*x^(1/3))-4*b*(-1/x^(1/6)*sin(2*a+2*b*x^(1/
3))+2*b^(1/2)*Pi^(1/2)*(cos(2*a)*FresnelC(2*x^(1/6)*b^(1/2)/Pi^(1/2))-sin(2
*a)*FresnelS(2*x^(1/6)*b^(1/2)/Pi^(1/2))))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.86

$$\int \frac{\cos^2\left(a+b\sqrt[3]{x}\right)}{x^{3/2}}\,dx = -\frac{2\left(4\,\pi bx\sqrt{\frac{b}{\pi}}\cos\left(2\,a\right)\mathrm{C}\left(2\,x^{\frac{1}{6}}\sqrt{\frac{b}{\pi}}\right)-4\,\pi bx\sqrt{\frac{b}{\pi}}\,\mathrm{S}\left(2\,x^{\frac{1}{6}}\sqrt{\frac{b}{\pi}}\right)\sin\left(2\,a\right)-4\,bx^{\frac{5}{6}}\cos\left(bx^{\frac{1}{3}}+a\right)\sin\left(bx^{\frac{1}{3}}+a\right)+\cdots}{x}$$

```
[In] integrate(cos(a+b*x^(1/3))^2/x^(3/2),x, algorithm="fricas")
```

```
[Out] -2*(4*pi*b*x*sqrt(b/pi)*cos(2*a)*fresnel_cos(2*x^(1/6)*sqrt(b/pi)) - 4*pi*b
*x*sqrt(b/pi)*fresnel_sin(2*x^(1/6)*sqrt(b/pi))*sin(2*a) - 4*b*x^(5/6)*cos(
b*x^(1/3) + a)*sin(b*x^(1/3) + a) + sqrt(x)*cos(b*x^(1/3) + a)^2)/x
```

Sympy [F]

$$\int \frac{\cos^2\left(a+b\sqrt[3]{x}\right)}{x^{3/2}}\,dx = \int \frac{\cos^2\left(a+b\sqrt[3]{x}\right)}{x^{\frac{3}{2}}}\,dx$$

```
[In] integrate(cos(a+b*x**(1/3))**2/x**(3/2),x)
```

```
[Out] Integral(cos(a + b*x**(1/3))**2/x**(3/2), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.45 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.76

$$\int \frac{\cos^2\left(a+b\sqrt[3]{x}\right)}{x^{3/2}}\,dx = \frac{3\sqrt{2}\left(\left(-(i-1)\sqrt{2}\Gamma\left(-\frac{3}{2},2i\,bx^{\frac{1}{3}}\right)+(i+1)\sqrt{2}\Gamma\left(-\frac{3}{2},-2i\,bx^{\frac{1}{3}}\right)\right)\cos\left(2\,a\right)+\left(-(i\cdots\right.\right.}{4\sqrt{x}}$$

```
[In] integrate(cos(a+b*x^(1/3))^2/x^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(3*sqrt(2)*((-(I - 1)*sqrt(2)*gamma(-3/2, 2*I*b*x^(1/3)) + (I + 1)*sqrt
(2)*gamma(-3/2, -2*I*b*x^(1/3)))*cos(2*a) + (-(I + 1)*sqrt(2)*gamma(-3/2, 2
*I*b*x^(1/3)) + (I - 1)*sqrt(2)*gamma(-3/2, -2*I*b*x^(1/3)))*sin(2*a))*sqrt
(b*x^(1/3))*b*x^(1/3) - 4)/sqrt(x)
```

Giac [F]

$$\int \frac{\cos^2\left(a+b\sqrt[3]{x}\right)}{x^{3/2}}\,dx=\int \frac{\cos\left(bx^{\frac{1}{3}}+a\right)^2}{x^{\frac{3}{2}}}\,dx$$

```
[In] integrate(cos(a+b*x^(1/3))^2/x^(3/2),x, algorithm="giac")
[Out] integrate(cos(b*x^(1/3) + a)^2/x^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2\left(a+b\sqrt[3]{x}\right)}{x^{3/2}}\,dx=\int \frac{\cos\left(a+b\,x^{1/3}\right)^2}{x^{3/2}}\,dx$$

```
[In] int(cos(a + b*x^(1/3))^2/x^(3/2),x)
[Out] int(cos(a + b*x^(1/3))^2/x^(3/2), x)
```

3.59 $\int \frac{\cos^2\left(a+b\sqrt[3]{x}\right)}{x^{5/2}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 228

$$\int \frac{\cos^2\left(a+b\sqrt[3]{x}\right)}{x^{5/2}}\,dx = -\frac{16b^2}{105x^{5/6}}+\frac{256b^4}{315\sqrt[6]{x}}-\frac{2\cos^2\left(a+b\sqrt[3]{x}\right)}{3x^{3/2}}$$
$$+\frac{32b^2\cos^2\left(a+b\sqrt[3]{x}\right)}{105x^{5/6}}-\frac{512b^4\cos^2\left(a+b\sqrt[3]{x}\right)}{315\sqrt[6]{x}}$$
$$-\frac{512}{315}b^{9/2}\sqrt{\pi}\cos(2a)\,\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt[6]{x}}{\sqrt{\pi}}\right)-\frac{512}{315}b^{9/2}\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt[6]{x}}{\sqrt{\pi}}\right)\sin(2a)+\frac{8b\cos\left(a+b\sqrt[3]{x}\right)\sin\left(\right.}{21x^{7/6}}$$

```
[Out] -16/105*b^2/x^(5/6)+256/315*b^4/x^(1/6)-2/3*cos(a+b*x^(1/3))^2/x^(3/2)+32/1
05*b^2*cos(a+b*x^(1/3))^2/x^(5/6)-512/315*b^4*cos(a+b*x^(1/3))^2/x^(1/6)+8/
21*b*cos(a+b*x^(1/3))*sin(a+b*x^(1/3))/x^(7/6)-512/315*b^(9/2)*cos(2*a)*Fre
snelS(2*x^(1/6)*b^(1/2)/Pi^(1/2))*Pi^(1/2)-512/315*b^(9/2)*FresnelC(2*x^(1/
6)*b^(1/2)/Pi^(1/2))*sin(2*a)*Pi^(1/2)-128/315*b^3*cos(a+b*x^(1/3))*sin(a+b
*x^(1/3))/x^(1/2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {3497, 3395, 30, 3394, 12, 3387, 3386, 3432, 3385, 3433}

$$\int \frac{\cos^2\left(a+b\sqrt[3]{x}\right)}{x^{5/2}}\,dx = -\frac{512}{315}\sqrt{\pi}b^{9/2}\sin(2a)\,\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt[6]{x}}{\sqrt{\pi}}\right)$$
$$-\frac{512}{315}\sqrt{\pi}b^{9/2}\cos(2a)\,\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt[6]{x}}{\sqrt{\pi}}\right)-\frac{512b^4\cos^2\left(a+b\sqrt[3]{x}\right)}{315\sqrt[6]{x}}-\frac{128b^3\sin\left(a+b\sqrt[3]{x}\right)\cos\left(a+b\sqrt[3]{x}\right)}{315\sqrt{x}}+\frac{32b}{}$$

```
[In] Int[Cos[a + b*x^(1/3)]^2/x^(5/2),x]
```

```
[Out] (-16*b^2)/(105*x^(5/6)) + (256*b^4)/(315*x^(1/6)) - (2*Cos[a + b*x^(1/3)]^2
)/(3*x^(3/2)) + (32*b^2*Cos[a + b*x^(1/3)]^2)/(105*x^(5/6)) - (512*b^4*Cos[
a + b*x^(1/3)]^2)/(315*x^(1/6)) - (512*b^(9/2)*Sqrt[Pi]*Cos[2*a]*FresnelS[(
2*Sqrt[b]*x^(1/6))/Sqrt[Pi]])/315 - (512*b^(9/2)*Sqrt[Pi]*FresnelC[(2*Sqrt[
b]*x^(1/6))/Sqrt[Pi]]*Sin[2*a])/315 + (8*b*Cos[a + b*x^(1/3)]*Sin[a + b*x^(
1/3)])/(21*x^(7/6)) - (128*b^3*Cos[a + b*x^(1/3)]*Sin[a + b*x^(1/3)])/(315*
Sqrt[x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3394

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Dist[f*(n/(d*(m + 1
))), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 3395

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(c + d*x)^(m + 1)*((b*Sin[e + f*x])^n/(d*(m + 1))), x] + (Dist[b
^2*f^2*n*((n - 1)/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[f^2*(n^2/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[b*f*n*(c + d*x)^(m + 2)*Cos[e +
f*x]*((b*Sin[e + f*x])^(n - 1)/(d^2*(m + 1)*(m + 2))), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3497

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Module[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a +
b*Cos[c + d*x^(k*n)])^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, m}, x]
&& IntegerQ[p] && FractionQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 3\text{Subst}\left(\int \frac{\cos^2(a+bx)}{x^{11/2}}\,dx, x, \sqrt[3]{x}\right)\\
&= -\frac{2\cos^2\left(a+b\sqrt[3]{x}\right)}{3x^{3/2}} + \frac{8b\cos\left(a+b\sqrt[3]{x}\right)\sin\left(a+b\sqrt[3]{x}\right)}{21x^{7/6}}\\
&\quad + \frac{1}{21}\left(8b^2\right)\text{Subst}\left(\int \frac{1}{x^{7/2}}\,dx, x, \sqrt[3]{x}\right) - \frac{1}{21}\left(16b^2\right)\text{Subst}\left(\int \frac{\cos^2(a+bx)}{x^{7/2}}\,dx, x, \sqrt[3]{x}\right)\\
&= -\frac{16b^2}{105x^{5/6}} - \frac{2\cos^2\left(a+b\sqrt[3]{x}\right)}{3x^{3/2}} + \frac{32b^2\cos^2\left(a+b\sqrt[3]{x}\right)}{105x^{5/6}}\\
&\quad + \frac{8b\cos\left(a+b\sqrt[3]{x}\right)\sin\left(a+b\sqrt[3]{x}\right)}{21x^{7/6}} - \frac{128b^3\cos\left(a+b\sqrt[3]{x}\right)\sin\left(a+b\sqrt[3]{x}\right)}{315\sqrt{x}}\\
&\quad - \frac{1}{315}\left(128b^4\right)\text{Subst}\left(\int \frac{1}{x^{3/2}}\,dx, x, \sqrt[3]{x}\right) + \frac{1}{315}\left(256b^4\right)\text{Subst}\left(\int \frac{\cos^2(a+bx)}{x^{3/2}}\,dx, x, \sqrt[3]{x}\right)
\end{aligned}$$

$$= -\frac{16b^2}{105x^{5/6}} + \frac{256b^4}{315\sqrt[6]{x}} - \frac{2\cos^2\left(a+b\sqrt[3]{x}\right)}{3x^{3/2}} + \frac{32b^2\cos^2\left(a+b\sqrt[3]{x}\right)}{105x^{5/6}} - \frac{512b^4\cos^2\left(a+b\sqrt[3]{x}\right)}{315\sqrt[6]{x}} + \frac{8b\cos\left(a+b\sqrt[3]{x}\right)\sin\left(a+b\sqrt[3]{x}\right)}{21x^{7/6}} - \frac{128b^3\cos\left(a+b\sqrt[3]{x}\right)\sin\left(a+b\sqrt[3]{x}\right)}{315\sqrt{x}} + \frac{1}{315}\left(1024b^5\right)\text{Subst}\left(\int -\frac{\sin(2a+2bx)}{2\sqrt{x}}\,dx, x, \sqrt[3]{x}\right)$$

$$= -\frac{16b^2}{105x^{5/6}} + \frac{256b^4}{315\sqrt[6]{x}} - \frac{2\cos^2\left(a+b\sqrt[3]{x}\right)}{3x^{3/2}} + \frac{32b^2\cos^2\left(a+b\sqrt[3]{x}\right)}{105x^{5/6}} - \frac{512b^4\cos^2\left(a+b\sqrt[3]{x}\right)}{315\sqrt[6]{x}} + \frac{8b\cos\left(a+b\sqrt[3]{x}\right)\sin\left(a+b\sqrt[3]{x}\right)}{21x^{7/6}} - \frac{128b^3\cos\left(a+b\sqrt[3]{x}\right)\sin\left(a+b\sqrt[3]{x}\right)}{315\sqrt{x}} - \frac{1}{315}\left(512b^5\right)\text{Subst}\left(\int \frac{\sin(2a+2bx)}{\sqrt{x}}\,dx, x, \sqrt[3]{x}\right)$$

$$= -\frac{16b^2}{105x^{5/6}} + \frac{256b^4}{315\sqrt[6]{x}} - \frac{2\cos^2\left(a+b\sqrt[3]{x}\right)}{3x^{3/2}} + \frac{32b^2\cos^2\left(a+b\sqrt[3]{x}\right)}{105x^{5/6}} - \frac{512b^4\cos^2\left(a+b\sqrt[3]{x}\right)}{315\sqrt[6]{x}} + \frac{8b\cos\left(a+b\sqrt[3]{x}\right)\sin\left(a+b\sqrt[3]{x}\right)}{21x^{7/6}} - \frac{128b^3\cos\left(a+b\sqrt[3]{x}\right)\sin\left(a+b\sqrt[3]{x}\right)}{315\sqrt{x}} - \frac{1}{315}\left(512b^5\cos(2a)\right)\text{Subst}\left(\int \frac{\sin(2bx)}{\sqrt{x}}\,dx, x, \sqrt[3]{x}\right) - \frac{1}{315}\left(512b^5\sin(2a)\right)\text{Subst}\left(\int \frac{\cos(2bx)}{\sqrt{x}}\,dx,\right.$$

$$= -\frac{16b^2}{105x^{5/6}} + \frac{256b^4}{315\sqrt[6]{x}} - \frac{2\cos^2\left(a+b\sqrt[3]{x}\right)}{3x^{3/2}} + \frac{32b^2\cos^2\left(a+b\sqrt[3]{x}\right)}{105x^{5/6}} - \frac{512b^4\cos^2\left(a+b\sqrt[3]{x}\right)}{315\sqrt[6]{x}} + \frac{8b\cos\left(a+b\sqrt[3]{x}\right)\sin\left(a+b\sqrt[3]{x}\right)}{21x^{7/6}} - \frac{128b^3\cos\left(a+b\sqrt[3]{x}\right)\sin\left(a+b\sqrt[3]{x}\right)}{315\sqrt{x}} - \frac{1}{315}\left(1024b^5\cos(2a)\right)\text{Subst}\left(\int \sin\left(2bx^2\right)\,dx, x, \sqrt[6]{x}\right) - \frac{1}{315}\left(1024b^5\sin(2a)\right)\text{Subst}\left(\int \cos\left(2bx\right.\right.$$

$$= -\frac{16b^2}{105x^{5/6}} + \frac{256b^4}{315\sqrt[6]{x}} - \frac{2\cos^2\left(a+b\sqrt[3]{x}\right)}{3x^{3/2}} + \frac{32b^2\cos^2\left(a+b\sqrt[3]{x}\right)}{105x^{5/6}} - \frac{512b^4\cos^2\left(a+b\sqrt[3]{x}\right)}{315\sqrt[6]{x}} - \frac{512}{315}b^{9/2}\sqrt{\pi}\cos(2a)\,\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt[6]{x}}{\sqrt{\pi}}\right) - \frac{512}{315}b^{9/2}\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt[6]{x}}{\sqrt{\pi}}\right)\sin(2a) + \frac{8b\cos\left(a+\right.}{}$$

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.81

$$\int \frac{\cos^2\left(a+b\sqrt[3]{x}\right)}{x^{5/2}}\,dx = \frac{-105-105\cos\left(2\left(a+b\sqrt[3]{x}\right)\right)+48b^2x^{2/3}\cos\left(2\left(a+b\sqrt[3]{x}\right)\right)-256b^4x^{4/3}\cos\left(2\left(a+b\right.\right.}{}$$

```
[In] Integrate[Cos[a + b*x^(1/3)]^2/x^(5/2),x]
```

```
[Out] (-105 - 105*Cos[2*(a + b*x^(1/3))] + 48*b^2*x^(2/3)*Cos[2*(a + b*x^(1/3))]
- 256*b^4*x^(4/3)*Cos[2*(a + b*x^(1/3))] - 512*b^(9/2)*Sqrt[Pi]*x^(3/2)*Cos
[2*a]*FresnelS[(2*Sqrt[b]*x^(1/6))/Sqrt[Pi]] - 512*b^(9/2)*Sqrt[Pi]*x^(3/2)
*FresnelC[(2*Sqrt[b]*x^(1/6))/Sqrt[Pi]]*Sin[2*a] + 60*b*x^(1/3)*Sin[2*(a +
b*x^(1/3))] - 64*b^3*x*Sin[2*(a + b*x^(1/3))])/(315*x^(3/2))
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.64

method	result
derivativedivides	$-\frac{1}{3x^{\frac{3}{2}}}-\frac{\cos\left(2a+2b\,x^{\frac{1}{3}}\right)}{3x^{\frac{3}{2}}}-\frac{4b\left(-\frac{\sin\left(2a+2b\,x^{\frac{1}{3}}\right)}{7x^{\frac{7}{6}}}+\frac{4b\left(-\frac{\cos\left(2a+2b\,x^{\frac{1}{3}}\right)}{5x^{\frac{5}{6}}}-\frac{4b\left(-\frac{\sin\left(2a+2b\,x^{\frac{1}{3}}\right)}{3\sqrt{x}}+\frac{4b\left(-\frac{\cos\left(2a+2b\,x^{\frac{1}{3}}\right)}{x^{\frac{1}{6}}}\cdots\right.}{\cdots}\right)}{\cdots}\right)}{7\cdots}\right)}{3\cdots}$
default	$-\frac{1}{3x^{\frac{3}{2}}}-\frac{\cos\left(2a+2b\,x^{\frac{1}{3}}\right)}{3x^{\frac{3}{2}}}-\frac{4b\left(-\frac{\sin\left(2a+2b\,x^{\frac{1}{3}}\right)}{7x^{\frac{7}{6}}}+\frac{4b\left(-\frac{\cos\left(2a+2b\,x^{\frac{1}{3}}\right)}{5x^{\frac{5}{6}}}-\frac{4b\left(-\frac{\sin\left(2a+2b\,x^{\frac{1}{3}}\right)}{3\sqrt{x}}+\frac{4b\left(-\frac{\cos\left(2a+2b\,x^{\frac{1}{3}}\right)}{x^{\frac{1}{6}}}\cdots\right.}{\cdots}\right)}{\cdots}\right)}{7\cdots}\right)}{3\cdots}$

```
[In] int(cos(a+b*x^(1/3))^2/x^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/x^(3/2)-1/3/x^(3/2)*cos(2*a+2*b*x^(1/3))-4/3*b*(-1/7/x^(7/6)*sin(2*a+2
*b*x^(1/3))+4/7*b*(-1/5/x^(5/6)*cos(2*a+2*b*x^(1/3))-4/5*b*(-1/3/x^(1/2)*si
n(2*a+2*b*x^(1/3))+4/3*b*(-1/x^(1/6)*cos(2*a+2*b*x^(1/3))-2*b^(1/2)*Pi^(1/2
)*(cos(2*a)*FresnelS(2*x^(1/6)*b^(1/2)/Pi^(1/2))+sin(2*a)*FresnelC(2*x^(1/6
)*b^(1/2)/Pi^(1/2))))))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.68

$$\int \frac{\cos^2\left(a+b\sqrt[3]{x}\right)}{x^{5/2}}\,dx =$$

$$-\frac{2\left(256\,\pi b^4x^2\sqrt{\frac{b}{\pi}}\cos\left(2\,a\right)\mathrm{S}\left(2\,x^{\frac{1}{6}}\sqrt{\frac{b}{\pi}}\right)+256\,\pi b^4x^2\sqrt{\frac{b}{\pi}}\,\mathrm{C}\left(2\,x^{\frac{1}{6}}\sqrt{\frac{b}{\pi}}\right)\sin\left(2\,a\right)-128\,b^4x^{\frac{11}{6}}+24\,b^2x^{\frac{7}{6}}+\left(256\right.\right.}{315\,x^2}$$

```
[In] integrate(cos(a+b*x^(1/3))^2/x^(5/2),x, algorithm="fricas")
```

```
[Out] -2/315*(256*pi*b^4*x^2*sqrt(b/pi)*cos(2*a)*fresnel_sin(2*x^(1/6)*sqrt(b/pi)
) + 256*pi*b^4*x^2*sqrt(b/pi)*fresnel_cos(2*x^(1/6)*sqrt(b/pi))*sin(2*a) -
128*b^4*x^(11/6) + 24*b^2*x^(7/6) + (256*b^4*x^(11/6) - 48*b^2*x^(7/6) + 10
5*sqrt(x))*cos(b*x^(1/3) + a)^2 + 4*(16*b^3*x^(3/2) - 15*b*x^(5/6))*cos(b*x
^(1/3) + a)*sin(b*x^(1/3) + a))/x^2
```

Sympy [F]

$$\int \frac{\cos^2\left(a+b\sqrt[3]{x}\right)}{x^{5/2}}\,dx = \int \frac{\cos^2\left(a+b\sqrt[3]{x}\right)}{x^{\frac{5}{2}}}\,dx$$

```
[In] integrate(cos(a+b*x**(1/3))**2/x**(5/2),x)
```

```
[Out] Integral(cos(a + b*x**(1/3))**2/x**(5/2), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.45 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.39

$$\int \frac{\cos^2\left(a+b\sqrt[3]{x}\right)}{x^{5/2}}\,dx = -\frac{18\sqrt{2}\left(\left((i+1)\sqrt{2}\Gamma\left(-\frac{9}{2},2i\,bx^{\frac{1}{3}}\right)-(i-1)\sqrt{2}\Gamma\left(-\frac{9}{2},-2i\,bx^{\frac{1}{3}}\right)\right)\cos\left(2\,a\right)+\left(-(i-1)\sqrt{2}\Gamma\left(-\frac{9}{2},2i\,bx^{\frac{1}{3}}\right)\right.\right.}{3\,x^{\frac{3}{2}}}$$

```
[In] integrate(cos(a+b*x^(1/3))^2/x^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*(18*sqrt(2)*(((I + 1)*sqrt(2)*gamma(-9/2, 2*I*b*x^(1/3)) - (I - 1)*sqr
t(2)*gamma(-9/2, -2*I*b*x^(1/3)))*cos(2*a) + (-(I - 1)*sqrt(2)*gamma(-9/2,
2*I*b*x^(1/3)) + (I + 1)*sqrt(2)*gamma(-9/2, -2*I*b*x^(1/3)))*sin(2*a))*sqr
t(b*x^(1/3))*b^4*x^(4/3) + 1)/x^(3/2)
```

Giac [F]

$$\int \frac{\cos^2\left(a+b\sqrt[3]{x}\right)}{x^{5/2}}\,dx = \int \frac{\cos\left(bx^{\frac{1}{3}}+a\right)^2}{x^{\frac{5}{2}}}\,dx$$

```
[In] integrate(cos(a+b*x^(1/3))^2/x^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x^(1/3) + a)^2/x^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2\left(a+b\sqrt[3]{x}\right)}{x^{5/2}}\,dx = \int \frac{\cos\left(a+b\,x^{1/3}\right)^2}{x^{5/2}}\,dx$$

```
[In] int(cos(a + b*x^(1/3))^2/x^(5/2),x)
```

```
[Out] int(cos(a + b*x^(1/3))^2/x^(5/2), x)
```

3.60 $\int \frac{\cos^2\left(a+b\sqrt[3]{x}\right)}{x^{7/2}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 328

$$\begin{aligned}\int \frac{\cos^2\left(a+b\sqrt[3]{x}\right)}{x^{7/2}}\,dx &= -\frac{16b^2}{715x^{11/6}}+\frac{256b^4}{45045x^{7/6}}-\frac{4096b^6}{675675\sqrt{x}}\\
&-\frac{2\cos^2\left(a+b\sqrt[3]{x}\right)}{5x^{5/2}}+\frac{32b^2\cos^2\left(a+b\sqrt[3]{x}\right)}{715x^{11/6}}-\frac{512b^4\cos^2\left(a+b\sqrt[3]{x}\right)}{45045x^{7/6}}\\
&+\frac{8192b^6\cos^2\left(a+b\sqrt[3]{x}\right)}{675675\sqrt{x}}+\frac{32768b^{15/2}\sqrt{\pi}\cos(2a)\operatorname{FresnelC}\left(\frac{2\sqrt{b}\sqrt[6]{x}}{\sqrt{\pi}}\right)}{675675}\\
&-\frac{32768b^{15/2}\sqrt{\pi}\operatorname{FresnelS}\left(\frac{2\sqrt{b}\sqrt[6]{x}}{\sqrt{\pi}}\right)\sin(2a)}{675675}\\
&+\frac{8b\cos\left(a+b\sqrt[3]{x}\right)\sin\left(a+b\sqrt[3]{x}\right)}{65x^{13/6}}-\frac{128b^3\cos\left(a+b\sqrt[3]{x}\right)\sin\left(a+b\sqrt[3]{x}\right)}{6435x^{3/2}}\\
&+\frac{2048b^5\cos\left(a+b\sqrt[3]{x}\right)\sin\left(a+b\sqrt[3]{x}\right)}{225225x^{5/6}}-\frac{32768b^7\cos\left(a+b\sqrt[3]{x}\right)\sin\left(a+b\sqrt[3]{x}\right)}{675675\sqrt[6]{x}}\end{aligned}$$

```
[Out] -16/715*b^2/x^(11/6)+256/45045*b^4/x^(7/6)-2/5*cos(a+b*x^(1/3))^2/x^(5/2)+3
2/715*b^2*cos(a+b*x^(1/3))^2/x^(11/6)-512/45045*b^4*cos(a+b*x^(1/3))^2/x^(7
/6)+8/65*b*cos(a+b*x^(1/3))*sin(a+b*x^(1/3))/x^(13/6)-128/6435*b^3*cos(a+b*
x^(1/3))*sin(a+b*x^(1/3))/x^(3/2)+2048/225225*b^5*cos(a+b*x^(1/3))*sin(a+b*
x^(1/3))/x^(5/6)-32768/675675*b^7*cos(a+b*x^(1/3))*sin(a+b*x^(1/3))/x^(1/6)
+32768/675675*b^(15/2)*cos(2*a)*FresnelC(2*x^(1/6)*b^(1/2)/Pi^(1/2))*Pi^(1/
2)-32768/675675*b^(15/2)*FresnelS(2*x^(1/6)*b^(1/2)/Pi^(1/2))*sin(2*a)*Pi^(
1/2)-4096/675675*b^6/x^(1/2)+8192/675675*b^6*cos(a+b*x^(1/3))^2/x^(1/2)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3497, 3395, 30, 3393, 3387, 3386, 3432, 3385, 3433}

$$\int \frac{\cos^2\left(a+b\sqrt[3]{x}\right)}{x^{7/2}}\,dx = \frac{32768\sqrt{\pi}b^{15/2}\cos(2a)\,\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt[6]{x}}{\sqrt{\pi}}\right)}{675675} - \frac{32768\sqrt{\pi}b^{15/2}\sin(2a)\,\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt[6]{x}}{\sqrt{\pi}}\right)}{675675} - \frac{32768b^7\sin\left(a+b\sqrt[3]{x}\right)\cos\left(a+b\sqrt[3]{x}\right)}{675675\sqrt[6]{x}} + \frac{8192b^6\cos^2\left(a+b\sqrt[3]{x}\right)}{675675\sqrt{x}} + \frac{2048b^5\sin\left(a+b\sqrt[3]{x}\right)\cos\left(a+b\sqrt[3]{x}\right)}{225225x^{5/6}} - \frac{512b^4\cos^2\left(a+b\sqrt[3]{x}\right)}{45045x^{7/6}} - \frac{128b^3\sin\left(a+b\sqrt[3]{x}\right)\cos\left(a+b\sqrt[3]{x}\right)}{6435x^{3/2}} + \frac{32b^2\cos^2\left(a+b\sqrt[3]{x}\right)}{715x^{11/6}} - \frac{2\cos^2\left(a+b\sqrt[3]{x}\right)}{5x^{5/2}} + \frac{8b\sin\left(a+b\sqrt[3]{x}\right)\cos\left(a+b\sqrt[3]{x}\right)}{65x^{13/6}} - \frac{4096b^6}{675675\sqrt{x}} + \frac{256b^4}{45045x^{7/6}} - \frac{16b^2}{715x^{11/6}}$$

```
[In] Int[Cos[a + b*x^(1/3)]^2/x^(7/2),x]
```

```
[Out] (-16*b^2)/(715*x^(11/6)) + (256*b^4)/(45045*x^(7/6)) - (4096*b^6)/(675675*S
qrt[x]) - (2*Cos[a + b*x^(1/3)]^2)/(5*x^(5/2)) + (32*b^2*Cos[a + b*x^(1/3)]
^2)/(715*x^(11/6)) - (512*b^4*Cos[a + b*x^(1/3)]^2)/(45045*x^(7/6)) + (8192
*b^6*Cos[a + b*x^(1/3)]^2)/(675675*Sqrt[x]) + (32768*b^(15/2)*Sqrt[Pi]*Cos[
2*a]*FresnelC[(2*Sqrt[b]*x^(1/6))/Sqrt[Pi]])/675675 - (32768*b^(15/2)*Sqrt[
Pi]*FresnelS[(2*Sqrt[b]*x^(1/6))/Sqrt[Pi]]*Sin[2*a])/675675 + (8*b*Cos[a +
b*x^(1/3)]*Sin[a + b*x^(1/3)])/(65*x^(13/6)) - (128*b^3*Cos[a + b*x^(1/3)]*
Sin[a + b*x^(1/3)])/(6435*x^(3/2)) + (2048*b^5*Cos[a + b*x^(1/3)]*Sin[a + b
*x^(1/3)])/(225225*x^(5/6)) - (32768*b^7*Cos[a + b*x^(1/3)]*Sin[a + b*x^(1/
3)])/(675675*x^(1/6))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3395

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(c + d*x)^(m + 1)*((b*Sin[e + f*x])^n/(d*(m + 1))), x] + (Dist[b
^2*f^2*n*((n - 1)/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[f^2*(n^2/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[b*f*n*(c + d*x)^(m + 2)*Cos[e +
f*x]*((b*Sin[e + f*x])^(n - 1)/(d^2*(m + 1)*(m + 2))), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3497

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Module[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a +
b*Cos[c + d*x^(k*n)])^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, m}, x]
&& IntegerQ[p] && FractionQ[n]
```

Rubi steps

$$\text{integral} = 3\text{Subst}\left(\int \frac{\cos^2(a+bx)}{x^{17/2}}\,dx, x, \sqrt[3]{x}\right)$$

$$= -\frac{2\cos^2\left(a+b\sqrt[3]{x}\right)}{5x^{5/2}} + \frac{8b\cos\left(a+b\sqrt[3]{x}\right)\sin\left(a+b\sqrt[3]{x}\right)}{65x^{13/6}} + \frac{1}{65}\left(8b^2\right)\text{Subst}\left(\int \frac{1}{x^{13/2}}\,dx, x, \sqrt[3]{x}\right) - \frac{1}{65}\left(16b^2\right)\text{Subst}\left(\int \frac{\cos^2(a+bx)}{x^{13/2}}\,dx, x, \sqrt[3]{x}\right)$$

$$= -\frac{16b^2}{715x^{11/6}} - \frac{2\cos^2\left(a+b\sqrt[3]{x}\right)}{5x^{5/2}} + \frac{32b^2\cos^2\left(a+b\sqrt[3]{x}\right)}{715x^{11/6}} + \frac{8b\cos\left(a+b\sqrt[3]{x}\right)\sin\left(a+b\sqrt[3]{x}\right)}{65x^{13/6}} - \frac{128b^3\cos\left(a+b\sqrt[3]{x}\right)\sin\left(a+b\sqrt[3]{x}\right)}{6435x^{3/2}} - \frac{\left(128b^4\right)\text{Subst}\left(\int \frac{1}{x^{9/2}}\,dx, x, \sqrt[3]{x}\right)}{6435} + \frac{\left(256b^4\right)\text{Subst}\left(\int \frac{\cos^2(a+bx)}{x^{9/2}}\,dx, x, \sqrt[3]{x}\right)}{6435}$$

$$= -\frac{16b^2}{715x^{11/6}} + \frac{256b^4}{45045x^{7/6}} - \frac{2\cos^2\left(a+b\sqrt[3]{x}\right)}{5x^{5/2}} + \frac{32b^2\cos^2\left(a+b\sqrt[3]{x}\right)}{715x^{11/6}} - \frac{512b^4\cos^2\left(a+b\sqrt[3]{x}\right)}{45045x^{7/6}} + \frac{8b\cos\left(a+b\sqrt[3]{x}\right)\sin\left(a+b\sqrt[3]{x}\right)}{65x^{13/6}} - \frac{128b^3\cos\left(a+b\sqrt[3]{x}\right)\sin\left(a+b\sqrt[3]{x}\right)}{6435x^{3/2}} + \frac{2048b^5\cos\left(a+b\sqrt[3]{x}\right)\sin\left(a+b\sqrt[3]{x}\right)}{225225x^{5/6}} + \frac{\left(2048b^6\right)\text{Subst}\left(\int \frac{1}{x^{5/2}}\,dx, x, \sqrt[3]{x}\right)}{225225} - \frac{\left(4096b^6\right)\text{Subst}\left(\int \frac{\cos^2(a+bx)}{x^{5/2}}\,dx, x, \sqrt[3]{x}\right)}{225225}$$

$$= -\frac{16b^2}{715x^{11/6}} + \frac{256b^4}{45045x^{7/6}} - \frac{4096b^6}{675675\sqrt{x}} - \frac{2\cos^2\left(a+b\sqrt[3]{x}\right)}{5x^{5/2}} + \frac{32b^2\cos^2\left(a+b\sqrt[3]{x}\right)}{715x^{11/6}} - \frac{512b^4\cos^2\left(a+b\sqrt[3]{x}\right)}{45045x^{7/6}} + \frac{8192b^6\cos^2\left(a+b\sqrt[3]{x}\right)}{675675\sqrt{x}} + \frac{8b\cos\left(a+b\sqrt[3]{x}\right)\sin\left(a+b\sqrt[3]{x}\right)}{65x^{13/6}} - \frac{128b^3\cos\left(a+b\sqrt[3]{x}\right)\sin\left(a+b\sqrt[3]{x}\right)}{6435x^{3/2}} + \frac{2048b^5\cos\left(a+b\sqrt[3]{x}\right)\sin\left(a+b\sqrt[3]{x}\right)}{225225x^{5/6}} - \frac{32768b^7\cos\left(a+b\sqrt[3]{x}\right)\sin\left(a+b\sqrt[3]{x}\right)}{675675\sqrt[6]{x}} - \frac{\left(32768b^8\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}}\,dx, x, \sqrt[3]{x}\right)}{675675} + \frac{\left(65536b^8\right)\text{Subst}\left(\int \frac{\cos^2(a+bx)}{\sqrt{x}}\,dx, x, \sqrt[3]{x}\right)}{675675}$$

$$= -\frac{16b^2}{715x^{11/6}} + \frac{256b^4}{45045x^{7/6}} - \frac{4096b^6}{675675\sqrt{x}} - \frac{65536b^8\sqrt[6]{x}}{675675} - \frac{2\cos^2\left(a+b\sqrt[3]{x}\right)}{5x^{5/2}} + \frac{32b^2\cos^2\left(a+b\sqrt[3]{x}\right)}{715x^{11/6}} - \frac{512b^4\cos^2\left(a+b\sqrt[3]{x}\right)}{45045x^{7/6}} + \frac{8192b^6\cos^2\left(a+b\sqrt[3]{x}\right)}{675675\sqrt{x}} + \frac{8b\cos\left(a+b\sqrt[3]{x}\right)\sin\left(a+b\sqrt[3]{x}\right)}{65x^{13/6}} - \frac{128b^3\cos\left(a+b\sqrt[3]{x}\right)\sin\left(a+b\sqrt[3]{x}\right)}{6435x^{3/2}} + \frac{2048b^5\cos\left(a+b\sqrt[3]{x}\right)\sin\left(a+b\sqrt[3]{x}\right)}{225225x^{5/6}} - \frac{32768b^7\cos\left(a+b\sqrt[3]{x}\right)\sin\left(a+b\sqrt[3]{x}\right)}{675675\sqrt[6]{x}} + \frac{(65536b^8)\,\text{Subst}\left(\int\left(\frac{1}{2\sqrt{x}}+\frac{\cos(2a+2bx)}{2\sqrt{x}}\right)dx, x, \sqrt[3]{x}\right)}{675675}$$

$$= -\frac{16b^2}{715x^{11/6}} + \frac{256b^4}{45045x^{7/6}} - \frac{4096b^6}{675675\sqrt{x}} - \frac{2\cos^2\left(a+b\sqrt[3]{x}\right)}{5x^{5/2}} + \frac{32b^2\cos^2\left(a+b\sqrt[3]{x}\right)}{715x^{11/6}} - \frac{512b^4\cos^2\left(a+b\sqrt[3]{x}\right)}{45045x^{7/6}} + \frac{8192b^6\cos^2\left(a+b\sqrt[3]{x}\right)}{675675\sqrt{x}} + \frac{8b\cos\left(a+b\sqrt[3]{x}\right)\sin\left(a+b\sqrt[3]{x}\right)}{65x^{13/6}} - \frac{128b^3\cos\left(a+b\sqrt[3]{x}\right)\sin\left(a+b\sqrt[3]{x}\right)}{6435x^{3/2}} + \frac{2048b^5\cos\left(a+b\sqrt[3]{x}\right)\sin\left(a+b\sqrt[3]{x}\right)}{225225x^{5/6}} - \frac{32768b^7\cos\left(a+b\sqrt[3]{x}\right)\sin\left(a+b\sqrt[3]{x}\right)}{675675\sqrt[6]{x}} + \frac{(32768b^8)\,\text{Subst}\left(\int\frac{\cos(2a+2bx)}{\sqrt{x}}dx, x, \sqrt[3]{x}\right)}{675675}$$

$$= -\frac{16b^2}{715x^{11/6}} + \frac{256b^4}{45045x^{7/6}} - \frac{4096b^6}{675675\sqrt{x}} - \frac{2\cos^2\left(a+b\sqrt[3]{x}\right)}{5x^{5/2}} + \frac{32b^2\cos^2\left(a+b\sqrt[3]{x}\right)}{715x^{11/6}} - \frac{512b^4\cos^2\left(a+b\sqrt[3]{x}\right)}{45045x^{7/6}} + \frac{8192b^6\cos^2\left(a+b\sqrt[3]{x}\right)}{675675\sqrt{x}} + \frac{8b\cos\left(a+b\sqrt[3]{x}\right)\sin\left(a+b\sqrt[3]{x}\right)}{65x^{13/6}} - \frac{128b^3\cos\left(a+b\sqrt[3]{x}\right)\sin\left(a+b\sqrt[3]{x}\right)}{6435x^{3/2}} + \frac{2048b^5\cos\left(a+b\sqrt[3]{x}\right)\sin\left(a+b\sqrt[3]{x}\right)}{225225x^{5/6}} - \frac{32768b^7\cos\left(a+b\sqrt[3]{x}\right)\sin\left(a+b\sqrt[3]{x}\right)}{675675\sqrt[6]{x}} + \frac{(32768b^8\cos(2a))\,\text{Subst}\left(\int\frac{\cos(2bx)}{\sqrt{x}}dx, x, \sqrt[3]{x}\right)}{675675} - \frac{(32768b^8\sin(2a))\,\text{Subst}\left(\int\frac{\sin(2bx)}{\sqrt{x}}dx, x, \sqrt[3]{x}\right)}{675675}$$

$$
\begin{aligned}
&= -\frac{16b^2}{715x^{11/6}} + \frac{256b^4}{45045x^{7/6}} - \frac{4096b^6}{675675\sqrt{x}} - \frac{2\cos^2\left(a+b\sqrt[3]{x}\right)}{5x^{5/2}} \\
&+ \frac{32b^2\cos^2\left(a+b\sqrt[3]{x}\right)}{715x^{11/6}} - \frac{512b^4\cos^2\left(a+b\sqrt[3]{x}\right)}{45045x^{7/6}} + \frac{8192b^6\cos^2\left(a+b\sqrt[3]{x}\right)}{675675\sqrt{x}} \\
&+ \frac{8b\cos\left(a+b\sqrt[3]{x}\right)\sin\left(a+b\sqrt[3]{x}\right)}{65x^{13/6}} - \frac{128b^3\cos\left(a+b\sqrt[3]{x}\right)\sin\left(a+b\sqrt[3]{x}\right)}{6435x^{3/2}} \\
&+ \frac{2048b^5\cos\left(a+b\sqrt[3]{x}\right)\sin\left(a+b\sqrt[3]{x}\right)}{225225x^{5/6}} - \frac{32768b^7\cos\left(a+b\sqrt[3]{x}\right)\sin\left(a+b\sqrt[3]{x}\right)}{675675\sqrt[6]{x}} \\
&+ \frac{(65536b^8\cos(2a))\operatorname{Subst}\left(\int\cos\left(2bx^2\right)\,dx, x, \sqrt[6]{x}\right)}{675675} \\
&- \frac{(65536b^8\sin(2a))\operatorname{Subst}\left(\int\sin\left(2bx^2\right)\,dx, x, \sqrt[6]{x}\right)}{675675} \\
&= -\frac{16b^2}{715x^{11/6}} + \frac{256b^4}{45045x^{7/6}} - \frac{4096b^6}{675675\sqrt{x}} - \frac{2\cos^2\left(a+b\sqrt[3]{x}\right)}{5x^{5/2}} \\
&+ \frac{32b^2\cos^2\left(a+b\sqrt[3]{x}\right)}{715x^{11/6}} - \frac{512b^4\cos^2\left(a+b\sqrt[3]{x}\right)}{45045x^{7/6}} \\
&+ \frac{8192b^6\cos^2\left(a+b\sqrt[3]{x}\right)}{675675\sqrt{x}} + \frac{32768b^{15/2}\sqrt{\pi}\cos(2a)\operatorname{FresnelC}\left(\frac{2\sqrt{b}\sqrt[6]{x}}{\sqrt{\pi}}\right)}{675675} \\
&- \frac{32768b^{15/2}\sqrt{\pi}\operatorname{FresnelS}\left(\frac{2\sqrt{b}\sqrt[6]{x}}{\sqrt{\pi}}\right)\sin(2a)}{675675} \\
&+ \frac{8b\cos\left(a+b\sqrt[3]{x}\right)\sin\left(a+b\sqrt[3]{x}\right)}{65x^{13/6}} - \frac{128b^3\cos\left(a+b\sqrt[3]{x}\right)\sin\left(a+b\sqrt[3]{x}\right)}{6435x^{3/2}} \\
&+ \frac{2048b^5\cos\left(a+b\sqrt[3]{x}\right)\sin\left(a+b\sqrt[3]{x}\right)}{225225x^{5/6}} - \frac{32768b^7\cos\left(a+b\sqrt[3]{x}\right)\sin\left(a+b\sqrt[3]{x}\right)}{675675\sqrt[6]{x}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 249, normalized size of antiderivative = 0.76

$$
\int \frac{\cos^2\left(a+b\sqrt[3]{x}\right)}{x^{7/2}}\,dx = \frac{-135135 - 135135\cos\left(2\left(a+b\sqrt[3]{x}\right)\right) + 15120b^2x^{2/3}\cos\left(2\left(a+b\sqrt[3]{x}\right)\right) - 3840b^4x^{4/3}}{}
$$

```
[In] Integrate[Cos[a + b*x^(1/3)]^2/x^(7/2),x]
```

```
[Out] (-135135 - 135135*Cos[2*(a + b*x^(1/3))] + 15120*b^2*x^(2/3)*Cos[2*(a + b*x
^(1/3))] - 3840*b^4*x^(4/3)*Cos[2*(a + b*x^(1/3))] + 4096*b^6*x^2*Cos[2*(a
+ b*x^(1/3))] + 32768*b^(15/2)*Sqrt[Pi]*x^(5/2)*Cos[2*a]*FresnelC[(2*Sqrt[b
]*x^(1/6))/Sqrt[Pi]] - 32768*b^(15/2)*Sqrt[Pi]*x^(5/2)*FresnelS[(2*Sqrt[b]*
x^(1/6))/Sqrt[Pi]]*Sin[2*a] + 41580*b*x^(1/3)*Sin[2*(a + b*x^(1/3))] - 6720
*b^3*x*Sin[2*(a + b*x^(1/3))] + 3072*b^5*x^(5/3)*Sin[2*(a + b*x^(1/3))] - 1
6384*b^7*x^(7/3)*Sin[2*(a + b*x^(1/3))])/(675675*x^(5/2))
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.63

method	result
	$4b\left(-\frac{\cos\left(2a+2b\,x^{\frac{1}{3}}\right)}{11x^{\frac{11}{6}}}-\frac{4b\left(-\frac{\sin\left(2a+2b\,x^{\frac{1}{3}}\right)}{9x^{\frac{3}{2}}}+\frac{4b\left(-\frac{\cos\left(2a+2b\,x^{\frac{1}{3}}\right)}{7x^{\frac{7}{6}}}\right.}{}\right.}{}\right.$

```
[In] int(cos(a+b*x^(1/3))^2/x^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/5/x^(5/2)-1/5/x^(5/2)*cos(2*a+2*b*x^(1/3))-4/5*b*(-1/13/x^(13/6)*sin(2*a
+2*b*x^(1/3))+4/13*b*(-1/11/x^(11/6)*cos(2*a+2*b*x^(1/3))-4/11*b*(-1/9/x^(3
/2)*sin(2*a+2*b*x^(1/3))+4/9*b*(-1/7/x^(7/6)*cos(2*a+2*b*x^(1/3))-4/7*b*(-1
/5/x^(5/6)*sin(2*a+2*b*x^(1/3))+4/5*b*(-1/3/x^(1/2)*cos(2*a+2*b*x^(1/3))-4/
3*b*(-1/x^(1/6)*sin(2*a+2*b*x^(1/3))+2*b^(1/2)*Pi^(1/2)*(cos(2*a)*FresnelC(
2*x^(1/6)*b^(1/2)/Pi^(1/2))-sin(2*a)*FresnelS(2*x^(1/6)*b^(1/2)/Pi^(1/2)))
))))))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.59

$$\int \frac{\cos^2\left(a+b\sqrt[3]{x}\right)}{x^{7/2}}\,dx = \frac{2\left(16384\,\pi b^7x^3\sqrt{\frac{b}{\pi}}\cos\left(2\,a\right)\mathrm{C}\left(2\,x^{\frac{1}{6}}\sqrt{\frac{b}{\pi}}\right) - 16384\,\pi b^7x^3\sqrt{\frac{b}{\pi}}\,\mathrm{S}\left(2\,x^{\frac{1}{6}}\sqrt{\frac{b}{\pi}}\right)\sin\left(2\,a\right) - \right.}{}$$

```
[In] integrate(cos(a+b*x^(1/3))^2/x^(7/2),x, algorithm="fricas")
```

```
[Out] 2/675675*(16384*pi*b^7*x^3*sqrt(b/pi)*cos(2*a)*fresnel_cos(2*x^(1/6)*sqrt(b
/pi)) - 16384*pi*b^7*x^3*sqrt(b/pi)*fresnel_sin(2*x^(1/6)*sqrt(b/pi))*sin(2
*a) - 2048*b^6*x^(5/2) + 1920*b^4*x^(11/6) - 7560*b^2*x^(7/6) - (3840*b^4*x
^(11/6) - 15120*b^2*x^(7/6) - (4096*b^6*x^2 - 135135)*sqrt(x))*cos(b*x^(1/3
) + a)^2 + 4*(768*b^5*x^(13/6) - 1680*b^3*x^(3/2) - (4096*b^7*x^2 - 10395*b
)*x^(5/6))*cos(b*x^(1/3) + a)*sin(b*x^(1/3) + a))/x^3
```

Sympy [F]

$$\int \frac{\cos^2\left(a+b\sqrt[3]{x}\right)}{x^{7/2}}\,dx = \int \frac{\cos^2\left(a+b\sqrt[3]{x}\right)}{x^{\frac{7}{2}}}\,dx$$

```
[In] integrate(cos(a+b*x**(1/3))**2/x**(7/2),x)
```

```
[Out] Integral(cos(a + b*x**(1/3))**2/x**(7/2), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.45 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.27

$$\int \frac{\cos^2\left(a+b\sqrt[3]{x}\right)}{x^{7/2}}\,dx =$$
$$-\frac{240\sqrt{2}\left(\left(-(i-1)\sqrt{2}\Gamma\left(-\frac{15}{2},2i\,bx^{\frac{1}{3}}\right)+(i+1)\sqrt{2}\Gamma\left(-\frac{15}{2},-2i\,bx^{\frac{1}{3}}\right)\right)\cos\left(2\,a\right)+\left(-(i+1)\sqrt{2}\Gamma\left(-\frac{15}{2},2\right.\right.}{5\,x^{\frac{5}{2}}}$$

```
[In] integrate(cos(a+b*x^(1/3))^2/x^(7/2),x, algorithm="maxima")
```

```
[Out] -1/5*(240*sqrt(2)*((-(I - 1)*sqrt(2)*gamma(-15/2, 2*I*b*x^(1/3)) + (I + 1)*
sqrt(2)*gamma(-15/2, -2*I*b*x^(1/3)))*cos(2*a) + (-(I + 1)*sqrt(2)*gamma(-1
5/2, 2*I*b*x^(1/3)) + (I - 1)*sqrt(2)*gamma(-15/2, -2*I*b*x^(1/3)))*sin(2*a
))*sqrt(b*x^(1/3))*b^7*x^(7/3) + 1)/x^(5/2)
```

Giac [F]

$$\int \frac{\cos^2\left(a+b\sqrt[3]{x}\right)}{x^{7/2}}\,dx = \int \frac{\cos\left(bx^{\frac{1}{3}}+a\right)^2}{x^{\frac{7}{2}}}\,dx$$

```
[In] integrate(cos(a+b*x^(1/3))^2/x^(7/2),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x^(1/3) + a)^2/x^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2\left(a+b\sqrt[3]{x}\right)}{x^{7/2}}\,dx = \int \frac{\cos\left(a+b\,x^{1/3}\right)^2}{x^{7/2}}\,dx$$

```
[In] int(cos(a + b*x^(1/3))^2/x^(7/2),x)
```

```
[Out] int(cos(a + b*x^(1/3))^2/x^(7/2), x)
```

3.61 $\int \cos^3\left(\sqrt[3]{x}\right)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 86

$$\int \cos^3\left(\sqrt[3]{x}\right)\,dx = 4\sqrt[3]{x}\cos\left(\sqrt[3]{x}\right)+\frac{2}{3}\sqrt[3]{x}\cos^3\left(\sqrt[3]{x}\right)-\frac{14}{3}\sin\left(\sqrt[3]{x}\right)+2x^{2/3}\sin\left(\sqrt[3]{x}\right)+x^{2/3}\cos^2\left(\sqrt[3]{x}\right)\sin\left(\sqrt[3]{x}\right)+\frac{2}{9}\sin^3\left(\sqrt[3]{x}\right)$$

```
[Out] 4*x^(1/3)*cos(x^(1/3))+2/3*x^(1/3)*cos(x^(1/3))^3-14/3*sin(x^(1/3))+2*x^(2/
3)*sin(x^(1/3))+x^(2/3)*cos(x^(1/3))^2*sin(x^(1/3))+2/9*sin(x^(1/3))^3
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {3443, 3392, 3377, 2717, 2713}

$$\int \cos^3\left(\sqrt[3]{x}\right)\,dx = 2x^{2/3}\sin\left(\sqrt[3]{x}\right)+x^{2/3}\sin\left(\sqrt[3]{x}\right)\cos^2\left(\sqrt[3]{x}\right)+\frac{2}{9}\sin^3\left(\sqrt[3]{x}\right)-\frac{14}{3}\sin\left(\sqrt[3]{x}\right)+\frac{2}{3}\sqrt[3]{x}\cos^3\left(\sqrt[3]{x}\right)+4\sqrt[3]{x}\cos\left(\sqrt[3]{x}\right)$$

```
[In] Int[Cos[x^(1/3)]^3,x]
```

```
[Out] 4*x^(1/3)*Cos[x^(1/3)] + (2*x^(1/3)*Cos[x^(1/3)]^3)/3 - (14*Sin[x^(1/3)])/3
 + 2*x^(2/3)*Sin[x^(1/3)] + x^(2/3)*Cos[x^(1/3)]^2*Sin[x^(1/3)] + (2*Sin[x^
(1/3)]^3)/9
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
```

```
&& IGtQ[(n - 1)/2, 0]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3443

```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_.), x_S
ymbol] :> Dist[1/(n*f), Subst[Int[x^(1/n - 1)*(a + b*Cos[c + d*x])^p, x], x
, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && Integer
Q[1/n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 3\text{Subst}\left(\int x^2\cos^3(x)\,dx, x, \sqrt[3]{x}\right)\\
&= \frac{2}{3}\sqrt[3]{x}\cos^3\left(\sqrt[3]{x}\right)+x^{2/3}\cos^2\left(\sqrt[3]{x}\right)\sin\left(\sqrt[3]{x}\right)\\
&\quad -\frac{2}{3}\text{Subst}\left(\int\cos^3(x)\,dx, x, \sqrt[3]{x}\right)+2\text{Subst}\left(\int x^2\cos(x)\,dx, x, \sqrt[3]{x}\right)\\
&= \frac{2}{3}\sqrt[3]{x}\cos^3\left(\sqrt[3]{x}\right)+2x^{2/3}\sin\left(\sqrt[3]{x}\right)+x^{2/3}\cos^2\left(\sqrt[3]{x}\right)\sin\left(\sqrt[3]{x}\right)\\
&\quad +\frac{2}{3}\text{Subst}\left(\int\left(1-x^2\right)\,dx, x, -\sin\left(\sqrt[3]{x}\right)\right)-4\text{Subst}\left(\int x\sin(x)\,dx, x, \sqrt[3]{x}\right)\\
&= 4\sqrt[3]{x}\cos\left(\sqrt[3]{x}\right)+\frac{2}{3}\sqrt[3]{x}\cos^3\left(\sqrt[3]{x}\right)-\frac{2}{3}\sin\left(\sqrt[3]{x}\right)+2x^{2/3}\sin\left(\sqrt[3]{x}\right)\\
&\quad +x^{2/3}\cos^2\left(\sqrt[3]{x}\right)\sin\left(\sqrt[3]{x}\right)+\frac{2}{9}\sin^3\left(\sqrt[3]{x}\right)-4\text{Subst}\left(\int\cos(x)\,dx, x, \sqrt[3]{x}\right)
\end{aligned}$$

$$= 4\sqrt[3]{x}\cos\left(\sqrt[3]{x}\right) + \frac{2}{3}\sqrt[3]{x}\cos^3\left(\sqrt[3]{x}\right) - \frac{14}{3}\sin\left(\sqrt[3]{x}\right) + 2x^{2/3}\sin\left(\sqrt[3]{x}\right) + x^{2/3}\cos^2\left(\sqrt[3]{x}\right)\sin\left(\sqrt[3]{x}\right) + \frac{2}{9}\sin^3\left(\sqrt[3]{x}\right)$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.77

$$\int \cos^3\left(\sqrt[3]{x}\right)\,dx = \frac{1}{36}\left(162\sqrt[3]{x}\cos\left(\sqrt[3]{x}\right) + 6\sqrt[3]{x}\cos\left(3\sqrt[3]{x}\right) + 81\left(-2+x^{2/3}\right)\sin\left(\sqrt[3]{x}\right) + \left(-2+9x^{2/3}\right)\sin\left(3\sqrt[3]{x}\right)\right)$$

```
[In] Integrate[Cos[x^(1/3)]^3,x]
[Out] (162*x^(1/3)*Cos[x^(1/3)] + 6*x^(1/3)*Cos[3*x^(1/3)] + 81*(-2 + x^(2/3))*Si
n[x^(1/3)] + (-2 + 9*x^(2/3))*Sin[3*x^(1/3)])/36
```

Maple [A] (verified)

Time = 1.30 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.67

method	result
derivativedivides	$x^{\frac{2}{3}}\left(2+\cos^2\left(x^{\frac{1}{3}}\right)\right)\sin\left(x^{\frac{1}{3}}\right) - 4\sin\left(x^{\frac{1}{3}}\right) + 4x^{\frac{1}{3}}\cos\left(x^{\frac{1}{3}}\right) + \frac{2x^{\frac{1}{3}}\left(\cos^3\left(x^{\frac{1}{3}}\right)\right)}{3} - \frac{2\left(2+\cos^2\left(x^{\frac{1}{3}}\right.\right.}{9}$
default	$x^{\frac{2}{3}}\left(2+\cos^2\left(x^{\frac{1}{3}}\right)\right)\sin\left(x^{\frac{1}{3}}\right) - 4\sin\left(x^{\frac{1}{3}}\right) + 4x^{\frac{1}{3}}\cos\left(x^{\frac{1}{3}}\right) + \frac{2x^{\frac{1}{3}}\left(\cos^3\left(x^{\frac{1}{3}}\right)\right)}{3} - \frac{2\left(2+\cos^2\left(x^{\frac{1}{3}}\right.\right.}{9}$

```
[In] int(cos(x^(1/3))^3,x,method=_RETURNVERBOSE)
[Out] x^(2/3)*(2+cos(x^(1/3))^2)*sin(x^(1/3))-4*sin(x^(1/3))+4*x^(1/3)*cos(x^(1/3
))+2/3*x^(1/3)*cos(x^(1/3))^3-2/9*(2+cos(x^(1/3))^2)*sin(x^(1/3))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.56

$$\int \cos^3\left(\sqrt[3]{x}\right)\,dx = \frac{2}{3}\,x^{\frac{1}{3}}\cos\left(x^{\frac{1}{3}}\right)^3 + \frac{1}{9}\left(\left(9\,x^{\frac{2}{3}}-2\right)\cos\left(x^{\frac{1}{3}}\right)^2 + 18\,x^{\frac{2}{3}} - 40\right)\sin\left(x^{\frac{1}{3}}\right) + 4\,x^{\frac{1}{3}}\cos\left(x^{\frac{1}{3}}\right)$$

```
[In] integrate(cos(x^(1/3))^3,x, algorithm="fricas")
```

```
[Out] 2/3*x^(1/3)*cos(x^(1/3))^3 + 1/9*((9*x^(2/3) - 2)*cos(x^(1/3))^2 + 18*x^(2/
3) - 40)*sin(x^(1/3)) + 4*x^(1/3)*cos(x^(1/3))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 513 vs. $2(85) = 170$.

Time = 0.99 (sec) , antiderivative size = 513, normalized size of antiderivative = 5.97

$$\int \cos^3\left(\sqrt[3]{x}\right)\,dx = \frac{54x^{\frac{2}{3}}\tan^5\left(\frac{\sqrt[3]{x}}{2}\right)}{9\tan^6\left(\frac{\sqrt[3]{x}}{2}\right)+27\tan^4\left(\frac{\sqrt[3]{x}}{2}\right)+27\tan^2\left(\frac{\sqrt[3]{x}}{2}\right)+9} + \frac{36x^{\frac{2}{3}}\tan^3\left(\frac{\sqrt[3]{x}}{2}\right)}{9\tan^6\left(\frac{\sqrt[3]{x}}{2}\right)+27\tan^4\left(\frac{\sqrt[3]{x}}{2}\right)+27\tan^2\left(\frac{\sqrt[3]{x}}{2}\right)+9} + \frac{54x^{\frac{2}{3}}\tan\left(\frac{\sqrt[3]{x}}{2}\right)}{9\tan^6\left(\frac{\sqrt[3]{x}}{2}\right)+27\tan^4\left(\frac{\sqrt[3]{x}}{2}\right)+27\tan^2\left(\frac{\sqrt[3]{x}}{2}\right)+9} - \frac{42\sqrt[3]{x}\tan^6\left(\frac{\sqrt[3]{x}}{2}\right)}{9\tan^6\left(\frac{\sqrt[3]{x}}{2}\right)+27\tan^4\left(\frac{\sqrt[3]{x}}{2}\right)+27\tan^2\left(\frac{\sqrt[3]{x}}{2}\right)+9} - \frac{18\sqrt[3]{x}\tan^4\left(\frac{\sqrt[3]{x}}{2}\right)}{9\tan^6\left(\frac{\sqrt[3]{x}}{2}\right)+27\tan^4\left(\frac{\sqrt[3]{x}}{2}\right)+27\tan^2\left(\frac{\sqrt[3]{x}}{2}\right)+9} + \frac{18\sqrt[3]{x}\tan^2\left(\frac{\sqrt[3]{x}}{2}\right)}{9\tan^6\left(\frac{\sqrt[3]{x}}{2}\right)+27\tan^4\left(\frac{\sqrt[3]{x}}{2}\right)+27\tan^2\left(\frac{\sqrt[3]{x}}{2}\right)+9} + \frac{42\sqrt[3]{x}}{9\tan^6\left(\frac{\sqrt[3]{x}}{2}\right)+27\tan^4\left(\frac{\sqrt[3]{x}}{2}\right)+27\tan^2\left(\frac{\sqrt[3]{x}}{2}\right)+9} - \frac{84\tan^5\left(\frac{\sqrt[3]{x}}{2}\right)}{9\tan^6\left(\frac{\sqrt[3]{x}}{2}\right)+27\tan^4\left(\frac{\sqrt[3]{x}}{2}\right)+27\tan^2\left(\frac{\sqrt[3]{x}}{2}\right)+9} - \frac{152\tan^3\left(\frac{\sqrt[3]{x}}{2}\right)}{9\tan^6\left(\frac{\sqrt[3]{x}}{2}\right)+27\tan^4\left(\frac{\sqrt[3]{x}}{2}\right)+27\tan^2\left(\frac{\sqrt[3]{x}}{2}\right)+9} - \frac{84\tan\left(\frac{\sqrt[3]{x}}{2}\right)}{9\tan^6\left(\frac{\sqrt[3]{x}}{2}\right)+27\tan^4\left(\frac{\sqrt[3]{x}}{2}\right)+27\tan^2\left(\frac{\sqrt[3]{x}}{2}\right)+9}$$

```
[In] integrate(cos(x**(1/3))**3,x)
```

```
[Out] 54*x**(2/3)*tan(x**(1/3)/2)**5/(9*tan(x**(1/3)/2)**6 + 27*tan(x**(1/3)/2)**
4 + 27*tan(x**(1/3)/2)**2 + 9) + 36*x**(2/3)*tan(x**(1/3)/2)**3/(9*tan(x**(
1/3)/2)**6 + 27*tan(x**(1/3)/2)**4 + 27*tan(x**(1/3)/2)**2 + 9) + 54*x**(2/
3)*tan(x**(1/3)/2)/(9*tan(x**(1/3)/2)**6 + 27*tan(x**(1/3)/2)**4 + 27*tan(x
**(1/3)/2)**2 + 9) - 42*x**(1/3)*tan(x**(1/3)/2)**6/(9*tan(x**(1/3)/2)**6 +
 27*tan(x**(1/3)/2)**4 + 27*tan(x**(1/3)/2)**2 + 9) - 18*x**(1/3)*tan(x**(1
/3)/2)**4/(9*tan(x**(1/3)/2)**6 + 27*tan(x**(1/3)/2)**4 + 27*tan(x**(1/3)/2
)**2 + 9) + 18*x**(1/3)*tan(x**(1/3)/2)**2/(9*tan(x**(1/3)/2)**6 + 27*tan(x
**(1/3)/2)**4 + 27*tan(x**(1/3)/2)**2 + 9) + 42*x**(1/3)/(9*tan(x**(1/3)/2)
**6 + 27*tan(x**(1/3)/2)**4 + 27*tan(x**(1/3)/2)**2 + 9) - 84*tan(x**(1/3)/
2)**5/(9*tan(x**(1/3)/2)**6 + 27*tan(x**(1/3)/2)**4 + 27*tan(x**(1/3)/2)**2
 + 9) - 152*tan(x**(1/3)/2)**3/(9*tan(x**(1/3)/2)**6 + 27*tan(x**(1/3)/2)**
4 + 27*tan(x**(1/3)/2)**2 + 9) - 84*tan(x**(1/3)/2)/(9*tan(x**(1/3)/2)**6 +
 27*tan(x**(1/3)/2)**4 + 27*tan(x**(1/3)/2)**2 + 9)
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.55

$$\int \cos^3\left(\sqrt[3]{x}\right)\,dx = \frac{1}{36}\left(9\,x^{\frac{2}{3}}-2\right)\sin\left(3\,x^{\frac{1}{3}}\right)+\frac{9}{4}\left(x^{\frac{2}{3}}-2\right)\sin\left(x^{\frac{1}{3}}\right) + \frac{1}{6}\,x^{\frac{1}{3}}\cos\left(3\,x^{\frac{1}{3}}\right)+\frac{9}{2}\,x^{\frac{1}{3}}\cos\left(x^{\frac{1}{3}}\right)$$

```
[In] integrate(cos(x^(1/3))^3,x, algorithm="maxima")
```

```
[Out] 1/36*(9*x^(2/3) - 2)*sin(3*x^(1/3)) + 9/4*(x^(2/3) - 2)*sin(x^(1/3)) + 1/6*
x^(1/3)*cos(3*x^(1/3)) + 9/2*x^(1/3)*cos(x^(1/3))
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.55

$$\int \cos^3\left(\sqrt[3]{x}\right)\,dx = \frac{1}{36}\left(9\,x^{\frac{2}{3}}-2\right)\sin\left(3\,x^{\frac{1}{3}}\right)+\frac{9}{4}\left(x^{\frac{2}{3}}-2\right)\sin\left(x^{\frac{1}{3}}\right) + \frac{1}{6}\,x^{\frac{1}{3}}\cos\left(3\,x^{\frac{1}{3}}\right)+\frac{9}{2}\,x^{\frac{1}{3}}\cos\left(x^{\frac{1}{3}}\right)$$

```
[In] integrate(cos(x^(1/3))^3,x, algorithm="giac")
```

```
[Out] 1/36*(9*x^(2/3) - 2)*sin(3*x^(1/3)) + 9/4*(x^(2/3) - 2)*sin(x^(1/3)) + 1/6*
x^(1/3)*cos(3*x^(1/3)) + 9/2*x^(1/3)*cos(x^(1/3))
```

Mupad [B] (verification not implemented)

Time = 13.56 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.72

$$\int \cos^3\left(\sqrt[3]{x}\right)\,dx = 4\,x^{1/3}\,\cos\left(x^{1/3}\right) - \frac{2\cos\left(x^{1/3}\right)^2\,\sin\left(x^{1/3}\right)}{9} - \frac{40\,\sin\left(x^{1/3}\right)}{9} + 2\,x^{2/3}\,\sin\left(x^{1/3}\right) + \frac{2\,x^{1/3}\cos\left(x^{1/3}\right)^3}{3} + x^{2/3}\cos\left(x^{1/3}\right)^2\,\sin\left(x^{1/3}\right)$$

```
[In] int(cos(x^(1/3))^3,x)
```

```
[Out] 4*x^(1/3)*cos(x^(1/3)) - (2*cos(x^(1/3))^2*sin(x^(1/3)))/9 - (40*sin(x^(1/3
)))/9 + 2*x^(2/3)*sin(x^(1/3)) + (2*x^(1/3)*cos(x^(1/3))^3)/3 + x^(2/3)*cos
(x^(1/3))^2*sin(x^(1/3))
```

3.62 $\int \frac{\cos\left(\sqrt[6]{x}\right)}{x^{5/6}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 8

$$\int \frac{\cos\left(\sqrt[6]{x}\right)}{x^{5/6}}\,dx = 6\sin\left(\sqrt[6]{x}\right)$$

```
[Out] 6*sin(x^(1/6))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3461, 2717}

$$\int \frac{\cos\left(\sqrt[6]{x}\right)}{x^{5/6}}\,dx = 6\sin\left(\sqrt[6]{x}\right)$$

```
[In] Int[Cos[x^(1/6)]/x^(5/6),x]
```

```
[Out] 6*Sin[x^(1/6)]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
```

```
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}\text{integral} &= 6\text{Subst}\left(\int \cos(x)\,dx, x, \sqrt[6]{x}\right)\\ &= 6\sin\left(\sqrt[6]{x}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\sqrt[6]{x}\right)}{x^{5/6}}\,dx = 6\sin\left(\sqrt[6]{x}\right)$$

```
[In] Integrate[Cos[x^(1/6)]/x^(5/6),x]
[Out] 6*Sin[x^(1/6)]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$6\sin\left(x^{\frac{1}{6}}\right)$	7
default	$6\sin\left(x^{\frac{1}{6}}\right)$	7
meijerg	$6\sin\left(x^{\frac{1}{6}}\right)$	7

```
[In] int(cos(x^(1/6))/x^(5/6),x,method=_RETURNVERBOSE)
[Out] 6*sin(x^(1/6))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{\cos \left(\sqrt[6]{x}\right)}{x^{5/6}}\,dx = 6\,\sin\left(x^{\frac{1}{6}}\right)$$

```
[In] integrate(cos(x^(1/6))/x^(5/6),x, algorithm="fricas")
[Out] 6*sin(x^(1/6))
```

Sympy [A] (verification not implemented)

Time = 6.71 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

$$\int \frac{\cos \left(\sqrt[6]{x}\right)}{x^{5/6}}\,dx = 6\sin\left(\sqrt[6]{x}\right)$$

```
[In] integrate(cos(x**(1/6))/x**(5/6),x)
[Out] 6*sin(x**(1/6))
```

Maxima [A] (verification not implemented)

none

Time = 0.41 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{\cos \left(\sqrt[6]{x}\right)}{x^{5/6}}\,dx = 6\,\sin\left(x^{\frac{1}{6}}\right)$$

```
[In] integrate(cos(x^(1/6))/x^(5/6),x, algorithm="maxima")
[Out] 6*sin(x^(1/6))
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{\cos \left(\sqrt[6]{x}\right)}{x^{5/6}}\,dx = 6\,\sin\left(x^{\frac{1}{6}}\right)$$

```
[In] integrate(cos(x^(1/6))/x^(5/6),x, algorithm="giac")
[Out] 6*sin(x^(1/6))
```

Mupad [B] (verification not implemented)

Time = 13.59 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{\cos\left(\sqrt[6]{x}\right)}{x^{5/6}}\,dx = 6\,\sin\left(x^{1/6}\right)$$

```
[In] int(cos(x^(1/6))/x^(5/6),x)
[Out] 6*sin(x^(1/6))
```

3.63 $\int (ex)^m (b\cos(c+dx^n))^p \, dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int (ex)^m (b\cos(c+dx^n))^p \, dx = \text{Int}\left((ex)^m (b\cos(c+dx^n))^p, x\right)$$

```
[Out] Unintegrable((e*x)^m*(b*cos(c+d*x^n))^p,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (ex)^m (b\cos(c+dx^n))^p \, dx = \int (ex)^m (b\cos(c+dx^n))^p \, dx$$

```
[In] Int[(e*x)^m*(b*Cos[c + d*x^n])^p,x]
```

```
[Out] Defer[Int][(e*x)^m*(b*Cos[c + d*x^n])^p, x]
```

Rubi steps

$$\text{integral} = \int (ex)^m (b\cos(c+dx^n))^p \, dx$$

Mathematica [N/A]

Not integrable

Time = 1.53 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (ex)^m (b\cos(c+dx^n))^p \, dx = \int (ex)^m (b\cos(c+dx^n))^p \, dx$$

```
[In] Integrate[(e*x)^m*(b*Cos[c + d*x^n])^p,x]
[Out] Integrate[(e*x)^m*(b*Cos[c + d*x^n])^p, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.33 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int (ex)^m (b\cos(c+d\,x^n))^p dx$$

```
[In] int((e*x)^m*(b*cos(c+d*x^n))^p,x)
[Out] int((e*x)^m*(b*cos(c+d*x^n))^p,x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (ex)^m (b\cos(c+dx^n))^p \, dx = \int (ex)^m (b\cos(dx^n+c))^p \, dx$$

```
[In] integrate((e*x)^m*(b*cos(c+d*x^n))^p,x, algorithm="fricas")
[Out] integral((e*x)^m*(b*cos(d*x^n + c))^p, x)
```

Sympy [N/A]

Not integrable

Time = 8.54 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int (ex)^m (b\cos(c+dx^n))^p \, dx = \int (b\cos(c+dx^n))^p (ex)^m \, dx$$

```
[In] integrate((e*x)**m*(b*cos(c+d*x**n))**p,x)
[Out] Integral((b*cos(c + d*x**n))**p*(e*x)**m, x)
```

Maxima [N/A]

Not integrable

Time = 1.22 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (ex)^m (b\cos(c+dx^n))^p \, dx = \int (ex)^m (b\cos(dx^n+c))^p \, dx$$

```
[In] integrate((e*x)^m*(b*cos(c+d*x^n))^p,x, algorithm="maxima")
[Out] integrate((e*x)^m*(b*cos(d*x^n + c))^p, x)
```

Giac [N/A]

Not integrable

Time = 0.94 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (ex)^m (b\cos(c+dx^n))^p \, dx = \int (ex)^m (b\cos(dx^n+c))^p \, dx$$

```
[In] integrate((e*x)^m*(b*cos(c+d*x^n))^p,x, algorithm="giac")
[Out] integrate((e*x)^m*(b*cos(d*x^n + c))^p, x)
```

Mupad [N/A]

Not integrable

Time = 13.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (ex)^m (b\cos(c+dx^n))^p \, dx = \int (e\,x)^m \left(b\,\cos\left(c+d\,x^n\right)\right)^p dx$$

```
[In] int((e*x)^m*(b*cos(c + d*x^n))^p,x)
[Out] int((e*x)^m*(b*cos(c + d*x^n))^p, x)
```

3.64 $\int (ex)^m (a+b\cos(c+dx^n))^p \, dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int (ex)^m (a+b\cos(c+dx^n))^p \, dx = \text{Int}((ex)^m (a+b\cos(c+dx^n))^p, x)$$

```
[Out] Unintegrable((e*x)^m*(a+b*cos(c+d*x^n))^p,x)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (ex)^m (a+b\cos(c+dx^n))^p \, dx = \int (ex)^m (a+b\cos(c+dx^n))^p \, dx$$

```
[In] Int[(e*x)^m*(a + b*Cos[c + d*x^n])^p,x]
```

```
[Out] Defer[Int][(e*x)^m*(a + b*Cos[c + d*x^n])^p, x]
```

Rubi steps

$$\text{integral} = \int (ex)^m (a+b\cos(c+dx^n))^p \, dx$$

Mathematica [N/A]

Not integrable

Time = 1.67 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (ex)^m (a+b\cos(c+dx^n))^p \, dx = \int (ex)^m (a+b\cos(c+dx^n))^p \, dx$$

```
[In] Integrate[(e*x)^m*(a + b*Cos[c + d*x^n])^p,x]
[Out] Integrate[(e*x)^m*(a + b*Cos[c + d*x^n])^p, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int (ex)^m (a+b\cos(c+d\,x^n))^p dx$$

```
[In] int((e*x)^m*(a+b*cos(c+d*x^n))^p,x)
[Out] int((e*x)^m*(a+b*cos(c+d*x^n))^p,x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (ex)^m (a+b\cos(c+dx^n))^p \, dx = \int (ex)^m (b\cos(dx^n+c)+a)^p \, dx$$

```
[In] integrate((e*x)^m*(a+b*cos(c+d*x^n))^p,x, algorithm="fricas")
[Out] integral((e*x)^m*(b*cos(d*x^n + c) + a)^p, x)
```

Sympy [N/A]

Not integrable

Time = 27.06 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int (ex)^m (a+b\cos(c+dx^n))^p \, dx = \int (ex)^m (a+b\cos(c+dx^n))^p \, dx$$

```
[In] integrate((e*x)**m*(a+b*cos(c+d*x**n))**p,x)
[Out] Integral((e*x)**m*(a + b*cos(c + d*x**n))**p, x)
```

Maxima [N/A]

Not integrable

Time = 1.61 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (ex)^m (a+b\cos(c+dx^n))^p \, dx = \int (ex)^m (b\cos(dx^n+c)+a)^p \, dx$$

```
[In] integrate((e*x)^m*(a+b*cos(c+d*x^n))^p,x, algorithm="maxima")
[Out] integrate((e*x)^m*(b*cos(d*x^n + c) + a)^p, x)
```

Giac [N/A]

Not integrable

Time = 5.77 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (ex)^m (a+b\cos(c+dx^n))^p \, dx = \int (ex)^m (b\cos(dx^n+c)+a)^p \, dx$$

```
[In] integrate((e*x)^m*(a+b*cos(c+d*x^n))^p,x, algorithm="giac")
[Out] integrate((e*x)^m*(b*cos(d*x^n + c) + a)^p, x)
```

Mupad [N/A]

Not integrable

Time = 13.10 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (ex)^m (a+b\cos(c+dx^n))^p \, dx = \int (e\,x)^m \left(a+b\,\cos\left(c+d\,x^n\right)\right)^p dx$$

```
[In] int((e*x)^m*(a + b*cos(c + d*x^n))^p,x)
[Out] int((e*x)^m*(a + b*cos(c + d*x^n))^p, x)
```

3.65 $\int (ex)^{-1+n} (b\cos(c+dx^n))^p \, dx$

Optimal result

Integrand size = 20, antiderivative size = 93

$$\int (ex)^{-1+n} (b\cos(c+dx^n))^p \, dx = -\frac{x^{-n}(ex)^n (b\cos(c+dx^n))^{1+p}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+p}{2},\frac{3+p}{2},\cos^2(c+dx^n)\right)\sin(c+dx^n)}{bden(1+p)\sqrt{\sin^2(c+dx^n)}}$$

```
[Out] -(e*x)^n*(b*cos(c+d*x^n))^(p+1)*hypergeom([1/2, 1/2+1/2*p],[3/2+1/2*p],cos(
c+d*x^n)^2)*sin(c+d*x^n)/b/d/e/n/(p+1)/(x^n)/(sin(c+d*x^n)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {3463, 3461, 2722}

$$\int (ex)^{-1+n} (b\cos(c+dx^n))^p \, dx = -\frac{x^{-n}(ex)^n \sin(c+dx^n)(b\cos(c+dx^n))^{p+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{p+1}{2},\frac{p+3}{2},\cos^2(dx^n+c)\right)}{bden(p+1)\sqrt{\sin^2(c+dx^n)}}$$

```
[In] Int[(e*x)^(-1 + n)*(b*Cos[c + d*x^n])^p,x]
```

```
[Out] -(((e*x)^n*(b*Cos[c + d*x^n])^(1 + p)*Hypergeometric2F1[1/2, (1 + p)/2, (3
+ p)/2, Cos[c + d*x^n]^2]*Sin[c + d*x^n])/(b*d*e*n*(1 + p)*x^n*Sqrt[Sin[c +
 d*x^n]^2]))
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
```

```
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3463

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*((e_)*(x_))^(m_), x_
Symbol] :> Dist[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(a
+ b*Cos[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && Inte
gerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{(x^{-n}(ex)^n)\int x^{-1+n}(b\cos(c+dx^n))^p\,dx}{e}\\ &= \frac{(x^{-n}(ex)^n)\,\text{Subst}\left(\int(b\cos(c+dx))^p\,dx,x,x^n\right)}{en}\\ &= -\frac{x^{-n}(ex)^n\,(b\cos(c+dx^n))^{1+p}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+p}{2},\frac{3+p}{2},\cos^2(c+dx^n)\right)\sin(c+dx^n)}{bden(1+p)\sqrt{\sin^2(c+dx^n)}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.96

$$\int (ex)^{-1+n}\,(b\cos(c+dx^n))^p\,dx = -\frac{x^{1-n}(ex)^{-1+n}\,(b\cos(c+dx^n))^p\cot(c+dx^n)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+p}{2},\frac{3+p}{2},\cos^2(c+dx^n)\right)\sqrt{\sin^2(c+}}{dn(1+p)}$$

```
[In] Integrate[(e*x)^(-1 + n)*(b*Cos[c + d*x^n])^p,x]
```

```
[Out] -((x^(1 - n)*(e*x)^(-1 + n)*(b*Cos[c + d*x^n])^p*Cot[c + d*x^n]*Hypergeomet
ric2F1[1/2, (1 + p)/2, (3 + p)/2, Cos[c + d*x^n]^2]*Sqrt[Sin[c + d*x^n]^2])
/(d*n*(1 + p)))
```

Maple [F]

$$\int (ex)^{-1+n} (b\cos (c+d\,x^{n}))^{p}dx$$

```
[In] int((e*x)^(-1+n)*(b*cos(c+d*x^n))^p,x)
[Out] int((e*x)^(-1+n)*(b*cos(c+d*x^n))^p,x)
```

Fricas [F]

$$\int (ex)^{-1+n} (b\cos (c+dx^n))^p \, dx = \int (ex)^{n-1} (b\cos (dx^n+c))^p \, dx$$

```
[In] integrate((e*x)^(-1+n)*(b*cos(c+d*x^n))^p,x, algorithm="fricas")
[Out] integral((e*x)^(n - 1)*(b*cos(d*x^n + c))^p, x)
```

Sympy [F]

$$\int (ex)^{-1+n} (b\cos (c+dx^n))^p \, dx = \int (b\cos (c+dx^n))^p (ex)^{n-1} \, dx$$

```
[In] integrate((e*x)**(-1+n)*(b*cos(c+d*x**n))**p,x)
[Out] Integral((b*cos(c + d*x**n))**p*(e*x)**(n - 1), x)
```

Maxima [F]

$$\int (ex)^{-1+n} (b\cos (c+dx^n))^p \, dx = \int (ex)^{n-1} (b\cos (dx^n+c))^p \, dx$$

```
[In] integrate((e*x)^(-1+n)*(b*cos(c+d*x^n))^p,x, algorithm="maxima")
[Out] integrate((e*x)^(n - 1)*(b*cos(d*x^n + c))^p, x)
```

Giac [F]

$$\int (ex)^{-1+n}(b\cos(c+dx^n))^p\,dx = \int (ex)^{n-1}(b\cos(dx^n+c))^p\,dx$$

```
[In] integrate((e*x)^(-1+n)*(b*cos(c+d*x^n))^p,x, algorithm="giac")
[Out] integrate((e*x)^(n - 1)*(b*cos(d*x^n + c))^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^{-1+n}(b\cos(c+dx^n))^p\,dx = \int (e\,x)^{n-1}\,(b\,\cos(c+d\,x^n))^p\,dx$$

```
[In] int((e*x)^(n - 1)*(b*cos(c + d*x^n))^p,x)
[Out] int((e*x)^(n - 1)*(b*cos(c + d*x^n))^p, x)
```

3.66 $\int (ex)^{-1+2n} (b\cos(c+dx^n))^p \, dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int (ex)^{-1+2n} (b\cos(c+dx^n))^p \, dx = \frac{x^{-2n}(ex)^{2n}\mathrm{Int}(x^{-1+2n}(b\cos(c+dx^n))^p, x)}{e}$$

```
[Out] (e*x)^(2*n)*Unintegrable(x^(-1+2*n)*(b*cos(c+d*x^n))^p,x)/e/(x^(2*n))
```

Rubi [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (ex)^{-1+2n} (b\cos(c+dx^n))^p \, dx = \int (ex)^{-1+2n} (b\cos(c+dx^n))^p \, dx$$

```
[In] Int[(e*x)^(-1 + 2*n)*(b*Cos[c + d*x^n])^p,x]
```

```
[Out] ((e*x)^(2*n)*Defer[Int][x^(-1 + 2*n)*(b*Cos[c + d*x^n])^p, x])/(e*x^(2*n))
```

Rubi steps

$$\text{integral} = \frac{\left(x^{-2n}(ex)^{2n}\right)\int x^{-1+2n}(b\cos(c+dx^n))^p \, dx}{e}$$

Mathematica [N/A]

Not integrable

Time = 1.75 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (ex)^{-1+2n}(b\cos(c+dx^n))^p\,dx=\int (ex)^{-1+2n}(b\cos(c+dx^n))^p\,dx$$

```
[In] Integrate[(e*x)^(-1 + 2*n)*(b*Cos[c + d*x^n])^p,x]
[Out] Integrate[(e*x)^(-1 + 2*n)*(b*Cos[c + d*x^n])^p, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.40 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int (ex)^{-1+2n}(b\cos(c+d\,x^n))^p dx$$

```
[In] int((e*x)^(-1+2*n)*(b*cos(c+d*x^n))^p,x)
[Out] int((e*x)^(-1+2*n)*(b*cos(c+d*x^n))^p,x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (ex)^{-1+2n}(b\cos(c+dx^n))^p\,dx=\int (ex)^{2n-1}(b\cos(dx^n+c))^p\,dx$$

```
[In] integrate((e*x)^(-1+2*n)*(b*cos(c+d*x^n))^p,x, algorithm="fricas")
[Out] integral((e*x)^(2*n - 1)*(b*cos(d*x^n + c))^p, x)
```

Sympy [N/A]

Not integrable

Time = 8.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int (ex)^{-1+2n}(b\cos(c+dx^n))^p\,dx=\int (b\cos(c+dx^n))^p(ex)^{2n-1}\,dx$$

```
[In] integrate((e*x)**(-1+2*n)*(b*cos(c+d*x**n))**p,x)
[Out] Integral((b*cos(c + d*x**n))**p*(e*x)**(2*n - 1), x)
```

Maxima [N/A]

Not integrable

Time = 1.42 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (ex)^{-1+2n}(b\cos(c+dx^n))^p\,dx = \int (ex)^{2n-1}(b\cos(dx^n+c))^p\,dx$$

```
[In] integrate((e*x)^(-1+2*n)*(b*cos(c+d*x^n))^p,x, algorithm="maxima")
[Out] integrate((e*x)^(2*n - 1)*(b*cos(d*x^n + c))^p, x)
```

Giac [N/A]

Not integrable

Time = 0.94 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (ex)^{-1+2n}(b\cos(c+dx^n))^p\,dx = \int (ex)^{2n-1}(b\cos(dx^n+c))^p\,dx$$

```
[In] integrate((e*x)^(-1+2*n)*(b*cos(c+d*x^n))^p,x, algorithm="giac")
[Out] integrate((e*x)^(2*n - 1)*(b*cos(d*x^n + c))^p, x)
```

Mupad [N/A]

Not integrable

Time = 13.33 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (ex)^{-1+2n}(b\cos(c+dx^n))^p\,dx = \int (e\,x)^{2n-1}\,(b\,\cos(c+d\,x^n))^p\,dx$$

```
[In] int((e*x)^(2*n - 1)*(b*cos(c + d*x^n))^p,x)
[Out] int((e*x)^(2*n - 1)*(b*cos(c + d*x^n))^p, x)
```

3.67 $\int (ex)^{-1+n} (a+b\cos(c+dx^n))^p \, dx$

Optimal result

Integrand size = 22, antiderivative size = 131

$$\int (ex)^{-1+n} (a+b\cos(c+dx^n))^p \, dx = \frac{\sqrt{2}x^{-n}(ex)^n \operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2},-p,\frac{3}{2},\frac{1}{2}(1-\cos(c+dx^n)),\frac{b(1-\cos(c+dx^n))}{a+b}\right)(a+b\cos(c+dx^n))^p\left(\frac{a+b\cos(c+dx^n)}{a+b}\right.}{den\sqrt{1+\cos(c+dx^n)}}$$

```
[Out] (e*x)^n*AppellF1(1/2,-p,1/2,3/2,b*(1-cos(c+d*x^n))/(a+b),1/2-1/2*cos(c+d*x^
n))*(a+b*cos(c+d*x^n))^p*sin(c+d*x^n)*2^(1/2)/d/e/n/(x^n)/(((a+b*cos(c+d*x^
n))/(a+b))^p)/(1+cos(c+d*x^n))^(1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {3463, 3461, 2744, 144, 143}

$$\int (ex)^{-1+n} (a+b\cos(c+dx^n))^p \, dx = \frac{\sqrt{2}x^{-n}(ex)^n \sin(c+dx^n)(a+b\cos(c+dx^n))^p\left(\frac{a+b\cos(c+dx^n)}{a+b}\right)^{-p}\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2},-p,\frac{3}{2},\frac{1}{2}(1-\cos(dx^n+c\right.}{den\sqrt{\cos(c+dx^n)+1}}$$

```
[In] Int[(e*x)^(-1 + n)*(a + b*Cos[c + d*x^n])^p,x]
```

```
[Out] (Sqrt[2]*(e*x)^n*AppellF1[1/2, 1/2, -p, 3/2, (1 - Cos[c + d*x^n])/2, (b*(1
- Cos[c + d*x^n]))/(a + b)]*(a + b*Cos[c + d*x^n])^p*Sin[c + d*x^n])/(d*e*n
*x^n*Sqrt[1 + Cos[c + d*x^n]]*((a + b*Cos[c + d*x^n])/(a + b))^p)
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 2744

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]]), Subst[Int[(a + b*x)
^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c, d
, n}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*n]
```

Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3463

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*((e_)*(x_))^(m_), x_
Symbol] :> Dist[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(a
+ b*Cos[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && Inte
gerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = \frac{(x^{-n}(ex)^n) \int x^{-1+n}(a + b\cos(c + dx^n))^p \, dx}{e}$$

$$= \frac{(x^{-n}(ex)^n)\,\text{Subst}\left(\int (a+b\cos(c+dx))^p\,dx, x, x^n\right)}{en}$$

$$= -\frac{(x^{-n}(ex)^n\sin(c+dx^n))\,\text{Subst}\left(\int \frac{(a+bx)^p}{\sqrt{1-x}\sqrt{1+x}}\,dx, x, \cos(c+dx^n)\right)}{den\sqrt{1-\cos(c+dx^n)}\sqrt{1+\cos(c+dx^n)}}$$

$$= -\frac{\left(x^{-n}(ex)^n(a+b\cos(c+dx^n))^p\left(-\frac{a+b\cos(c+dx^n)}{-a-b}\right)^{-p}\sin(c+dx^n)\right)\text{Subst}\left(\int \frac{\left(-\frac{a}{-a-b}-\frac{bx}{-a-b}\right)^p}{\sqrt{1-x}\sqrt{1+x}}\,dx,\right.}{den\sqrt{1-\cos(c+dx^n)}\sqrt{1+\cos(c+dx^n)}}$$

$$= \frac{\sqrt{2}x^{-n}(ex)^n\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-p,\frac{3}{2},\frac{1}{2}(1-\cos(c+dx^n)),\frac{b(1-\cos(c+dx^n))}{a+b}\right)(a+b\cos(c+dx^n))^p\left(\frac{a+b}{}\right.}{den\sqrt{1+\cos(c+dx^n)}}$$

Mathematica [A] (verified)

Time = 0.82 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.14

$$\int (ex)^{-1+n}(a+b\cos(c+dx^n))^p\,dx =$$

$$-\frac{x^{-n}(ex)^n\,\text{AppellF1}\left(1+p,\frac{1}{2},\frac{1}{2},2+p,\frac{a+b\cos(c+dx^n)}{a-b},\frac{a+b\cos(c+dx^n)}{a+b}\right)\sqrt{-\frac{b(-1+\cos(c+dx^n))}{a+b}}\sqrt{\frac{b(1+\cos(c+dx^n))}{-a+b}}(a-}{bden(1+p)}$$

```
[In] Integrate[(e*x)^(-1 + n)*(a + b*Cos[c + d*x^n])^p,x]
```

```
[Out] -(((e*x)^n*AppellF1[1 + p, 1/2, 1/2, 2 + p, (a + b*Cos[c + d*x^n])/(a - b),
 (a + b*Cos[c + d*x^n])/(a + b)]*Sqrt[-((b*(-1 + Cos[c + d*x^n]))/(a + b))]
*Sqrt[(b*(1 + Cos[c + d*x^n]))/(-a + b)]*(a + b*Cos[c + d*x^n])^(1 + p)*Csc
[c + d*x^n])/(b*d*e*n*(1 + p)*x^n))
```

Maple [F]

$$\int (ex)^{-1+n}(a+b\cos(c+d\,x^n))^p dx$$

```
[In] int((e*x)^(-1+n)*(a+b*cos(c+d*x^n))^p,x)
```

```
[Out] int((e*x)^(-1+n)*(a+b*cos(c+d*x^n))^p,x)
```

Fricas [F]

$$\int (ex)^{-1+n}\left(a+b\cos\left(c+dx^n\right)\right)^p\,dx=\int (ex)^{n-1}\left(b\cos\left(dx^n+c\right)+a\right)^p\,dx$$

```
[In] integrate((e*x)^(-1+n)*(a+b*cos(c+d*x^n))^p,x, algorithm="fricas")
[Out] integral((e*x)^(n - 1)*(b*cos(d*x^n + c) + a)^p, x)
```

Sympy [F]

$$\int (ex)^{-1+n}\left(a+b\cos\left(c+dx^n\right)\right)^p\,dx=\int (ex)^{n-1}\left(a+b\cos\left(c+dx^n\right)\right)^p\,dx$$

```
[In] integrate((e*x)**(-1+n)*(a+b*cos(c+d*x**n))**p,x)
[Out] Integral((e*x)**(n - 1)*(a + b*cos(c + d*x**n))**p, x)
```

Maxima [F]

$$\int (ex)^{-1+n}\left(a+b\cos\left(c+dx^n\right)\right)^p\,dx=\int (ex)^{n-1}\left(b\cos\left(dx^n+c\right)+a\right)^p\,dx$$

```
[In] integrate((e*x)^(-1+n)*(a+b*cos(c+d*x^n))^p,x, algorithm="maxima")
[Out] integrate((e*x)^(n - 1)*(b*cos(d*x^n + c) + a)^p, x)
```

Giac [F]

$$\int (ex)^{-1+n}\left(a+b\cos\left(c+dx^n\right)\right)^p\,dx=\int (ex)^{n-1}\left(b\cos\left(dx^n+c\right)+a\right)^p\,dx$$

```
[In] integrate((e*x)^(-1+n)*(a+b*cos(c+d*x^n))^p,x, algorithm="giac")
[Out] integrate((e*x)^(n - 1)*(b*cos(d*x^n + c) + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^{-1+n} (a + b\cos(c + dx^n))^p \, dx = \int (e\,x)^{n-1} (a + b\,\cos(c + d\,x^n))^p \, dx$$

```
[In] int((e*x)^(n - 1)*(a + b*cos(c + d*x^n))^p,x)
[Out] int((e*x)^(n - 1)*(a + b*cos(c + d*x^n))^p, x)
```

3.68 $\int (ex)^{-1+2n} (a+b\cos(c+dx^n))^p \, dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int (ex)^{-1+2n} (a+b\cos(c+dx^n))^p \, dx = \frac{x^{-2n}(ex)^{2n}\mathrm{Int}(x^{-1+2n}(a+b\cos(c+dx^n))^p\,,x)}{e}$$

```
[Out] (e*x)^(2*n)*Unintegrable(x^(-1+2*n)*(a+b*cos(c+d*x^n))^p,x)/e/(x^(2*n))
```

Rubi [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (ex)^{-1+2n} (a+b\cos(c+dx^n))^p \, dx = \int (ex)^{-1+2n} (a+b\cos(c+dx^n))^p \, dx$$

```
[In] Int[(e*x)^(-1 + 2*n)*(a + b*Cos[c + d*x^n])^p,x]
```

```
[Out] ((e*x)^(2*n)*Defer[Int][x^(-1 + 2*n)*(a + b*Cos[c + d*x^n])^p, x])/(e*x^(2*
n))
```

Rubi steps

$$\text{integral} = \frac{(x^{-2n}(ex)^{2n}) \int x^{-1+2n}(a+b\cos(c+dx^n))^p \, dx}{e}$$

Mathematica [N/A]

Not integrable

Time = 1.85 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int (ex)^{-1+2n}(a+b\cos(c+dx^n))^p\,dx = \int (ex)^{-1+2n}(a+b\cos(c+dx^n))^p\,dx$$

```
[In] Integrate[(e*x)^(-1 + 2*n)*(a + b*Cos[c + d*x^n])^p,x]
[Out] Integrate[(e*x)^(-1 + 2*n)*(a + b*Cos[c + d*x^n])^p, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.37 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int (ex)^{-1+2n}(a+b\cos(c+d\,x^n))^p dx$$

```
[In] int((e*x)^(-1+2*n)*(a+b*cos(c+d*x^n))^p,x)
[Out] int((e*x)^(-1+2*n)*(a+b*cos(c+d*x^n))^p,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int (ex)^{-1+2n}(a+b\cos(c+dx^n))^p\,dx = \int (ex)^{2n-1}(b\cos(dx^n+c)+a)^p\,dx$$

```
[In] integrate((e*x)^(-1+2*n)*(a+b*cos(c+d*x^n))^p,x, algorithm="fricas")
[Out] integral((e*x)^(2*n - 1)*(b*cos(d*x^n + c) + a)^p, x)
```

Sympy [N/A]

Not integrable

Time = 25.85 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int (ex)^{-1+2n}(a+b\cos(c+dx^n))^p\,dx = \int (ex)^{2n-1}(a+b\cos(c+dx^n))^p\,dx$$

```
[In] integrate((e*x)**(-1+2*n)*(a+b*cos(c+d*x**n))**p,x)
[Out] Integral((e*x)**(2*n - 1)*(a + b*cos(c + d*x**n))**p, x)
```

Maxima [N/A]

Not integrable

Time = 1.68 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int (ex)^{-1+2n} (a + b\cos(c + dx^n))^p \, dx = \int (ex)^{2n-1} (b\cos(dx^n + c) + a)^p \, dx$$

```
[In] integrate((e*x)^(-1+2*n)*(a+b*cos(c+d*x^n))^p,x, algorithm="maxima")
[Out] integrate((e*x)^(2*n - 1)*(b*cos(d*x^n + c) + a)^p, x)
```

Giac [N/A]

Not integrable

Time = 5.99 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int (ex)^{-1+2n} (a + b\cos(c + dx^n))^p \, dx = \int (ex)^{2n-1} (b\cos(dx^n + c) + a)^p \, dx$$

```
[In] integrate((e*x)^(-1+2*n)*(a+b*cos(c+d*x^n))^p,x, algorithm="giac")
[Out] integrate((e*x)^(2*n - 1)*(b*cos(d*x^n + c) + a)^p, x)
```

Mupad [N/A]

Not integrable

Time = 14.40 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int (ex)^{-1+2n} (a + b\cos(c + dx^n))^p \, dx = \int (e\,x)^{2\,n-1} \,(a + b\,\cos(c + d\,x^n))^p \, dx$$

```
[In] int((e*x)^(2*n - 1)*(a + b*cos(c + d*x^n))^p,x)
[Out] int((e*x)^(2*n - 1)*(a + b*cos(c + d*x^n))^p, x)
```

3.69 $\int \frac{\cos(a+bx^n)}{x}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 26

$$\int \frac{\cos(a+bx^n)}{x}\,dx = \frac{\cos(a)\operatorname{CosIntegral}(bx^n)}{n} - \frac{\sin(a)\operatorname{Si}(bx^n)}{n}$$

```
[Out] Ci(b*x^n)*cos(a)/n-Si(b*x^n)*sin(a)/n
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3459, 3457, 3456}

$$\int \frac{\cos(a+bx^n)}{x}\,dx = \frac{\cos(a)\operatorname{CosIntegral}(bx^n)}{n} - \frac{\sin(a)\operatorname{Si}(bx^n)}{n}$$

```
[In] Int[Cos[a + b*x^n]/x,x]
```

```
[Out] (Cos[a]*CosIntegral[b*x^n])/n - (Sin[a]*SinIntegral[b*x^n])/n
```

Rule 3456

```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3457

```
Int[Cos[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[CosIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3459

```
Int[Cos[(c_) + (d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Dist[Cos[c], Int[Cos[d*x
^n]/x, x], x] - Dist[Sin[c], Int[Sin[d*x^n]/x, x], x] /; FreeQ[{c, d, n}, x
]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \cos(a)\int\frac{\cos(bx^n)}{x}\,dx-\sin(a)\int\frac{\sin(bx^n)}{x}\,dx\\ &= \frac{\cos(a)\,\text{CosIntegral}(bx^n)}{n}-\frac{\sin(a)\text{Si}(bx^n)}{n}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int\frac{\cos(a+bx^n)}{x}\,dx=\frac{\cos(a)\,\text{CosIntegral}(bx^n)-\sin(a)\text{Si}(bx^n)}{n}$$

```
[In] Integrate[Cos[a + b*x^n]/x,x]
[Out] (Cos[a]*CosIntegral[b*x^n] - Sin[a]*SinIntegral[b*x^n])/n
```

Maple [A] (verified)

Time = 1.14 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.96

method	result	size
derivativedivides	$\frac{-\text{Si}(b\,x^n)\sin(a)+\text{Ci}(b\,x^n)\cos(a)}{n}$	25
default	$\frac{-\text{Si}(b\,x^n)\sin(a)+\text{Ci}(b\,x^n)\cos(a)}{n}$	25
risch	$\frac{i\mathrm{e}^{-ia}\pi\ \text{csgn}(b\,x^n)}{2n}-\frac{i\mathrm{e}^{-ia}\,\text{Si}(b\,x^n)}{n}-\frac{\mathrm{e}^{-ia}\,\text{Ei}_1(-ib\,x^n)}{2n}-\frac{\mathrm{e}^{ia}\,\text{Ei}_1(-ib\,x^n)}{2n}$	75
meijerg	$\frac{\sqrt{\pi}\left(\frac{2\gamma+2n\ln(x)+\ln\left(b^2\right)}{\sqrt{\pi}}-\frac{2\gamma}{\sqrt{\pi}}-\frac{2\ln(2)}{\sqrt{\pi}}-\frac{2\ln\left(\frac{b\,x^n}{2}\right)}{\sqrt{\pi}}+\frac{2\ \text{Ci}(b\,x^n)}{\sqrt{\pi}}\right)\cos(a)}{2n}-\frac{\text{Si}(b\,x^n)\sin(a)}{n}$	79

```
[In] int(cos(a+b*x^n)/x,x,method=_RETURNVERBOSE)
[Out] 1/n*(-Si(b*x^n)*sin(a)+Ci(b*x^n)*cos(a))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{\cos(a+bx^n)}{x}\,dx = \frac{\cos(a)\operatorname{Ci}(bx^n) - \sin(a)\operatorname{Si}(bx^n)}{n}$$

```
[In] integrate(cos(a+b*x^n)/x,x, algorithm="fricas")
[Out] (cos(a)*cos_integral(b*x^n) - sin(a)*sin_integral(b*x^n))/n
```

Sympy [F]

$$\int \frac{\cos(a+bx^n)}{x}\,dx = \int \frac{\cos(a+bx^n)}{x}\,dx$$

```
[In] integrate(cos(a+b*x**n)/x,x)
[Out] Integral(cos(a + b*x**n)/x, x)
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.40 (sec) , antiderivative size = 90, normalized size of antiderivative = 3.46

$$\int \frac{\cos(a+bx^n)}{x}\,dx = \frac{\left(\operatorname{Ei}(i\,bx^n) + \operatorname{Ei}(-i\,bx^n) + \operatorname{Ei}\left(i\,be^{\left(n\overline{\log(x)}\right)}\right) + \operatorname{Ei}\left(-i\,be^{\left(n\overline{\log(x)}\right)}\right)\right)\cos(a) + \left(i\operatorname{Ei}(i\,bx^n) - i\operatorname{Ei}(-i\,bx^n) + \right.}{4\,n}$$

```
[In] integrate(cos(a+b*x^n)/x,x, algorithm="maxima")
[Out] 1/4*((Ei(I*b*x^n) + Ei(-I*b*x^n) + Ei(I*b*e^(n*conjugate(log(x)))) + Ei(-I*
b*e^(n*conjugate(log(x)))))*cos(a) + (I*Ei(I*b*x^n) - I*Ei(-I*b*x^n) + I*Ei
(I*b*e^(n*conjugate(log(x)))) - I*Ei(-I*b*e^(n*conjugate(log(x)))))*sin(a))
/n
```

Giac [F]

$$\int \frac{\cos(a+bx^n)}{x}\,dx = \int \frac{\cos(bx^n+a)}{x}\,dx$$

```
[In] integrate(cos(a+b*x^n)/x,x, algorithm="giac")
[Out] integrate(cos(b*x^n + a)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(a+bx^n)}{x}\,dx = \int \frac{\cos(a+b\,x^n)}{x}\,dx$$

```
[In] int(cos(a + b*x^n)/x,x)
[Out] int(cos(a + b*x^n)/x, x)
```

3.70 $\int \frac{\cos^2(a+bx^n)}{x}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 43

$$\int \frac{\cos^2(a+bx^n)}{x}\,dx = \frac{\cos(2a)\,\text{CosIntegral}(2bx^n)}{2n} + \frac{\log(x)}{2} - \frac{\sin(2a)\text{Si}(2bx^n)}{2n}$$

```
[Out] 1/2*Ci(2*b*x^n)*cos(2*a)/n+1/2*ln(x)-1/2*Si(2*b*x^n)*sin(2*a)/n
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3507, 3459, 3457, 3456}

$$\int \frac{\cos^2(a+bx^n)}{x}\,dx = \frac{\cos(2a)\,\text{CosIntegral}(2bx^n)}{2n} - \frac{\sin(2a)\text{Si}(2bx^n)}{2n} + \frac{\log(x)}{2}$$

```
[In] Int[Cos[a + b*x^n]^2/x,x]
```

```
[Out] (Cos[2*a]*CosIntegral[2*b*x^n])/(2*n) + Log[x]/2 - (Sin[2*a]*SinIntegral[2*
b*x^n])/(2*n)
```

Rule 3456

```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3457

```
Int[Cos[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[CosIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3459

```
Int[Cos[(c_) + (d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Dist[Cos[c], Int[Cos[d*x
^n]/x, x], x] - Dist[Sin[c], Int[Sin[d*x^n]/x, x], x] /; FreeQ[{c, d, n}, x
]
```

Rule 3507

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_)*((e_.)*(x_))^(m_.), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Cos[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{1}{2x} + \frac{\cos(2a + 2bx^n)}{2x}\right) dx \\
&= \frac{\log(x)}{2} + \frac{1}{2} \int \frac{\cos(2a + 2bx^n)}{x} dx \\
&= \frac{\log(x)}{2} + \frac{1}{2}\cos(2a) \int \frac{\cos(2bx^n)}{x} dx - \frac{1}{2}\sin(2a) \int \frac{\sin(2bx^n)}{x} dx \\
&= \frac{\cos(2a)\,\text{CosIntegral}(2bx^n)}{2n} + \frac{\log(x)}{2} - \frac{\sin(2a)\text{Si}(2bx^n)}{2n}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \frac{\cos^2(a + bx^n)}{x} dx = \frac{\cos(2a)\,\text{CosIntegral}(2bx^n) + n\log(x) - \sin(2a)\text{Si}(2bx^n)}{2n}$$

```
[In] Integrate[Cos[a + b*x^n]^2/x,x]
[Out] (Cos[2*a]*CosIntegral[2*b*x^n] + n*Log[x] - Sin[2*a]*SinIntegral[2*b*x^n])/
(2*n)
```

Maple [A] (verified)

Time = 1.11 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$\frac{-\frac{\mathrm{Si}(2b\,x^{n})\sin(2a)}{2}+\frac{\mathrm{Ci}(2b\,x^{n})\cos(2a)}{2}+\frac{\ln(b\,x^{n})}{2}}{n}$	40
default	$\frac{-\frac{\mathrm{Si}(2b\,x^{n})\sin(2a)}{2}+\frac{\mathrm{Ci}(2b\,x^{n})\cos(2a)}{2}+\frac{\ln(b\,x^{n})}{2}}{n}$	40
risch	$\frac{\ln(x)}{2}+\frac{i\mathrm{e}^{-2ia}\pi\,\mathrm{csgn}(b\,x^{n})}{4n}-\frac{i\mathrm{e}^{-2ia}\,\mathrm{Si}(2b\,x^{n})}{2n}-\frac{\mathrm{e}^{-2ia}\,\mathrm{Ei}_{1}(-2ib\,x^{n})}{4n}-\frac{\mathrm{e}^{2ia}\,\mathrm{Ei}_{1}(-2ib\,x^{n})}{4n}$	80

```
[In] int(cos(a+b*x^n)^2/x,x,method=_RETURNVERBOSE)
[Out] 1/n*(-1/2*Si(2*b*x^n)*sin(2*a)+1/2*Ci(2*b*x^n)*cos(2*a)+1/2*ln(b*x^n))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int \frac{\cos^2(a+bx^n)}{x}\,dx=\frac{\cos(2a)\,\mathrm{Ci}(2bx^n)+n\log(x)-\sin(2a)\,\mathrm{Si}(2bx^n)}{2n}$$

```
[In] integrate(cos(a+b*x^n)^2/x,x, algorithm="fricas")
[Out] 1/2*(cos(2*a)*cos_integral(2*b*x^n) + n*log(x) - sin(2*a)*sin_integral(2*b*
x^n))/n
```

Sympy [F]

$$\int \frac{\cos^2(a+bx^n)}{x}\,dx=\int \frac{\cos^2(a+bx^n)}{x}\,dx$$

```
[In] integrate(cos(a+b*x**n)**2/x,x)
[Out] Integral(cos(a + b*x**n)**2/x, x)
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.41 (sec) , antiderivative size = 99, normalized size of antiderivative = 2.30

$$\int \frac{\cos^2(a+bx^n)}{x}\,dx$$
$$=\frac{\left(\mathrm{Ei}(2i\,bx^n)+\mathrm{Ei}(-2i\,bx^n)+\mathrm{Ei}\left(2i\,be^{\left(n\overline{\log(x)}\right)}\right)+\mathrm{Ei}\left(-2i\,be^{\left(n\overline{\log(x)}\right)}\right)\right)\cos(2a)+4n\log(x)+\left(i\,\mathrm{Ei}(2i\,bx\right.}{8n}$$

```
[In] integrate(cos(a+b*x^n)^2/x,x, algorithm="maxima")
```

```
[Out] 1/8*((Ei(2*I*b*x^n) + Ei(-2*I*b*x^n) + Ei(2*I*b*e^(n*conjugate(log(x)))) +
Ei(-2*I*b*e^(n*conjugate(log(x)))))*cos(2*a) + 4*n*log(x) + (I*Ei(2*I*b*x^n
) - I*Ei(-2*I*b*x^n) + I*Ei(2*I*b*e^(n*conjugate(log(x)))) - I*Ei(-2*I*b*e^
(n*conjugate(log(x)))))*sin(2*a))/n
```

Giac [F]

$$\int \frac{\cos^2(a+bx^n)}{x}\,dx = \int \frac{\cos(bx^n+a)^2}{x}\,dx$$

```
[In] integrate(cos(a+b*x^n)^2/x,x, algorithm="giac")
```

```
[Out] integrate(cos(b*x^n + a)^2/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(a+bx^n)}{x}\,dx = \int \frac{\cos(a+b\,x^n)^2}{x}\,dx$$

```
[In] int(cos(a + b*x^n)^2/x,x)
```

```
[Out] int(cos(a + b*x^n)^2/x, x)
```

3.71 $\int \frac{\cos^3(a+bx^n)}{x} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 67

$$\int \frac{\cos^3(a+bx^n)}{x} \, dx = \frac{3\cos(a)\,\text{CosIntegral}(bx^n)}{4n} + \frac{\cos(3a)\,\text{CosIntegral}(3bx^n)}{4n} - \frac{3\sin(a)\text{Si}(bx^n)}{4n} - \frac{\sin(3a)\text{Si}(3bx^n)}{4n}$$

```
[Out] 3/4*Ci(b*x^n)*cos(a)/n+1/4*Ci(3*b*x^n)*cos(3*a)/n-3/4*Si(b*x^n)*sin(a)/n-1/
4*Si(3*b*x^n)*sin(3*a)/n
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3507, 3459, 3457, 3456}

$$\int \frac{\cos^3(a+bx^n)}{x} \, dx = \frac{3\cos(a)\,\text{CosIntegral}(bx^n)}{4n} + \frac{\cos(3a)\,\text{CosIntegral}(3bx^n)}{4n} - \frac{3\sin(a)\text{Si}(bx^n)}{4n} - \frac{\sin(3a)\text{Si}(3bx^n)}{4n}$$

```
[In] Int[Cos[a + b*x^n]^3/x,x]
```

```
[Out] (3*Cos[a]*CosIntegral[b*x^n])/(4*n) + (Cos[3*a]*CosIntegral[3*b*x^n])/(4*n)
 - (3*Sin[a]*SinIntegral[b*x^n])/(4*n) - (Sin[3*a]*SinIntegral[3*b*x^n])/(4
*n)
```

Rule 3456

```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3457

```
Int[Cos[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[CosIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3459

```
Int[Cos[(c_) + (d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Dist[Cos[c], Int[Cos[d*x
^n]/x, x], x] - Dist[Sin[c], Int[Sin[d*x^n]/x, x], x] /; FreeQ[{c, d, n}, x
]
```

Rule 3507

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_)*((e_.)*(x_))^(m_.), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Cos[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(\frac{3\cos\left(a+bx^n\right)}{4x}+\frac{\cos\left(3a+3bx^n\right)}{4x}\right)dx\\ &= \frac{1}{4}\int \frac{\cos\left(3a+3bx^n\right)}{x}\,dx+\frac{3}{4}\int \frac{\cos\left(a+bx^n\right)}{x}\,dx\\ &= \frac{1}{4}(3\cos(a))\int \frac{\cos\left(bx^n\right)}{x}\,dx+\frac{1}{4}\cos(3a)\int \frac{\cos\left(3bx^n\right)}{x}\,dx\\ &\quad-\frac{1}{4}(3\sin(a))\int \frac{\sin\left(bx^n\right)}{x}\,dx-\frac{1}{4}\sin(3a)\int \frac{\sin\left(3bx^n\right)}{x}\,dx\\ &= \frac{3\cos(a)\operatorname{CosIntegral}\left(bx^n\right)}{4n}+\frac{\cos(3a)\operatorname{CosIntegral}\left(3bx^n\right)}{4n}-\frac{3\sin(a)\operatorname{Si}(bx^n)}{4n}-\frac{\sin(3a)\operatorname{Si}(3bx^n)}{4n}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.79

$$\int \frac{\cos^3\left(a+bx^n\right)}{x}\,dx = \frac{3\cos(a)\operatorname{CosIntegral}\left(bx^n\right)+\cos(3a)\operatorname{CosIntegral}\left(3bx^n\right)-3\sin(a)\operatorname{Si}(bx^n)-\sin(3a)\operatorname{Si}(3bx^n)}{4n}$$

```
[In] Integrate[Cos[a + b*x^n]^3/x,x]
[Out] (3*Cos[a]*CosIntegral[b*x^n] + Cos[3*a]*CosIntegral[3*b*x^n] - 3*Sin[a]*Sin
Integral[b*x^n] - Sin[3*a]*SinIntegral[3*b*x^n])/(4*n)
```

Maple [A] (verified)

Time = 1.61 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.78

method	result
derivativedivides	$\frac{-\frac{\mathrm{Si}(3b\,x^{n})\sin(3a)}{4}+\frac{\mathrm{Ci}(3b\,x^{n})\cos(3a)}{4}-\frac{3\ \mathrm{Si}(b\,x^{n})\sin(a)}{4}+\frac{3\ \mathrm{Ci}(b\,x^{n})\cos(a)}{4}}{n}$
default	$\frac{-\frac{\mathrm{Si}(3b\,x^{n})\sin(3a)}{4}+\frac{\mathrm{Ci}(3b\,x^{n})\cos(3a)}{4}-\frac{3\ \mathrm{Si}(b\,x^{n})\sin(a)}{4}+\frac{3\ \mathrm{Ci}(b\,x^{n})\cos(a)}{4}}{n}$
risch	$\frac{i\mathrm{e}^{-3ia}\pi\ \mathrm{csgn}(b\,x^{n})}{8n}-\frac{i\mathrm{e}^{-3ia}\,\mathrm{Si}(3b\,x^{n})}{4n}-\frac{\mathrm{e}^{-3ia}\,\mathrm{Ei}_{1}(-3ib\,x^{n})}{8n}+\frac{3i\mathrm{e}^{-ia}\pi\ \mathrm{csgn}(b\,x^{n})}{8n}-\frac{3i\mathrm{e}^{-ia}\,\mathrm{Si}(b\,x^{n})}{4n}-\frac{3\,\mathrm{e}^{-ia}\,\mathrm{Ei}}{8}$

```
[In] int(cos(a+b*x^n)^3/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/n*(-1/4*Si(3*b*x^n)*sin(3*a)+1/4*Ci(3*b*x^n)*cos(3*a)-3/4*Si(b*x^n)*sin(a
)+3/4*Ci(b*x^n)*cos(a))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.76

$$\int \frac{\cos^3(a+bx^n)}{x}\,dx = \frac{\cos(3a)\,\mathrm{Ci}(3bx^n)+3\cos(a)\,\mathrm{Ci}(bx^n)-\sin(3a)\,\mathrm{Si}(3bx^n)-3\sin(a)\,\mathrm{Si}(bx^n)}{4n}$$

```
[In] integrate(cos(a+b*x^n)^3/x,x, algorithm="fricas")
```

```
[Out] 1/4*(cos(3*a)*cos_integral(3*b*x^n) + 3*cos(a)*cos_integral(b*x^n) - sin(3*
a)*sin_integral(3*b*x^n) - 3*sin(a)*sin_integral(b*x^n))/n
```

Sympy [F]

$$\int \frac{\cos^3(a+bx^n)}{x}\,dx = \int \frac{\cos^3(a+bx^n)}{x}\,dx$$

```
[In] integrate(cos(a+b*x**n)**3/x,x)
```

```
[Out] Integral(cos(a + b*x**n)**3/x, x)
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.51 (sec) , antiderivative size = 180, normalized size of antiderivative = 2.69

$$\int \frac{\cos^3(a+bx^n)}{x}\,dx = \frac{\left(\mathrm{Ei}(3i\,bx^n)+\mathrm{Ei}(-3i\,bx^n)+\mathrm{Ei}\left(3i\,be^{\left(n\overline{\log(x)}\right)}\right)+\mathrm{Ei}\left(-3i\,be^{\left(n\overline{\log(x)}\right)}\right)\right)\cos(3\,a)+3\left(\mathrm{Ei}(i\,bx^n)+\mathrm{Ei}(-i\right.}{}$$

```
[In] integrate(cos(a+b*x^n)^3/x,x, algorithm="maxima")
```

```
[Out] 1/16*((Ei(3*I*b*x^n) + Ei(-3*I*b*x^n) + Ei(3*I*b*e^(n*conjugate(log(x)))) +
 Ei(-3*I*b*e^(n*conjugate(log(x)))))*cos(3*a) + 3*(Ei(I*b*x^n) + Ei(-I*b*x^
n) + Ei(I*b*e^(n*conjugate(log(x)))) + Ei(-I*b*e^(n*conjugate(log(x)))))*co
s(a) + (I*Ei(3*I*b*x^n) - I*Ei(-3*I*b*x^n) + I*Ei(3*I*b*e^(n*conjugate(log(
x)))) - I*Ei(-3*I*b*e^(n*conjugate(log(x)))))*sin(3*a) - 3*(-I*Ei(I*b*x^n)
+ I*Ei(-I*b*x^n) - I*Ei(I*b*e^(n*conjugate(log(x)))) + I*Ei(-I*b*e^(n*conju
gate(log(x)))))*sin(a))/n
```

Giac [F]

$$\int \frac{\cos^3(a+bx^n)}{x}\,dx = \int \frac{\cos(bx^n+a)^3}{x}\,dx$$

```
[In] integrate(cos(a+b*x^n)^3/x,x, algorithm="giac")
```

```
[Out] integrate(cos(b*x^n + a)^3/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^3(a+bx^n)}{x}\,dx = \int \frac{\cos(a+b\,x^n)^3}{x}\,dx$$

```
[In] int(cos(a + b*x^n)^3/x,x)
```

```
[Out] int(cos(a + b*x^n)^3/x, x)
```

3.72 $\int \frac{\cos^4(a+bx^n)}{x} dx$

Optimal result

Integrand size = 14, antiderivative size = 79

$$\int \frac{\cos^4(a+bx^n)}{x} dx = \frac{\cos(2a)\,\text{CosIntegral}(2bx^n)}{2n} + \frac{\cos(4a)\,\text{CosIntegral}(4bx^n)}{8n} + \frac{3\log(x)}{8} - \frac{\sin(2a)\text{Si}(2bx^n)}{2n} - \frac{\sin(4a)\text{Si}(4bx^n)}{8n}$$

```
[Out] 1/2*Ci(2*b*x^n)*cos(2*a)/n+1/8*Ci(4*b*x^n)*cos(4*a)/n+3/8*ln(x)-1/2*Si(2*b*
x^n)*sin(2*a)/n-1/8*Si(4*b*x^n)*sin(4*a)/n
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3507, 3459, 3457, 3456}

$$\int \frac{\cos^4(a+bx^n)}{x} dx = \frac{\cos(2a)\,\text{CosIntegral}(2bx^n)}{2n} + \frac{\cos(4a)\,\text{CosIntegral}(4bx^n)}{8n} - \frac{\sin(2a)\text{Si}(2bx^n)}{2n} - \frac{\sin(4a)\text{Si}(4bx^n)}{8n} + \frac{3\log(x)}{8}$$

```
[In] Int[Cos[a + b*x^n]^4/x,x]
```

```
[Out] (Cos[2*a]*CosIntegral[2*b*x^n])/(2*n) + (Cos[4*a]*CosIntegral[4*b*x^n])/(8*
n) + (3*Log[x])/8 - (Sin[2*a]*SinIntegral[2*b*x^n])/(2*n) - (Sin[4*a]*SinIn
tegral[4*b*x^n])/(8*n)
```

Rule 3456

```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3457

```
Int[Cos[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[CosIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3459

```
Int[Cos[(c_) + (d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Dist[Cos[c], Int[Cos[d*x
^n]/x, x], x] - Dist[Sin[c], Int[Sin[d*x^n]/x, x], x] /; FreeQ[{c, d, n}, x
]
```

Rule 3507

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_)*((e_.)*(x_))^(m_.), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Cos[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{3}{8x} + \frac{\cos(2a+2bx^n)}{2x} + \frac{\cos(4a+4bx^n)}{8x}\right) dx \\
&= \frac{3\log(x)}{8} + \frac{1}{8}\int \frac{\cos(4a+4bx^n)}{x}\,dx + \frac{1}{2}\int \frac{\cos(2a+2bx^n)}{x}\,dx \\
&= \frac{3\log(x)}{8} + \frac{1}{2}\cos(2a)\int \frac{\cos(2bx^n)}{x}\,dx + \frac{1}{8}\cos(4a)\int \frac{\cos(4bx^n)}{x}\,dx \\
&\quad - \frac{1}{2}\sin(2a)\int \frac{\sin(2bx^n)}{x}\,dx - \frac{1}{8}\sin(4a)\int \frac{\sin(4bx^n)}{x}\,dx \\
&= \frac{\cos(2a)\,\text{CosIntegral}(2bx^n)}{2n} + \frac{\cos(4a)\,\text{CosIntegral}(4bx^n)}{8n} \\
&\quad + \frac{3\log(x)}{8} - \frac{\sin(2a)\text{Si}(2bx^n)}{2n} - \frac{\sin(4a)\text{Si}(4bx^n)}{8n}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.84

$$
\int \frac{\cos^4(a+bx^n)}{x}\,dx = \frac{3\log(x)}{8} + \frac{4\cos(2a)\,\text{CosIntegral}(2bx^n) + \cos(4a)\,\text{CosIntegral}(4bx^n) - 4\sin(2a)\text{Si}(2bx^n) - \sin(4a)\text{Si}(4bx^n)}{8n}
$$

```
[In] Integrate[Cos[a + b*x^n]^4/x,x]
[Out] (3*Log[x])/8 + (4*Cos[2*a]*CosIntegral[2*b*x^n] + Cos[4*a]*CosIntegral[4*b*
x^n] - 4*Sin[2*a]*SinIntegral[2*b*x^n] - Sin[4*a]*SinIntegral[4*b*x^n])/(8*
n)
```

Maple [A] (verified)

Time = 2.83 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.84

method	result
derivativedivides	$\frac{-\frac{\mathrm{Si}(4b\,x^{n})\sin(4a)}{8}+\frac{\mathrm{Ci}(4b\,x^{n})\cos(4a)}{8}-\frac{\mathrm{Si}(2b\,x^{n})\sin(2a)}{2}+\frac{\mathrm{Ci}(2b\,x^{n})\cos(2a)}{2}+\frac{3\ln(b\,x^{n})}{8}}{n}$
default	$\frac{-\frac{\mathrm{Si}(4b\,x^{n})\sin(4a)}{8}+\frac{\mathrm{Ci}(4b\,x^{n})\cos(4a)}{8}-\frac{\mathrm{Si}(2b\,x^{n})\sin(2a)}{2}+\frac{\mathrm{Ci}(2b\,x^{n})\cos(2a)}{2}+\frac{3\ln(b\,x^{n})}{8}}{n}$
risch	$\frac{3\ln(x)}{8}+\frac{i\mathrm{e}^{-4ia}\pi\,\mathrm{csgn}(b\,x^{n})}{16n}-\frac{i\mathrm{e}^{-4ia}\,\mathrm{Si}(4b\,x^{n})}{8n}-\frac{\mathrm{e}^{-4ia}\,\mathrm{Ei}_1(-4ib\,x^{n})}{16n}+\frac{i\mathrm{e}^{-2ia}\pi\,\mathrm{csgn}(b\,x^{n})}{4n}-\frac{i\mathrm{e}^{-2ia}\,\mathrm{Si}(2b\,x^{n})}{2n}$

```
[In] int(cos(a+b*x^n)^4/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/n*(-1/8*Si(4*b*x^n)*sin(4*a)+1/8*Ci(4*b*x^n)*cos(4*a)-1/2*Si(2*b*x^n)*sin
(2*a)+1/2*Ci(2*b*x^n)*cos(2*a)+3/8*ln(b*x^n))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.78

$$\int \frac{\cos^4(a+bx^n)}{x}\,dx$$
$$=\frac{\cos(4a)\,\mathrm{Ci}(4bx^n)+4\cos(2a)\,\mathrm{Ci}(2bx^n)+3n\log(x)-\sin(4a)\,\mathrm{Si}(4bx^n)-4\sin(2a)\,\mathrm{Si}(2bx^n)}{8n}$$

```
[In] integrate(cos(a+b*x^n)^4/x,x, algorithm="fricas")
```

```
[Out] 1/8*(cos(4*a)*cos_integral(4*b*x^n) + 4*cos(2*a)*cos_integral(2*b*x^n) + 3*
n*log(x) - sin(4*a)*sin_integral(4*b*x^n) - 4*sin(2*a)*sin_integral(2*b*x^n
))/n
```

Sympy [F]

$$\int \frac{\cos^4(a+bx^n)}{x}\,dx = \int \frac{\cos^4(a+bx^n)}{x}\,dx$$

```
[In] integrate(cos(a+b*x**n)**4/x,x)
```

```
[Out] Integral(cos(a + b*x**n)**4/x, x)
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.51 (sec) , antiderivative size = 189, normalized size of antiderivative = 2.39

$$\int \frac{\cos^4(a+bx^n)}{x}\,dx = \frac{\left(\text{Ei}(4i\,bx^n)+\text{Ei}(-4i\,bx^n)+\text{Ei}\left(4i\,be^{\left(n\overline{\log(x)}\right)}\right)+\text{Ei}\left(-4i\,be^{\left(n\overline{\log(x)}\right)}\right)\right)\cos(4\,a)+4\left(\text{Ei}(2i\,bx^n)+\text{Ei}(-2\right.}{}$$

```
[In] integrate(cos(a+b*x^n)^4/x,x, algorithm="maxima")
```

```
[Out] 1/32*((Ei(4*I*b*x^n) + Ei(-4*I*b*x^n) + Ei(4*I*b*e^(n*conjugate(log(x))) +
 Ei(-4*I*b*e^(n*conjugate(log(x)))))*cos(4*a) + 4*(Ei(2*I*b*x^n) + Ei(-2*I*
b*x^n) + Ei(2*I*b*e^(n*conjugate(log(x)))) + Ei(-2*I*b*e^(n*conjugate(log(x
)))))*cos(2*a) + 12*n*log(x) + (I*Ei(4*I*b*x^n) - I*Ei(-4*I*b*x^n) + I*Ei(4
*I*b*e^(n*conjugate(log(x)))) - I*Ei(-4*I*b*e^(n*conjugate(log(x)))))*sin(4
*a) - 4*(-I*Ei(2*I*b*x^n) + I*Ei(-2*I*b*x^n) - I*Ei(2*I*b*e^(n*conjugate(lo
g(x)))) + I*Ei(-2*I*b*e^(n*conjugate(log(x)))))*sin(2*a))/n
```

Giac [F]

$$\int \frac{\cos^4(a+bx^n)}{x}\,dx = \int \frac{\cos(bx^n+a)^4}{x}\,dx$$

```
[In] integrate(cos(a+b*x^n)^4/x,x, algorithm="giac")
```

```
[Out] integrate(cos(b*x^n + a)^4/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^4(a+bx^n)}{x}\,dx = \int \frac{\cos(a+b\,x^n)^4}{x}\,dx$$

```
[In] int(cos(a + b*x^n)^4/x,x)
```

```
[Out] int(cos(a + b*x^n)^4/x, x)
```

3.73 $\int \cos (a+b x^n) \, dx$

Optimal result

Integrand size = 8, antiderivative size = 83

$$\int \cos (a+b x^n) \, dx=-\frac{e^{ia} x(-ibx^n)^{-1/n} \Gamma\left(\frac{1}{n},-ibx^n\right)}{2n}-\frac{e^{-ia} x(ibx^n)^{-1/n} \Gamma\left(\frac{1}{n},ibx^n\right)}{2n}$$

```
[Out] -1/2*exp(I*a)*x*GAMMA(1/n,-I*b*x^n)/n/((-I*b*x^n)^(1/n))-1/2*x*GAMMA(1/n,I*
b*x^n)/exp(I*a)/n/((I*b*x^n)^(1/n))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3447, 2239}

$$\int \cos (a+b x^n) \, dx=-\frac{e^{ia} x(-ibx^n)^{-1/n} \Gamma\left(\frac{1}{n},-ibx^n\right)}{2n}-\frac{e^{-ia} x(ibx^n)^{-1/n} \Gamma\left(\frac{1}{n},ibx^n\right)}{2n}$$

```
[In] Int[Cos[a + b*x^n],x]
```

```
[Out] -1/2*(E^(I*a)*x*Gamma[n^(-1), (-I)*b*x^n])/(n*((-I)*b*x^n)^n^(-1)) - (x*Gam
ma[n^(-1), I*b*x^n])/(2*E^(I*a)*n*(I*b*x^n)^n^(-1))
```

Rule 2239

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> Simp[(-F^a
)*(c + d*x)*(Gamma[1/n, (-b)*(c + d*x)^n*Log[F]]/(d*n*((-b)*(c + d*x)^n*Log
[F])^(1/n))), x] /; FreeQ[{F, a, b, c, d, n}, x] && !IntegerQ[2/n]
```

Rule 3447

```
Int[Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Dist[1/2, In
t[E^((-c)*I - d*I*(e + f*x)^n), x], x] + Dist[1/2, Int[E^(c*I + d*I*(e + f*
x)^n), x], x] /; FreeQ[{c, d, e, f, n}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}\int e^{-ia-ibx^n}\,dx + \frac{1}{2}\int e^{ia+ibx^n}\,dx\\ &= -\frac{e^{ia}x(-ibx^n)^{-1/n}\,\Gamma\left(\frac{1}{n},-ibx^n\right)}{2n} - \frac{e^{-ia}x(ibx^n)^{-1/n}\,\Gamma\left(\frac{1}{n},ibx^n\right)}{2n}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.11

$$\int \cos\left(a+bx^n\right)\,dx = -\frac{x(b^2x^{2n})^{-1/n}\left((-ibx^n)^{\frac{1}{n}}\,\Gamma\left(\frac{1}{n},ibx^n\right)(\cos(a)-i\sin(a))+(ibx^n)^{\frac{1}{n}}\,\Gamma\left(\frac{1}{n},-ibx^n\right)(\cos(a)+i\sin(a))\right)}{2n}$$

```
[In] Integrate[Cos[a + b*x^n],x]
[Out] -1/2*(x*(((-I)*b*x^n)^n^(-1)*Gamma[n^(-1), I*b*x^n]*(Cos[a] - I*Sin[a]) + (
I*b*x^n)^n^(-1)*Gamma[n^(-1), (-I)*b*x^n]*(Cos[a] + I*Sin[a])))/(n*(b^2*x^(
2*n))^n^(-1))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.52 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.90

method	result	size
meijerg	$x\,{}_1F_2\left(\frac{1}{2n};\frac{1}{2},1+\frac{1}{2n};-\frac{x^{2n}b^2}{4}\right)\cos(a)-\frac{b\,x^{1+n}\,{}_1F_2\left(\frac{1}{2}+\frac{1}{2n};\frac{3}{2},\frac{3}{2}+\frac{1}{2n};-\frac{x^{2n}b^2}{4}\right)\sin(a)}{1+n}$	75

```
[In] int(cos(a+b*x^n),x,method=_RETURNVERBOSE)
[Out] x*hypergeom([1/2/n],[1/2,1+1/2/n],-1/4*x^(2*n)*b^2)*cos(a)-b/(1+n)*x^(1+n)*
hypergeom([1/2+1/2/n],[3/2,3/2+1/2/n],-1/4*x^(2*n)*b^2)*sin(a)
```

Fricas [F]

$$\int \cos(a+bx^n)\ dx = \int \cos(bx^n+a)\ dx$$

```
[In] integrate(cos(a+b*x^n),x, algorithm="fricas")
[Out] integral(cos(b*x^n + a), x)
```

Sympy [F]

$$\int \cos(a+bx^n)\ dx = \int \cos(a+bx^n)\,dx$$

```
[In] integrate(cos(a+b*x**n),x)
[Out] Integral(cos(a + b*x**n), x)
```

Maxima [F]

$$\int \cos(a+bx^n)\ dx = \int \cos(bx^n+a)\ dx$$

```
[In] integrate(cos(a+b*x^n),x, algorithm="maxima")
[Out] integrate(cos(b*x^n + a), x)
```

Giac [F]

$$\int \cos(a+bx^n)\ dx = \int \cos(bx^n+a)\ dx$$

```
[In] integrate(cos(a+b*x^n),x, algorithm="giac")
[Out] integrate(cos(b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos(a+bx^n)\,dx=\int \cos(a+b\,x^n)\,dx$$

```
[In] int(cos(a + b*x^n),x)
[Out] int(cos(a + b*x^n), x)
```

3.74 $\int \cos^2(a+bx^n)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 102

$$\int \cos^2(a+bx^n)\,dx=\frac{x}{2}-\frac{2^{-2-\frac{1}{n}}e^{2ia}x(-ibx^n)^{-1/n}\,\Gamma\left(\frac{1}{n},-2ibx^n\right)}{n}-\frac{2^{-2-\frac{1}{n}}e^{-2ia}x(ibx^n)^{-1/n}\,\Gamma\left(\frac{1}{n},2ibx^n\right)}{n}$$

```
[Out] 1/2*x-2^(-2-1/n)*exp(2*I*a)*x*GAMMA(1/n,-2*I*b*x^n)/n/((-I*b*x^n)^(1/n))-2^
(-2-1/n)*x*GAMMA(1/n,2*I*b*x^n)/exp(2*I*a)/n/((I*b*x^n)^(1/n))
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3449, 3447, 2239}

$$\int \cos^2(a+bx^n)\,dx=-\frac{e^{2ia}2^{-\frac{1}{n}-2}x(-ibx^n)^{-1/n}\,\Gamma\left(\frac{1}{n},-2ibx^n\right)}{n}-\frac{e^{-2ia}2^{-\frac{1}{n}-2}x(ibx^n)^{-1/n}\,\Gamma\left(\frac{1}{n},2ibx^n\right)}{n}+\frac{x}{2}$$

```
[In] Int[Cos[a + b*x^n]^2,x]
```

```
[Out] x/2 - (2^(-2 - n^(-1))*E^((2*I)*a)*x*Gamma[n^(-1), (-2*I)*b*x^n])/(n*((-I)*
b*x^n)^n^(-1)) - (2^(-2 - n^(-1))*x*Gamma[n^(-1), (2*I)*b*x^n])/(E^((2*I)*a
)*n*(I*b*x^n)^n^(-1))
```

Rule 2239

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> Simp[(-F^a
)*(c + d*x)*(Gamma[1/n, (-b)*(c + d*x)^n*Log[F]]/(d*n*((-b)*(c + d*x)^n*Log
```

```
[F])^(1/n))), x] /; FreeQ[{F, a, b, c, d, n}, x] && !IntegerQ[2/n]
```

Rule 3447

```
Int[Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Dist[1/2, In
t[E^((-c)*I - d*I*(e + f*x)^n), x], x] + Dist[1/2, Int[E^(c*I + d*I*(e + f*
x)^n), x], x] /; FreeQ[{c, d, e, f, n}, x]
```

Rule 3449

```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_), x_Sy
mbol] :> Int[ExpandTrigReduce[(a + b*Cos[c + d*(e + f*x)^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, f, n}, x] && IGtQ[p, 1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(\frac{1}{2}+\frac{1}{2}\cos\left(2a+2bx^n\right)\right)\,dx\\ &= \frac{x}{2}+\frac{1}{2}\int \cos\left(2a+2bx^n\right)\,dx\\ &= \frac{x}{2}+\frac{1}{4}\int e^{-2ia-2ibx^n}\,dx+\frac{1}{4}\int e^{2ia+2ibx^n}\,dx\\ &= \frac{x}{2}-\frac{2^{-2-\frac{1}{n}}e^{2ia}x(-ibx^n)^{-1/n}\,\Gamma\left(\frac{1}{n},-2ibx^n\right)}{n}-\frac{2^{-2-\frac{1}{n}}e^{-2ia}x(ibx^n)^{-1/n}\,\Gamma\left(\frac{1}{n},2ibx^n\right)}{n}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.92

$$\int \cos^2\left(a+bx^n\right)\,dx = -\frac{x\left(-2n+2^{-1/n}e^{2ia}(-ibx^n)^{-1/n}\,\Gamma\left(\frac{1}{n},-2ibx^n\right)+2^{-1/n}e^{-2ia}(ibx^n)^{-1/n}\,\Gamma\left(\frac{1}{n},2ibx^n\right)\right)}{4n}$$

```
[In] Integrate[Cos[a + b*x^n]^2,x]
[Out] -1/4*(x*(-2*n + (E^((2*I)*a)*Gamma[n^(-1), (-2*I)*b*x^n])/(2^n^(-1)*((-I)*b
*x^n)^n^(-1)) + Gamma[n^(-1), (2*I)*b*x^n]/(2^n^(-1)*E^((2*I)*a)*(I*b*x^n)^
n^(-1))))/n
```

Maple [F]

$$\int \left(\cos^2\left(a + b\,x^n\right)\right) dx$$

```
[In] int(cos(a+b*x^n)^2,x)
[Out] int(cos(a+b*x^n)^2,x)
```

Fricas [F]

$$\int \cos^2\left(a + bx^n\right)\,dx = \int \cos\left(bx^n + a\right)^2\,dx$$

```
[In] integrate(cos(a+b*x^n)^2,x, algorithm="fricas")
[Out] integral(cos(b*x^n + a)^2, x)
```

Sympy [F]

$$\int \cos^2\left(a + bx^n\right)\,dx = \int \cos^2\left(a + bx^n\right)dx$$

```
[In] integrate(cos(a+b*x**n)**2,x)
[Out] Integral(cos(a + b*x**n)**2, x)
```

Maxima [F]

$$\int \cos^2\left(a + bx^n\right)\,dx = \int \cos\left(bx^n + a\right)^2\,dx$$

```
[In] integrate(cos(a+b*x^n)^2,x, algorithm="maxima")
[Out] 1/2*x + 1/2*integrate(cos(2*b*x^n + 2*a), x)
```

Giac [F]

$$\int \cos^2(a + bx^n) \, dx = \int \cos(bx^n + a)^2 \, dx$$

```
[In] integrate(cos(a+b*x^n)^2,x, algorithm="giac")
[Out] integrate(cos(b*x^n + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^2(a + bx^n) \, dx = \int \cos(a + b\,x^n)^2 \, dx$$

```
[In] int(cos(a + b*x^n)^2,x)
[Out] int(cos(a + b*x^n)^2, x)
```

3.75 $\int \cos^3(a+bx^n)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 179

$$\int \cos^3(a+bx^n)\,dx = -\frac{3e^{ia}x(-ibx^n)^{-1/n}\Gamma\left(\frac{1}{n},-ibx^n\right)}{8n} - \frac{3e^{-ia}x(ibx^n)^{-1/n}\Gamma\left(\frac{1}{n},ibx^n\right)}{8n} - \frac{3^{-1/n}e^{3ia}x(-ibx^n)^{-1/n}\Gamma\left(\frac{1}{n},-3ibx^n\right)}{8n} - \frac{3^{-1/n}e^{-3ia}x(ibx^n)^{-1/n}\Gamma\left(\frac{1}{n},3ibx^n\right)}{8n}$$

```
[Out] -3/8*exp(I*a)*x*GAMMA(1/n,-I*b*x^n)/n/((-I*b*x^n)^(1/n))-3/8*x*GAMMA(1/n,I*
b*x^n)/exp(I*a)/n/((I*b*x^n)^(1/n))-1/8*exp(3*I*a)*x*GAMMA(1/n,-3*I*b*x^n)/
(3^(1/n))/n/((-I*b*x^n)^(1/n))-1/8*x*GAMMA(1/n,3*I*b*x^n)/(3^(1/n))/exp(3*I
*a)/n/((I*b*x^n)^(1/n))
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3449, 3447, 2239}

$$\int \cos^3(a+bx^n)\,dx = -\frac{3e^{ia}x(-ibx^n)^{-1/n}\Gamma\left(\frac{1}{n},-ibx^n\right)}{8n} - \frac{e^{3ia}3^{-1/n}x(-ibx^n)^{-1/n}\Gamma\left(\frac{1}{n},-3ibx^n\right)}{8n} - \frac{3e^{-ia}x(ibx^n)^{-1/n}\Gamma\left(\frac{1}{n},ibx^n\right)}{8n} - \frac{e^{-3ia}3^{-1/n}x(ibx^n)^{-1/n}\Gamma\left(\frac{1}{n},3ibx^n\right)}{8n}$$

```
[In] Int[Cos[a + b*x^n]^3,x]
```

```
[Out] (-3*E^(I*a)*x*Gamma[n^(-1), (-I)*b*x^n])/(8*n*((-I)*b*x^n)^n^(-1)) - (3*x*G
amma[n^(-1), I*b*x^n])/(8*E^(I*a)*n*(I*b*x^n)^n^(-1)) - (E^((3*I)*a)*x*Gamm
```

```
a[n^(-1), (-3*I)*b*x^n])/(8*3^n^(-1)*n*((-I)*b*x^n)^n^(-1)) - (x*Gamma[n^(-
1), (3*I)*b*x^n])/(8*3^n^(-1)*E^((3*I)*a)*n*(I*b*x^n)^n^(-1))
```

Rule 2239

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> Simp[(-F^a
)*(c + d*x)*(Gamma[1/n, (-b)*(c + d*x)^n*Log[F]]/(d*n*((-b)*(c + d*x)^n*Log
[F])^(1/n))), x] /; FreeQ[{F, a, b, c, d, n}, x] &&  !IntegerQ[2/n]
```

Rule 3447

```
Int[Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Dist[1/2, In
t[E^((-c)*I - d*I*(e + f*x)^n), x], x] + Dist[1/2, Int[E^(c*I + d*I*(e + f*
x)^n), x], x] /; FreeQ[{c, d, e, f, n}, x]
```

Rule 3449

```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_), x_Sy
mbol] :> Int[ExpandTrigReduce[(a + b*Cos[c + d*(e + f*x)^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, f, n}, x] && IGtQ[p, 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{3}{4}\cos\left(a+bx^n\right)+\frac{1}{4}\cos\left(3a+3bx^n\right)\right)\,dx\\
&= \frac{1}{4}\int \cos\left(3a+3bx^n\right)\,dx+\frac{3}{4}\int \cos\left(a+bx^n\right)\,dx\\
&= \frac{1}{8}\int e^{-3ia-3ibx^n}\,dx+\frac{1}{8}\int e^{3ia+3ibx^n}\,dx+\frac{3}{8}\int e^{-ia-ibx^n}\,dx+\frac{3}{8}\int e^{ia+ibx^n}\,dx\\
&= -\frac{3e^{ia}x(-ibx^n)^{-1/n}\,\Gamma\left(\frac{1}{n},-ibx^n\right)}{8n}-\frac{3e^{-ia}x(ibx^n)^{-1/n}\,\Gamma\left(\frac{1}{n},ibx^n\right)}{8n}\\
&\quad-\frac{3^{-1/n}e^{3ia}x(-ibx^n)^{-1/n}\,\Gamma\left(\frac{1}{n},-3ibx^n\right)}{8n}-\frac{3^{-1/n}e^{-3ia}x(ibx^n)^{-1/n}\,\Gamma\left(\frac{1}{n},3ibx^n\right)}{8n}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.97

$$\int \cos^3\left(a+bx^n\right)\,dx =$$

$$-\frac{3^{-1/n}e^{-3ia}x(b^2x^{2n})^{-1/n}\left(3^{1+\frac{1}{n}}e^{4ia}(ibx^n)^{\frac{1}{n}}\,\Gamma\left(\frac{1}{n},-ibx^n\right)+3^{1+\frac{1}{n}}e^{2ia}(-ibx^n)^{\frac{1}{n}}\,\Gamma\left(\frac{1}{n},ibx^n\right)+e^{6ia}(ibx^n)^{\frac{1}{n}}\,\Gamma\left(\right.\right.}{8n}$$

```
[In] Integrate[Cos[a + b*x^n]^3,x]
```

```
[Out] -1/8*(x*(3^(1 + n^(-1))*E^((4*I)*a)*(I*b*x^n)^n^(-1)*Gamma[n^(-1), (-I)*b*x
^n] + 3^(1 + n^(-1))*E^((2*I)*a)*((-I)*b*x^n)^n^(-1)*Gamma[n^(-1), I*b*x^n]
 + E^((6*I)*a)*(I*b*x^n)^n^(-1)*Gamma[n^(-1), (-3*I)*b*x^n] + ((-I)*b*x^n)^
n^(-1)*Gamma[n^(-1), (3*I)*b*x^n]))/(3^n^(-1)*E^((3*I)*a)*n*(b^2*x^(2*n))^n
^(-1))
```

Maple [F]

$$\int \left(\cos^3\left(a + b\,x^n\right)\right) dx$$

```
[In] int(cos(a+b*x^n)^3,x)
[Out] int(cos(a+b*x^n)^3,x)
```

Fricas [F]

$$\int \cos^3\left(a + bx^n\right)\, dx = \int \cos\left(bx^n + a\right)^3\, dx$$

```
[In] integrate(cos(a+b*x^n)^3,x, algorithm="fricas")
[Out] integral(cos(b*x^n + a)^3, x)
```

Sympy [F]

$$\int \cos^3\left(a + bx^n\right)\, dx = \int \cos^3\left(a + bx^n\right) dx$$

```
[In] integrate(cos(a+b*x**n)**3,x)
[Out] Integral(cos(a + b*x**n)**3, x)
```

Maxima [F]

$$\int \cos^3\left(a + bx^n\right)\, dx = \int \cos\left(bx^n + a\right)^3\, dx$$

```
[In] integrate(cos(a+b*x^n)^3,x, algorithm="maxima")
[Out] integrate(cos(b*x^n + a)^3, x)
```

Giac [F]

$$\int \cos^3\left(a+bx^n\right)\,dx=\int \cos\left(bx^n+a\right)^3\,dx$$

```
[In] integrate(cos(a+b*x^n)^3,x, algorithm="giac")
[Out] integrate(cos(b*x^n + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^3\left(a+bx^n\right)\,dx=\int \cos\left(a+b\,x^n\right)^3\,dx$$

```
[In] int(cos(a + b*x^n)^3,x)
[Out] int(cos(a + b*x^n)^3, x)
```

3.76 $\int x^m \cos(a+bx^n)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 105

$$\int x^m \cos(a+bx^n)\,dx = -\frac{e^{ia}x^{1+m}(-ibx^n)^{-\frac{1+m}{n}}\Gamma\left(\frac{1+m}{n},-ibx^n\right)}{2n} - \frac{e^{-ia}x^{1+m}(ibx^n)^{-\frac{1+m}{n}}\Gamma\left(\frac{1+m}{n},ibx^n\right)}{2n}$$

```
[Out] -1/2*exp(I*a)*x^(1+m)*GAMMA((1+m)/n,-I*b*x^n)/n/((-I*b*x^n)^((1+m)/n))-1/2*
x^(1+m)*GAMMA((1+m)/n,I*b*x^n)/exp(I*a)/n/((I*b*x^n)^((1+m)/n))
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3505, 2250}

$$\int x^m \cos(a+bx^n)\,dx = -\frac{e^{ia}x^{m+1}(-ibx^n)^{-\frac{m+1}{n}}\Gamma\left(\frac{m+1}{n},-ibx^n\right)}{2n} - \frac{e^{-ia}x^{m+1}(ibx^n)^{-\frac{m+1}{n}}\Gamma\left(\frac{m+1}{n},ibx^n\right)}{2n}$$

```
[In] Int[x^m*Cos[a + b*x^n],x]
```

```
[Out] -1/2*(E^(I*a)*x^(1 + m)*Gamma[(1 + m)/n, (-I)*b*x^n])/(n*((-I)*b*x^n)^((1 +
 m)/n)) - (x^(1 + m)*Gamma[(1 + m)/n, I*b*x^n])/(2*E^(I*a)*n*(I*b*x^n)^((1
+ m)/n))
```

Rule 2250

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 3505

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Dist[1/2,
 Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] + Dist[1/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}\int e^{-ia-ibx^n}x^m\,dx + \frac{1}{2}\int e^{ia+ibx^n}x^m\,dx\\ &= -\frac{e^{ia}x^{1+m}(-ibx^n)^{-\frac{1+m}{n}}\,\Gamma\left(\frac{1+m}{n},-ibx^n\right)}{2n} - \frac{e^{-ia}x^{1+m}(ibx^n)^{-\frac{1+m}{n}}\,\Gamma\left(\frac{1+m}{n},ibx^n\right)}{2n}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.10

$$\int x^m\cos\left(a+bx^n\right)\,dx = -\frac{x^{1+m}(b^2x^{2n})^{-\frac{1+m}{n}}\left((-ibx^n)^{\frac{1+m}{n}}\,\Gamma\left(\frac{1+m}{n},ibx^n\right)(\cos(a)-i\sin(a))+(ibx^n)^{\frac{1+m}{n}}\,\Gamma\left(\frac{1+m}{n},-ibx^n\right)(\cos(a)+i\right.}{2n}$$

```
[In] Integrate[x^m*Cos[a + b*x^n],x]
```

```
[Out] -1/2*(x^(1 + m)*(((-I)*b*x^n)^((1 + m)/n)*Gamma[(1 + m)/n, I*b*x^n]*(Cos[a]
 - I*Sin[a]) + (I*b*x^n)^((1 + m)/n)*Gamma[(1 + m)/n, (-I)*b*x^n]*(Cos[a] +
 I*Sin[a])))/(n*(b^2*x^(2*n))^((1 + m)/n))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.78 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.06

method	result	size
meijerg	$\frac{x^{1+m}{}_1F_2\left(\frac{m}{2n}+\frac{1}{2n};\frac{1}{2},1+\frac{m}{2n}+\frac{1}{2n};-\frac{x^{2n}b^2}{4}\right)\cos(a)}{1+m}-\frac{b\,x^{1+m+n}{}_1F_2\left(\frac{1}{2}+\frac{m}{2n}+\frac{1}{2n};\frac{3}{2},\frac{3}{2}+\frac{m}{2n}+\frac{1}{2n};-\frac{x^{2n}b^2}{4}\right)\sin(a)}{1+m+n}$	111

```
[In] int(x^m*cos(a+b*x^n),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(1+m)*x^(1+m)*hypergeom([1/2/n*m+1/2/n],[1/2,1+1/2/n*m+1/2/n],-1/4*x^(2*n
)*b^2)*cos(a)-b/(1+m+n)*x^(1+m+n)*hypergeom([1/2+1/2/n*m+1/2/n],[3/2,3/2+1/
2/n*m+1/2/n],-1/4*x^(2*n)*b^2)*sin(a)
```

Fricas [F]

$$\int x^m \cos(a + bx^n) \, dx = \int x^m \cos(bx^n + a) \, dx$$

```
[In] integrate(x^m*cos(a+b*x^n),x, algorithm="fricas")
```

```
[Out] integral(x^m*cos(b*x^n + a), x)
```

Sympy [F]

$$\int x^m \cos(a + bx^n) \, dx = \int x^m \cos(a + bx^n) \, dx$$

```
[In] integrate(x**m*cos(a+b*x**n),x)
```

```
[Out] Integral(x**m*cos(a + b*x**n), x)
```

Maxima [F]

$$\int x^m \cos(a + bx^n) \, dx = \int x^m \cos(bx^n + a) \, dx$$

```
[In] integrate(x^m*cos(a+b*x^n),x, algorithm="maxima")
```

```
[Out] integrate(x^m*cos(b*x^n + a), x)
```

Giac [F]

$$\int x^m \cos(a + bx^n) \, dx = \int x^m \cos(bx^n + a) \, dx$$

```
[In] integrate(x^m*cos(a+b*x^n),x, algorithm="giac")
```

```
[Out] integrate(x^m*cos(b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^m \cos(a+bx^n) \, dx = \int x^m \cos(a+b\,x^n) \, dx$$

```
[In] int(x^m*cos(a + b*x^n),x)
[Out] int(x^m*cos(a + b*x^n), x)
```

3.77 $\int x^m \cos^2(a + bx^n)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 141

$$\int x^m \cos^2(a + bx^n)\,dx = \frac{x^{1+m}}{2(1+m)} - \frac{2^{-\frac{1+m+2n}{n}} e^{2ia} x^{1+m} (-ibx^n)^{-\frac{1+m}{n}} \Gamma\left(\frac{1+m}{n}, -2ibx^n\right)}{n} - \frac{2^{-\frac{1+m+2n}{n}} e^{-2ia} x^{1+m} (ibx^n)^{-\frac{1+m}{n}} \Gamma\left(\frac{1+m}{n}, 2ibx^n\right)}{n}$$

```
[Out] 1/2*x^(1+m)/(1+m)-exp(2*I*a)*x^(1+m)*GAMMA((1+m)/n,-2*I*b*x^n)/(2^((1+m+2*n
)/n))/n/((-I*b*x^n)^((1+m)/n))-x^(1+m)*GAMMA((1+m)/n,2*I*b*x^n)/(2^((1+m+2*
n)/n))/exp(2*I*a)/n/((I*b*x^n)^((1+m)/n))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3507, 3505, 2250}

$$\int x^m \cos^2(a + bx^n)\,dx = -\frac{e^{2ia} 2^{-\frac{m+2n+1}{n}} x^{m+1} (-ibx^n)^{-\frac{m+1}{n}} \Gamma\left(\frac{m+1}{n}, -2ibx^n\right)}{n} - \frac{e^{-2ia} 2^{-\frac{m+2n+1}{n}} x^{m+1} (ibx^n)^{-\frac{m+1}{n}} \Gamma\left(\frac{m+1}{n}, 2ibx^n\right)}{n} + \frac{x^{m+1}}{2(m+1)}$$

```
[In] Int[x^m*Cos[a + b*x^n]^2,x]
```

```
[Out] x^(1 + m)/(2*(1 + m)) - (E^((2*I)*a)*x^(1 + m)*Gamma[(1 + m)/n, (-2*I)*b*x^
n])/(2^((1 + m + 2*n)/n)*n*((-I)*b*x^n)^((1 + m)/n)) - (x^(1 + m)*Gamma[(1
+ m)/n, (2*I)*b*x^n])/(2^((1 + m + 2*n)/n)*E^((2*I)*a)*n*(I*b*x^n)^((1 + m)
/n))
```

Rule 2250

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 3505

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Dist[1/2,
 Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] + Dist[1/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rule 3507

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_)*((e_.)*(x_))^(m_.), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Cos[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{x^m}{2} + \frac{1}{2}x^m \cos\left(2a + 2bx^n\right)\right) dx \\
&= \frac{x^{1+m}}{2(1+m)} + \frac{1}{2}\int x^m \cos\left(2a + 2bx^n\right) dx \\
&= \frac{x^{1+m}}{2(1+m)} + \frac{1}{4}\int e^{-2ia-2ibx^n} x^m \, dx + \frac{1}{4}\int e^{2ia+2ibx^n} x^m \, dx \\
&= \frac{x^{1+m}}{2(1+m)} - \frac{2^{-\frac{1+m+2n}{n}} e^{2ia} x^{1+m} (-ibx^n)^{-\frac{1+m}{n}} \Gamma\left(\frac{1+m}{n}, -2ibx^n\right)}{n} \\
&\quad - \frac{2^{-\frac{1+m+2n}{n}} e^{-2ia} x^{1+m} (ibx^n)^{-\frac{1+m}{n}} \Gamma\left(\frac{1+m}{n}, 2ibx^n\right)}{n}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.91

$$
\int x^m \cos^2\left(a + bx^n\right) dx = -\frac{x^{1+m}\left(-2n + 2^{-\frac{1+m}{n}} e^{2ia}(1+m)\left(-ibx^n\right)^{-\frac{1+m}{n}} \Gamma\left(\frac{1+m}{n}, -2ibx^n\right) + 2^{-\frac{1+m}{n}} e^{-2ia}(1+m)\left(ibx^n\right)^{-\frac{1+m}{n}} \Gamma\left(\frac{1+m}{n}\right.\right.}{4(1+m)n}
$$

```
[In] Integrate[x^m*Cos[a + b*x^n]^2,x]
```

```
[Out] -1/4*(x^(1 + m)*(-2*n + (E^((2*I)*a)*(1 + m)*Gamma[(1 + m)/n, (-2*I)*b*x^n]
)/(2^((1 + m)/n)*((-I)*b*x^n)^((1 + m)/n)) + ((1 + m)*Gamma[(1 + m)/n, (2*I
)*b*x^n])/(2^((1 + m)/n)*E^((2*I)*a)*(I*b*x^n)^((1 + m)/n))))/((1 + m)*n)
```

Maple [F]

$$\int x^{m} \left(\cos^{2}\left(a+b\,x^{n}\right)\right) dx$$

```
[In] int(x^m*cos(a+b*x^n)^2,x)
[Out] int(x^m*cos(a+b*x^n)^2,x)
```

Fricas [F]

$$\int x^m \cos^2\left(a+bx^n\right)\,dx=\int x^{m} \cos\left(bx^{n}+a\right)^{2}\,dx$$

```
[In] integrate(x^m*cos(a+b*x^n)^2,x, algorithm="fricas")
[Out] integral(x^m*cos(b*x^n + a)^2, x)
```

Sympy [F]

$$\int x^m \cos^2\left(a+bx^n\right)\,dx=\int x^{m} \cos^{2}\left(a+bx^{n}\right)dx$$

```
[In] integrate(x**m*cos(a+b*x**n)**2,x)
[Out] Integral(x**m*cos(a + b*x**n)**2, x)
```

Maxima [F]

$$\int x^m \cos^2\left(a+bx^n\right)\,dx=\int x^{m} \cos\left(bx^{n}+a\right)^{2}\,dx$$

```
[In] integrate(x^m*cos(a+b*x^n)^2,x, algorithm="maxima")
[Out] 1/2*(x*x^m + (m + 1)*integrate(x^m*cos(2*b*x^n + 2*a), x))/(m + 1)
```

Giac [F]

$$\int x^m \cos^2(a+bx^n)\, dx = \int x^m \cos(bx^n+a)^2\, dx$$

```
[In] integrate(x^m*cos(a+b*x^n)^2,x, algorithm="giac")
[Out] integrate(x^m*cos(b*x^n + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^m \cos^2(a+bx^n)\, dx = \int x^m \cos(a+b\,x^n)^2\, dx$$

```
[In] int(x^m*cos(a + b*x^n)^2,x)
[Out] int(x^m*cos(a + b*x^n)^2, x)
```

3.78 $\int x^m \cos^3(a+bx^n)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 229

$$\int x^m\cos^3(a+bx^n)\,dx=-\frac{3e^{ia}x^{1+m}(-ibx^n)^{-\frac{1+m}{n}}\Gamma\left(\frac{1+m}{n},-ibx^n\right)}{8n}-\frac{3e^{-ia}x^{1+m}(ibx^n)^{-\frac{1+m}{n}}\Gamma\left(\frac{1+m}{n},ibx^n\right)}{8n}-\frac{3^{-\frac{1+m}{n}}e^{3ia}x^{1+m}(-ibx^n)^{-\frac{1+m}{n}}\Gamma\left(\frac{1+m}{n},-3ibx^n\right)}{8n}-\frac{3^{-\frac{1+m}{n}}e^{-3ia}x^{1+m}(ibx^n)^{-\frac{1+m}{n}}\Gamma\left(\frac{1+m}{n},3ibx^n\right)}{8n}$$

```
[Out] -3/8*exp(I*a)*x^(1+m)*GAMMA((1+m)/n,-I*b*x^n)/n/((-I*b*x^n)^((1+m)/n))-3/8*
x^(1+m)*GAMMA((1+m)/n,I*b*x^n)/exp(I*a)/n/((I*b*x^n)^((1+m)/n))-1/8*exp(3*I
*a)*x^(1+m)*GAMMA((1+m)/n,-3*I*b*x^n)/(3^((1+m)/n))/n/((-I*b*x^n)^((1+m)/n)
)-1/8*x^(1+m)*GAMMA((1+m)/n,3*I*b*x^n)/(3^((1+m)/n))/exp(3*I*a)/n/((I*b*x^n
)^((1+m)/n))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used

= {3507, 3505, 2250}

$$\int x^m \cos^3(a+bx^n)\,dx = -\frac{3e^{ia}x^{m+1}(-ibx^n)^{-\frac{m+1}{n}}\,\Gamma\left(\frac{m+1}{n},-ibx^n\right)}{8n} - \frac{3e^{-ia}x^{m+1}(ibx^n)^{-\frac{m+1}{n}}\,\Gamma\left(\frac{m+1}{n},ibx^n\right)}{8n} - \frac{e^{3ia}3^{-\frac{m+1}{n}}x^{m+1}(-ibx^n)^{-\frac{m+1}{n}}\,\Gamma\left(\frac{m+1}{n},-3ibx^n\right)}{8n} - \frac{e^{-3ia}3^{-\frac{m+1}{n}}x^{m+1}(ibx^n)^{-\frac{m+1}{n}}\,\Gamma\left(\frac{m+1}{n},3ibx^n\right)}{8n}$$

```
[In] Int[x^m*Cos[a + b*x^n]^3,x]
```

```
[Out] (-3*E^(I*a)*x^(1 + m)*Gamma[(1 + m)/n, (-I)*b*x^n])/(8*n*((-I)*b*x^n)^((1 +
 m)/n)) - (3*x^(1 + m)*Gamma[(1 + m)/n, I*b*x^n])/(8*E^(I*a)*n*(I*b*x^n)^((
1 + m)/n)) - (E^((3*I)*a)*x^(1 + m)*Gamma[(1 + m)/n, (-3*I)*b*x^n])/(8*3^((
1 + m)/n)*n*((-I)*b*x^n)^((1 + m)/n)) - (x^(1 + m)*Gamma[(1 + m)/n, (3*I)*b
*x^n])/(8*3^((1 + m)/n)*E^((3*I)*a)*n*(I*b*x^n)^((1 + m)/n))
```

Rule 2250

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 3505

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Dist[1/2,
 Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] + Dist[1/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rule 3507

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_)*((e_.)*(x_))^(m_.), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Cos[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\text{integral} = \int \left(\frac{3}{4}x^m\cos(a+bx^n)+\frac{1}{4}x^m\cos(3a+3bx^n)\right)dx$$

$$= \frac{1}{4}\int x^m\cos(3a+3bx^n)\,dx + \frac{3}{4}\int x^m\cos(a+bx^n)\,dx$$

$$=\frac{1}{8}\int e^{-3ia-3ibx^n}x^m\,dx+\frac{1}{8}\int e^{3ia+3ibx^n}x^m\,dx+\frac{3}{8}\int e^{-ia-ibx^n}x^m\,dx+\frac{3}{8}\int e^{ia+ibx^n}x^m\,dx$$

$$=-\frac{3e^{ia}x^{1+m}(-ibx^n)^{-\frac{1+m}{n}}\Gamma\left(\frac{1+m}{n},-ibx^n\right)}{8n}-\frac{3e^{-ia}x^{1+m}(ibx^n)^{-\frac{1+m}{n}}\Gamma\left(\frac{1+m}{n},ibx^n\right)}{8n}$$
$$-\frac{3^{-\frac{1+m}{n}}e^{3ia}x^{1+m}(-ibx^n)^{-\frac{1+m}{n}}\Gamma\left(\frac{1+m}{n},-3ibx^n\right)}{8n}$$
$$-\frac{3^{-\frac{1+m}{n}}e^{-3ia}x^{1+m}(ibx^n)^{-\frac{1+m}{n}}\Gamma\left(\frac{1+m}{n},3ibx^n\right)}{8n}$$

Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 221, normalized size of antiderivative = 0.97

$$\int x^m\cos^3\left(a+bx^n\right)\,dx=$$
$$-\frac{3^{-\frac{1+m}{n}}e^{-3ia}x^{1+m}(b^2x^{2n})^{-\frac{1+m}{n}}\left(3^{\frac{1+m+n}{n}}e^{4ia}(ibx^n)^{\frac{1+m}{n}}\Gamma\left(\frac{1+m}{n},-ibx^n\right)+3^{\frac{1+m+n}{n}}e^{2ia}(-ibx^n)^{\frac{1+m}{n}}\Gamma\left(\frac{1+m}{n},ibx^n\right.\right.}{8n}$$

```
[In] Integrate[x^m*Cos[a + b*x^n]^3,x]
[Out] -1/8*(x^(1 + m)*(3^((1 + m + n)/n)*E^((4*I)*a)*(I*b*x^n)^((1 + m)/n)*Gamma[
(1 + m)/n, (-I)*b*x^n] + 3^((1 + m + n)/n)*E^((2*I)*a)*((-I)*b*x^n)^((1 + m
)/n)*Gamma[(1 + m)/n, I*b*x^n] + E^((6*I)*a)*(I*b*x^n)^((1 + m)/n)*Gamma[(1
 + m)/n, (-3*I)*b*x^n] + ((-I)*b*x^n)^((1 + m)/n)*Gamma[(1 + m)/n, (3*I)*b*
x^n]))/(3^((1 + m)/n)*E^((3*I)*a)*n*(b^2*x^(2*n))^((1 + m)/n))
```

Maple [F]

$$\int x^m\left(\cos^3\left(a+b\,x^n\right)\right)dx$$

```
[In] int(x^m*cos(a+b*x^n)^3,x)
[Out] int(x^m*cos(a+b*x^n)^3,x)
```

Fricas [F]

$$\int x^m\cos^3\left(a+bx^n\right)\,dx=\int x^m\cos\left(bx^n+a\right)^3\,dx$$

```
[In] integrate(x^m*cos(a+b*x^n)^3,x, algorithm="fricas")
[Out] integral(x^m*cos(b*x^n + a)^3, x)
```

Sympy [F]

$$\int x^m \cos^3(a+bx^n)\,dx = \int x^m \cos^3(a+bx^n)\,dx$$

```
[In] integrate(x**m*cos(a+b*x**n)**3,x)
[Out] Integral(x**m*cos(a + b*x**n)**3, x)
```

Maxima [F]

$$\int x^m \cos^3(a+bx^n)\,dx = \int x^m \cos(bx^n+a)^3\,dx$$

```
[In] integrate(x^m*cos(a+b*x^n)^3,x, algorithm="maxima")
[Out] integrate(x^m*cos(b*x^n + a)^3, x)
```

Giac [F]

$$\int x^m \cos^3(a+bx^n)\,dx = \int x^m \cos(bx^n+a)^3\,dx$$

```
[In] integrate(x^m*cos(a+b*x^n)^3,x, algorithm="giac")
[Out] integrate(x^m*cos(b*x^n + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^m \cos^3(a+bx^n)\,dx = \int x^m \cos(a+b\,x^n)^3\,dx$$

```
[In] int(x^m*cos(a + b*x^n)^3,x)
[Out] int(x^m*cos(a + b*x^n)^3, x)
```

3.79 $\int x^{-1-n} \cos(a+bx^n)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 47

$$\int x^{-1-n} \cos(a+bx^n)\,dx = -\frac{x^{-n}\cos(a+bx^n)}{n} - \frac{b\,\text{CosIntegral}(bx^n)\sin(a)}{n} - \frac{b\cos(a)\text{Si}(bx^n)}{n}$$

```
[Out] -cos(a+b*x^n)/n/(x^n)-b*cos(a)*Si(b*x^n)/n-b*Ci(b*x^n)*sin(a)/n
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {3461, 3378, 3384, 3380, 3383}

$$\int x^{-1-n} \cos(a+bx^n)\,dx = -\frac{b\sin(a)\,\text{CosIntegral}(bx^n)}{n} - \frac{b\cos(a)\text{Si}(bx^n)}{n} - \frac{x^{-n}\cos(a+bx^n)}{n}$$

```
[In] Int[x^(-1 - n)*Cos[a + b*x^n],x]
```

```
[Out] -(Cos[a + b*x^n]/(n*x^n)) - (b*CosIntegral[b*x^n]*Sin[a])/n - (b*Cos[a]*Sin
Integral[b*x^n])/n
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{\cos(a+bx)}{x^2}\,dx, x, x^n\right)}{n} \\
&= -\frac{x^{-n}\cos\left(a+bx^n\right)}{n} - \frac{b\text{Subst}\left(\int \frac{\sin(a+bx)}{x}\,dx, x, x^n\right)}{n} \\
&= -\frac{x^{-n}\cos\left(a+bx^n\right)}{n} - \frac{(b\cos(a))\text{Subst}\left(\int \frac{\sin(bx)}{x}\,dx, x, x^n\right)}{n} \\
&\quad - \frac{(b\sin(a))\text{Subst}\left(\int \frac{\cos(bx)}{x}\,dx, x, x^n\right)}{n} \\
&= -\frac{x^{-n}\cos\left(a+bx^n\right)}{n} - \frac{b\,\text{CosIntegral}\left(bx^n\right)\sin(a)}{n} - \frac{b\cos(a)\text{Si}(bx^n)}{n}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.96

$$\int x^{-1-n}\cos(a+bx^n)\,dx$$
$$=-\frac{x^{-n}(\cos(a+bx^n)+bx^n\,\text{CosIntegral}(bx^n)\sin(a)+bx^n\cos(a)\text{Si}(bx^n))}{n}$$

```
[In] Integrate[x^(-1 - n)*Cos[a + b*x^n],x]
[Out] -((Cos[a + b*x^n] + b*x^n*CosIntegral[b*x^n]*Sin[a] + b*x^n*Cos[a]*SinInteg
ral[b*x^n])/(n*x^n))
```

Maple [A] (verified)

Time = 1.36 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.96

method	result	size
default	$\frac{b\left(-\frac{\cos(a+b\,x^n)x^{-n}}{b}-\text{Si}(b\,x^n)\cos(a)-\text{Ci}(b\,x^n)\sin(a)\right)}{n}$	45
risch	$\frac{b\,\mathrm{e}^{-ia}\pi\ \text{csgn}(b\,x^n)}{2n}-\frac{b\,\mathrm{e}^{-ia}\,\text{Si}(b\,x^n)}{n}+\frac{ib\,\mathrm{e}^{-ia}\,\text{Ei}_1(-ib\,x^n)}{2n}-\frac{ib\,\mathrm{e}^{ia}\,\text{Ei}_1(-ib\,x^n)}{2n}-\frac{\cos(a+b\,x^n)x^{-n}}{n}$	97

```
[In] int(x^(-1-n)*cos(a+b*x^n),x,method=_RETURNVERBOSE)
[Out] 1/n*b*(-cos(a+b*x^n)/b/(x^n)-Si(b*x^n)*cos(a)-Ci(b*x^n)*sin(a))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.96

$$\int x^{-1-n}\cos(a+bx^n)\,dx=-\frac{bx^n\,\text{Ci}(bx^n)\sin(a)+bx^n\cos(a)\,\text{Si}(bx^n)+\cos(bx^n+a)}{nx^n}$$

```
[In] integrate(x^(-1-n)*cos(a+b*x^n),x, algorithm="fricas")
[Out] -(b*x^n*cos_integral(b*x^n)*sin(a) + b*x^n*cos(a)*sin_integral(b*x^n) + cos
(b*x^n + a))/(n*x^n)
```

Sympy [F]

$$\int x^{-1-n}\cos(a+bx^n)\,dx=\int x^{-n-1}\cos(a+bx^n)\,dx$$

```
[In] integrate(x**(-1-n)*cos(a+b*x**n),x)
[Out] Integral(x**(-n - 1)*cos(a + b*x**n), x)
```

Maxima [F]

$$\int x^{-1-n}\cos(a+bx^n)\,dx=\int x^{-n-1}\cos(bx^n+a)\,dx$$

```
[In] integrate(x^(-1-n)*cos(a+b*x^n),x, algorithm="maxima")
[Out] integrate(x^(-n - 1)*cos(b*x^n + a), x)
```

Giac [F]

$$\int x^{-1-n}\cos(a+bx^n)\,dx=\int x^{-n-1}\cos(bx^n+a)\,dx$$

```
[In] integrate(x^(-1-n)*cos(a+b*x^n),x, algorithm="giac")
[Out] integrate(x^(-n - 1)*cos(b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1-n}\cos(a+bx^n)\,dx=\int \frac{\cos(a+b\,x^n)}{x^{n+1}}\,dx$$

```
[In] int(cos(a + b*x^n)/x^(n + 1),x)
[Out] int(cos(a + b*x^n)/x^(n + 1), x)
```

3.80 $\int x^{-1-n} \cos^2(a+bx^n)\, dx$

Optimal result

Integrand size = 18, antiderivative size = 69

$$\int x^{-1-n}\cos^2(a+bx^n)\,dx = -\frac{x^{-n}}{2n}-\frac{x^{-n}\cos(2(a+bx^n))}{2n} - \frac{b\,\mathrm{CosIntegral}(2bx^n)\sin(2a)}{n}-\frac{b\cos(2a)\mathrm{Si}(2bx^n)}{n}$$

```
[Out] -1/2/n/(x^n)-1/2*cos(2*a+2*b*x^n)/n/(x^n)-b*cos(2*a)*Si(2*b*x^n)/n-b*Ci(2*b
*x^n)*sin(2*a)/n
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3507, 3461, 3378, 3384, 3380, 3383}

$$\int x^{-1-n}\cos^2(a+bx^n)\,dx = -\frac{b\sin(2a)\,\mathrm{CosIntegral}(2bx^n)}{n}-\frac{b\cos(2a)\mathrm{Si}(2bx^n)}{n} - \frac{x^{-n}\cos(2(a+bx^n))}{2n}-\frac{x^{-n}}{2n}$$

```
[In] Int[x^(-1 - n)*Cos[a + b*x^n]^2,x]
```

```
[Out] -1/2*1/(n*x^n) - Cos[2*(a + b*x^n)]/(2*n*x^n) - (b*CosIntegral[2*b*x^n]*Sin
[2*a])/n - (b*Cos[2*a]*SinIntegral[2*b*x^n])/n
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
```

```
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3507

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_)*((e_.)*(x_))^(m_.), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Cos[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{x^{-1-n}}{2}+\frac{1}{2}x^{-1-n}\cos\left(2a+2bx^n\right)\right)\,dx \\
&= -\frac{x^{-n}}{2n}+\frac{1}{2}\int x^{-1-n}\cos\left(2a+2bx^n\right)\,dx \\
&= -\frac{x^{-n}}{2n}+\frac{\text{Subst}\left(\int \frac{\cos(2a+2bx)}{x^2}\,dx,x,x^n\right)}{2n} \\
&= -\frac{x^{-n}}{2n}-\frac{x^{-n}\cos\left(2(a+bx^n)\right)}{2n}-\frac{b\text{Subst}\left(\int \frac{\sin(2a+2bx)}{x}\,dx,x,x^n\right)}{n}
\end{aligned}
$$

$$= -\frac{x^{-n}}{2n} - \frac{x^{-n}\cos\left(2(a+bx^n)\right)}{2n} - \frac{(b\cos(2a))\text{Subst}\left(\int \frac{\sin(2bx)}{x}\,dx, x, x^n\right)}{n} - \frac{(b\sin(2a))\text{Subst}\left(\int \frac{\cos(2bx)}{x}\,dx, x, x^n\right)}{n}$$

$$= -\frac{x^{-n}}{2n} - \frac{x^{-n}\cos\left(2(a+bx^n)\right)}{2n} - \frac{b\,\text{CosIntegral}\left(2bx^n\right)\sin(2a)}{n} - \frac{b\cos(2a)\text{Si}(2bx^n)}{n}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.77

$$\int x^{-1-n}\cos^2\left(a+bx^n\right)\,dx = -\frac{x^{-n}(\cos^2\left(a+bx^n\right) + bx^n\,\text{CosIntegral}\left(2bx^n\right)\sin(2a) + bx^n\cos(2a)\text{Si}(2bx^n))}{n}$$

```
[In] Integrate[x^(-1 - n)*Cos[a + b*x^n]^2,x]
[Out] -((Cos[a + b*x^n]^2 + b*x^n*CosIntegral[2*b*x^n]*Sin[2*a] + b*x^n*Cos[2*a]*
SinIntegral[2*b*x^n])/(n*x^n))
```

Maple [A] (verified)

Time = 2.57 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.94

method	result	size
default	$-\frac{x^{-n}}{2n}+\frac{b\left(-\frac{\cos\left(2a+2b\,x^{n}\right)x^{-n}}{2b}-\operatorname{Si}\left(2b\,x^{n}\right)\cos\left(2a\right)-\operatorname{Ci}\left(2b\,x^{n}\right)\sin\left(2a\right)\right)}{n}$	65
risch	$\frac{\left(b\,{\mathrm e}^{-2ia}\pi\,\operatorname{csgn}\left(b\,x^{n}\right)x^{n}+ib\,{\mathrm e}^{-2ia}\operatorname{Ei}_{1}\left(-2ib\,x^{n}\right)x^{n}-ib\,{\mathrm e}^{2ia}\operatorname{Ei}_{1}\left(-2ib\,x^{n}\right)x^{n}-2b\,{\mathrm e}^{-2ia}\operatorname{Si}\left(2b\,x^{n}\right)x^{n}-\cos\left(2a+2b\,x^{n}\right)-1\right)x^{-n}}{2n}$	103

```
[In] int(x^(-1-n)*cos(a+b*x^n)^2,x,method=_RETURNVERBOSE)
[Out] -1/2/n/(x^n)+1/n*b*(-1/2*cos(2*a+2*b*x^n)/(x^n)/b-Si(2*b*x^n)*cos(2*a)-Ci(2
*b*x^n)*sin(2*a))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.77

$$\int x^{-1-n}\cos^2\left(a+bx^n\right)\,dx = -\frac{bx^n\,\mathrm{Ci}\left(2\,bx^n\right)\sin\left(2\,a\right)+bx^n\cos\left(2\,a\right)\mathrm{Si}\left(2\,bx^n\right)+\cos\left(bx^n+a\right)^2}{nx^n}$$

```
[In] integrate(x^(-1-n)*cos(a+b*x^n)^2,x, algorithm="fricas")
[Out] -(b*x^n*cos_integral(2*b*x^n)*sin(2*a) + b*x^n*cos(2*a)*sin_integral(2*b*x^
n) + cos(b*x^n + a)^2)/(n*x^n)
```

Sympy [F]

$$\int x^{-1-n}\cos^2\left(a+bx^n\right)\,dx = \int x^{-n-1}\cos^2\left(a+bx^n\right)dx$$

```
[In] integrate(x**(-1-n)*cos(a+b*x**n)**2,x)
[Out] Integral(x**(-n - 1)*cos(a + b*x**n)**2, x)
```

Maxima [F]

$$\int x^{-1-n}\cos^2\left(a+bx^n\right)\,dx = \int x^{-n-1}\cos\left(bx^n+a\right)^2\,dx$$

```
[In] integrate(x^(-1-n)*cos(a+b*x^n)^2,x, algorithm="maxima")
[Out] 1/2*(n*x^n*integrate(cos(2*b*x^n + 2*a)/(x*x^n), x) - 1)/(n*x^n)
```

Giac [F]

$$\int x^{-1-n}\cos^2\left(a+bx^n\right)\,dx = \int x^{-n-1}\cos\left(bx^n+a\right)^2\,dx$$

```
[In] integrate(x^(-1-n)*cos(a+b*x^n)^2,x, algorithm="giac")
[Out] integrate(x^(-n - 1)*cos(b*x^n + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1-n}\cos^2\left(a+bx^n\right)\,dx=\int\frac{\cos\left(a+b\,x^n\right)^2}{x^{n+1}}\,dx$$

```
[In] int(cos(a + b*x^n)^2/x^(n + 1),x)
[Out] int(cos(a + b*x^n)^2/x^(n + 1), x)
```

3.81 $\int x^{-1-n}\cos^3(a+bx^n)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 113

$$\int x^{-1-n}\cos^3(a+bx^n)\,dx = -\frac{3x^{-n}\cos(a+bx^n)}{4n} - \frac{x^{-n}\cos(3(a+bx^n))}{4n} - \frac{3b\,\text{CosIntegral}(bx^n)\sin(a)}{4n} - \frac{3b\,\text{CosIntegral}(3bx^n)\sin(3a)}{4n} - \frac{3b\cos(a)\text{Si}(bx^n)}{4n} - \frac{3b\cos(3a)\text{Si}(3bx^n)}{4n}$$

```
[Out] -3/4*cos(a+b*x^n)/n/(x^n)-1/4*cos(3*a+3*b*x^n)/n/(x^n)-3/4*b*cos(a)*Si(b*x^
n)/n-3/4*b*cos(3*a)*Si(3*b*x^n)/n-3/4*b*Ci(b*x^n)*sin(a)/n-3/4*b*Ci(3*b*x^n
)*sin(3*a)/n
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3507, 3461, 3378, 3384, 3380, 3383}

$$\int x^{-1-n}\cos^3(a+bx^n)\,dx = -\frac{3b\sin(a)\,\text{CosIntegral}(bx^n)}{4n} - \frac{3b\sin(3a)\,\text{CosIntegral}(3bx^n)}{4n} - \frac{3b\cos(a)\text{Si}(bx^n)}{4n} - \frac{3b\cos(3a)\text{Si}(3bx^n)}{4n} - \frac{3x^{-n}\cos(a+bx^n)}{4n} - \frac{x^{-n}\cos(3(a+bx^n))}{4n}$$

```
[In] Int[x^(-1 - n)*Cos[a + b*x^n]^3,x]
```

```
[Out] (-3*Cos[a + b*x^n])/(4*n*x^n) - Cos[3*(a + b*x^n)]/(4*n*x^n) - (3*b*CosInte
gral[b*x^n]*Sin[a])/(4*n) - (3*b*CosIntegral[3*b*x^n]*Sin[3*a])/(4*n) - (3*
```

```
b*Cos[a]*SinIntegral[b*x^n])/(4*n) - (3*b*Cos[3*a]*SinIntegral[3*b*x^n])/(4
*n)
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3507

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_)*((e_.)*(x_))^(m_.), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Cos[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\text{integral} = \int \left(\frac{3}{4}x^{-1-n}\cos\left(a+bx^n\right)+\frac{1}{4}x^{-1-n}\cos\left(3a+3bx^n\right)\right)\,dx$$

$$= \frac{1}{4}\int x^{-1-n}\cos\left(3a+3bx^n\right)\,dx + \frac{3}{4}\int x^{-1-n}\cos\left(a+bx^n\right)\,dx$$

$$= \frac{\text{Subst}\left(\int \frac{\cos(3a+3bx)}{x^2}\,dx, x, x^n\right)}{4n} + \frac{3\text{Subst}\left(\int \frac{\cos(a+bx)}{x^2}\,dx, x, x^n\right)}{4n}$$

$$= -\frac{3x^{-n}\cos\left(a+bx^n\right)}{4n} - \frac{x^{-n}\cos\left(3(a+bx^n)\right)}{4n} - \frac{(3b)\text{Subst}\left(\int \frac{\sin(a+bx)}{x}\,dx, x, x^n\right)}{4n} - \frac{(3b)\text{Subst}\left(\int \frac{\sin(3a+3bx)}{x}\,dx, x, x^n\right)}{4n}$$

$$= -\frac{3x^{-n}\cos\left(a+bx^n\right)}{4n} - \frac{x^{-n}\cos\left(3(a+bx^n)\right)}{4n} - \frac{(3b\cos(a))\text{Subst}\left(\int \frac{\sin(bx)}{x}\,dx, x, x^n\right)}{4n} - \frac{(3b\cos(3a))\text{Subst}\left(\int \frac{\sin(3bx)}{x}\,dx, x, x^n\right)}{4n} - \frac{(3b\sin(a))\text{Subst}\left(\int \frac{\cos(bx)}{x}\,dx, x, x^n\right)}{4n} - \frac{(3b\sin(3a))\text{Subst}\left(\int \frac{\cos(3bx)}{x}\,dx, x, x^n\right)}{4n}$$

$$= -\frac{3x^{-n}\cos\left(a+bx^n\right)}{4n} - \frac{x^{-n}\cos\left(3(a+bx^n)\right)}{4n} - \frac{3b\,\text{CosIntegral}\left(bx^n\right)\sin(a)}{4n} - \frac{3b\,\text{CosIntegral}\left(3bx^n\right)\sin(3a)}{4n} - \frac{3b\cos(a)\text{Si}(bx^n)}{4n} - \frac{3b\cos(3a)\text{Si}(3bx^n)}{4n}$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.84

$$\int x^{-1-n}\cos^3\left(a+bx^n\right)\,dx = -\frac{x^{-n}\left(3\cos\left(a+bx^n\right)+\cos\left(3(a+bx^n)\right)+3bx^n\,\text{CosIntegral}\left(bx^n\right)\sin(a)+3bx^n\,\text{CosIntegral}\left(3bx^n\right)\sin(3a\right.}{4n}$$

```
[In] Integrate[x^(-1 - n)*Cos[a + b*x^n]^3,x]
```

```
[Out] -1/4*(3*Cos[a + b*x^n] + Cos[3*(a + b*x^n)] + 3*b*x^n*CosIntegral[b*x^n]*Si
n[a] + 3*b*x^n*CosIntegral[3*b*x^n]*Sin[3*a] + 3*b*x^n*Cos[a]*SinIntegral[b
*x^n] + 3*b*x^n*Cos[3*a]*SinIntegral[3*b*x^n])/(n*x^n)
```

Maple [A] (verified)

Time = 10.36 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.89

method	result
default	$\frac{3b\left(-\frac{\cos(a+b\,x^n)x^{-n}}{b}-\mathrm{Si}(b\,x^n)\cos(a)-\mathrm{Ci}(b\,x^n)\sin(a)\right)}{4n}+\frac{3b\left(-\frac{\cos(3a+3b\,x^n)x^{-n}}{3b}-\mathrm{Si}(3b\,x^n)\cos(3a)-\mathrm{Ci}(3b\,x^n)\sin(3a)\right)}{4n}$
risch	$-\frac{(-3b\,\mathrm{e}^{-3ia}\pi\,\mathrm{csgn}(b\,x^n)x^n-3b\,\mathrm{e}^{-ia}\pi\,\mathrm{csgn}(b\,x^n)x^n-3ib\,\mathrm{e}^{-3ia}\,\mathrm{Ei}_1(-3ib\,x^n)x^n-3ib\,\mathrm{e}^{-ia}\,\mathrm{Ei}_1(-ib\,x^n)x^n+3ib\,\mathrm{e}^{ia}\,\mathrm{Ei}_1(-ib\,x^n)x^n+3ib}{8n}$

```
[In] int(x^(-1-n)*cos(a+b*x^n)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 3/4/n*b*(-cos(a+b*x^n)/b/(x^n)-Si(b*x^n)*cos(a)-Ci(b*x^n)*sin(a))+3/4/n*b*(
-1/3*cos(3*a+3*b*x^n)/(x^n)/b-Si(3*b*x^n)*cos(3*a)-Ci(3*b*x^n)*sin(3*a))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.75

$$\int x^{-1-n}\cos^3\left(a+bx^n\right)\,dx =$$
$$-\frac{3\,bx^n\,\mathrm{Ci}\left(3\,bx^n\right)\sin\left(3\,a\right)+3\,bx^n\,\mathrm{Ci}\left(bx^n\right)\sin\left(a\right)+3\,bx^n\cos\left(3\,a\right)\mathrm{Si}\left(3\,bx^n\right)+3\,bx^n\cos\left(a\right)\mathrm{Si}\left(bx^n\right)+4\,\mathrm{cos}}{4\,nx^n}$$

```
[In] integrate(x^(-1-n)*cos(a+b*x^n)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(3*b*x^n*cos_integral(3*b*x^n)*sin(3*a) + 3*b*x^n*cos_integral(b*x^n)*
sin(a) + 3*b*x^n*cos(3*a)*sin_integral(3*b*x^n) + 3*b*x^n*cos(a)*sin_integr
al(b*x^n) + 4*cos(b*x^n + a)^3)/(n*x^n)
```

Sympy [F]

$$\int x^{-1-n}\cos^3\left(a+bx^n\right)\,dx = \int x^{-n-1}\cos^3\left(a+bx^n\right)dx$$

```
[In] integrate(x**(-1-n)*cos(a+b*x**n)**3,x)
```

```
[Out] Integral(x**(-n - 1)*cos(a + b*x**n)**3, x)
```

Maxima [F]

$$\int x^{-1-n}\cos^3(a+bx^n)\,dx = \int x^{-n-1}\cos(bx^n+a)^3\,dx$$

```
[In] integrate(x^(-1-n)*cos(a+b*x^n)^3,x, algorithm="maxima")
[Out] integrate(x^(-n - 1)*cos(b*x^n + a)^3, x)
```

Giac [F]

$$\int x^{-1-n}\cos^3(a+bx^n)\,dx = \int x^{-n-1}\cos(bx^n+a)^3\,dx$$

```
[In] integrate(x^(-1-n)*cos(a+b*x^n)^3,x, algorithm="giac")
[Out] integrate(x^(-n - 1)*cos(b*x^n + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1-n}\cos^3(a+bx^n)\,dx = \int \frac{\cos(a+b\,x^n)^3}{x^{n+1}}\,dx$$

```
[In] int(cos(a + b*x^n)^3/x^(n + 1),x)
[Out] int(cos(a + b*x^n)^3/x^(n + 1), x)
```

3.82 $\int x^{-1-2n} \cos(a+bx^n)\, dx$

Optimal result

Integrand size = 16, antiderivative size = 78

$$\int x^{-1-2n} \cos(a+bx^n)\, dx = -\frac{x^{-2n}\cos(a+bx^n)}{2n} - \frac{b^2\cos(a)\,\text{CosIntegral}(bx^n)}{2n} + \frac{bx^{-n}\sin(a+bx^n)}{2n} + \frac{b^2\sin(a)\text{Si}(bx^n)}{2n}$$

```
[Out] -1/2*b^2*Ci(b*x^n)*cos(a)/n-1/2*cos(a+b*x^n)/n/(x^(2*n))+1/2*b^2*Si(b*x^n)*
sin(a)/n+1/2*b*sin(a+b*x^n)/n/(x^n)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {3461, 3378, 3384, 3380, 3383}

$$\int x^{-1-2n} \cos(a+bx^n)\, dx = -\frac{b^2\cos(a)\,\text{CosIntegral}(bx^n)}{2n} + \frac{b^2\sin(a)\text{Si}(bx^n)}{2n} + \frac{bx^{-n}\sin(a+bx^n)}{2n} - \frac{x^{-2n}\cos(a+bx^n)}{2n}$$

```
[In] Int[x^(-1 - 2*n)*Cos[a + b*x^n],x]
```

```
[Out] -1/2*Cos[a + b*x^n]/(n*x^(2*n)) - (b^2*Cos[a]*CosIntegral[b*x^n])/(2*n) + (
b*Sin[a + b*x^n])/(2*n*x^n) + (b^2*Sin[a]*SinIntegral[b*x^n])/(2*n)
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
```

```
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{\cos(a+bx)}{x^3}\,dx, x, x^n\right)}{n}\\
&= -\frac{x^{-2n}\cos\left(a+bx^n\right)}{2n} - \frac{b\text{Subst}\left(\int \frac{\sin(a+bx)}{x^2}\,dx, x, x^n\right)}{2n}\\
&= -\frac{x^{-2n}\cos\left(a+bx^n\right)}{2n} + \frac{bx^{-n}\sin\left(a+bx^n\right)}{2n} - \frac{b^2\text{Subst}\left(\int \frac{\cos(a+bx)}{x}\,dx, x, x^n\right)}{2n}\\
&= -\frac{x^{-2n}\cos\left(a+bx^n\right)}{2n} + \frac{bx^{-n}\sin\left(a+bx^n\right)}{2n}\\
&\quad - \frac{\left(b^2\cos(a)\right)\text{Subst}\left(\int \frac{\cos(bx)}{x}\,dx, x, x^n\right)}{2n} + \frac{\left(b^2\sin(a)\right)\text{Subst}\left(\int \frac{\sin(bx)}{x}\,dx, x, x^n\right)}{2n}\\
&= -\frac{x^{-2n}\cos\left(a+bx^n\right)}{2n} - \frac{b^2\cos(a)\,\text{CosIntegral}\left(bx^n\right)}{2n} + \frac{bx^{-n}\sin\left(a+bx^n\right)}{2n} + \frac{b^2\sin(a)\text{Si}(bx^n)}{2n}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.90

$$\int x^{-1-2n} \cos \left(a+b x^n\right) \, dx = -\frac{x^{-2n} \left(\cos \left(a+b x^n\right)+b^2 x^{2n} \cos (a) \text{CosIntegral}\left(b x^n\right)-b x^n \sin \left(a+b x^n\right)-b^2 x^{2n} \sin (a) \text{Si}\left(b x^n\right)\right)}{2n}$$

```
[In] Integrate[x^(-1 - 2*n)*Cos[a + b*x^n],x]
[Out] -1/2*(Cos[a + b*x^n] + b^2*x^(2*n)*Cos[a]*CosIntegral[b*x^n] - b*x^n*Sin[a
+ b*x^n] - b^2*x^(2*n)*Sin[a]*SinIntegral[b*x^n])/(n*x^(2*n))
```

Maple [A] (verified)

Time = 1.54 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.83

method	result
default	$\frac{b^{2} \left(-\frac{\cos \left(a+b\,x^{n}\right) x^{-2 n}}{2 b^{2}}+\frac{\sin \left(a+b\,x^{n}\right) x^{-n}}{2 b}+\frac{\mathrm{Si}\left(b\,x^{n}\right) \sin \left(a\right)}{2}-\frac{\mathrm{Ci}\left(b\,x^{n}\right) \cos \left(a\right)}{2}\right)}{n}$
risch	$-\frac{\left(i b^{2} {\mathrm e}^{-i a} \pi \,\mathrm{csgn}\left(b\,x^{n}\right) x^{2 n}-2 i b^{2} {\mathrm e}^{-i a} \mathrm{Si}\left(b\,x^{n}\right) x^{2 n}-b^{2} {\mathrm e}^{-i a} \mathrm{Ei}_{1}\left(-i b\,x^{n}\right) x^{2 n}-b^{2} {\mathrm e}^{i a} \mathrm{Ei}_{1}\left(-i b\,x^{n}\right) x^{2 n}-2 \sin \left(a+b\,x^{n}\right) x^{n} b+2 \cos \left(a+b\,x^{n}\right)\right)}{4 n}$
meijerg	$b^{2} \sqrt{\pi}\left(-\frac{x^{2\left(\frac{-1-2 n}{2 n}+\frac{1}{2 n}\right) n} 2^{-\frac{-1-2 n}{n}-\frac{1}{n}}}{\sqrt{\pi}\, b^{2}}+\frac{\left(-1\right)^{-\frac{-1-2 n}{2 n}-\frac{1}{2 n}} \left(-\Psi\left(1-\frac{-1-2 n}{2 n}-\frac{1}{2 n}\right)-\Psi\left(\frac{1}{2}-\frac{-1-2 n}{2 n}-\frac{1}{2 n}\right)+2 n \ln\left(x\right)-2 \ln\left(2\right)+\ln\left(b^{2}\right)\right) \sqrt{2}\, 2^{-}}{2 \sqrt{\pi}\, \Gamma\left(-\frac{-1-2 n}{n}-\frac{1}{n}\right)}$

```
[In] int(x^(-1-2*n)*cos(a+b*x^n),x,method=_RETURNVERBOSE)
[Out] 1/n*b^2*(-1/2*cos(a+b*x^n)/b^2/(x^n)^2+1/2*sin(a+b*x^n)/b/(x^n)+1/2*Si(b*x^
n)*sin(a)-1/2*Ci(b*x^n)*cos(a))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.90

$$\int x^{-1-2n} \cos \left(a+b x^n\right) \, dx = -\frac{b^2 x^{2\,n} \cos \left(a\right) \operatorname{Ci}\left(b x^n\right) - b^2 x^{2\,n} \sin \left(a\right) \operatorname{Si}\left(b x^n\right) - b x^n \sin \left(b x^n + a\right) + \cos \left(b x^n + a\right)}{2\,n x^{2\,n}}$$

```
[In] integrate(x^(-1-2*n)*cos(a+b*x^n),x, algorithm="fricas")
[Out] -1/2*(b^2*x^(2*n)*cos(a)*cos_integral(b*x^n) - b^2*x^(2*n)*sin(a)*sin_integ
ral(b*x^n) - b*x^n*sin(b*x^n + a) + cos(b*x^n + a))/(n*x^(2*n))
```

Sympy [F]

$$\int x^{-1-2n}\cos (a+bx^n)\;dx=\int x^{-2n-1}\cos (a+bx^n)\,dx$$

```
[In] integrate(x**(-1-2*n)*cos(a+b*x**n),x)
[Out] Integral(x**(-2*n - 1)*cos(a + b*x**n), x)
```

Maxima [F]

$$\int x^{-1-2n}\cos (a+bx^n)\;dx=\int x^{-2\,n-1}\cos (bx^n+a)\;dx$$

```
[In] integrate(x^(-1-2*n)*cos(a+b*x^n),x, algorithm="maxima")
[Out] integrate(x^(-2*n - 1)*cos(b*x^n + a), x)
```

Giac [F]

$$\int x^{-1-2n}\cos (a+bx^n)\;dx=\int x^{-2\,n-1}\cos (bx^n+a)\;dx$$

```
[In] integrate(x^(-1-2*n)*cos(a+b*x^n),x, algorithm="giac")
[Out] integrate(x^(-2*n - 1)*cos(b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1-2n}\cos (a+bx^n)\;dx=\int \frac{\cos (a+b\,x^n)}{x^{2\,n+1}}\,dx$$

```
[In] int(cos(a + b*x^n)/x^(2*n + 1),x)
[Out] int(cos(a + b*x^n)/x^(2*n + 1), x)
```

3.83 $\int x^{-1-2n}\cos^2(a+bx^n)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 95

$$\int x^{-1-2n}\cos^2(a+bx^n)\,dx=-\frac{x^{-2n}}{4n}-\frac{x^{-2n}\cos(2(a+bx^n))}{4n}-\frac{b^2\cos(2a)\,\text{CosIntegral}(2bx^n)}{n}+\frac{bx^{-n}\sin(2(a+bx^n))}{2n}+\frac{b^2\sin(2a)\text{Si}(2bx^n)}{n}$$

```
[Out] -1/4/n/(x^(2*n))-b^2*Ci(2*b*x^n)*cos(2*a)/n-1/4*cos(2*a+2*b*x^n)/n/(x^(2*n)
)+b^2*Si(2*b*x^n)*sin(2*a)/n+1/2*b*sin(2*a+2*b*x^n)/n/(x^n)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3507, 3461, 3378, 3384, 3380, 3383}

$$\int x^{-1-2n}\cos^2(a+bx^n)\,dx=-\frac{b^2\cos(2a)\,\text{CosIntegral}(2bx^n)}{n}+\frac{b^2\sin(2a)\text{Si}(2bx^n)}{n}+\frac{bx^{-n}\sin(2(a+bx^n))}{2n}-\frac{x^{-2n}\cos(2(a+bx^n))}{4n}-\frac{x^{-2n}}{4n}$$

```
[In] Int[x^(-1 - 2*n)*Cos[a + b*x^n]^2,x]
```

```
[Out] -1/4*1/(n*x^(2*n)) - Cos[2*(a + b*x^n)]/(4*n*x^(2*n)) - (b^2*Cos[2*a]*CosIn
tegral[2*b*x^n])/n + (b*Sin[2*(a + b*x^n)])/(2*n*x^n) + (b^2*Sin[2*a]*SinIn
tegral[2*b*x^n])/n
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
```

```
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3507

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_)*((e_.)*(x_))^(m_.), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Cos[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{1}{2}x^{-1-2n}+\frac{1}{2}x^{-1-2n}\cos\left(2a+2bx^n\right)\right)\,dx \\
&= -\frac{x^{-2n}}{4n}+\frac{1}{2}\int x^{-1-2n}\cos\left(2a+2bx^n\right)\,dx \\
&= -\frac{x^{-2n}}{4n}+\frac{\text{Subst}\left(\int \frac{\cos(2a+2bx)}{x^3}\,dx,x,x^n\right)}{2n}
\end{aligned}$$

$$= -\frac{x^{-2n}}{4n} - \frac{x^{-2n}\cos\left(2(a+bx^n)\right)}{4n} - \frac{b\text{Subst}\left(\int \frac{\sin(2a+2bx)}{x^2}\,dx, x, x^n\right)}{2n}$$

$$= -\frac{x^{-2n}}{4n} - \frac{x^{-2n}\cos\left(2(a+bx^n)\right)}{4n} + \frac{bx^{-n}\sin\left(2(a+bx^n)\right)}{2n} - \frac{b^2\text{Subst}\left(\int \frac{\cos(2a+2bx)}{x}\,dx, x, x^n\right)}{n}$$

$$= -\frac{x^{-2n}}{4n} - \frac{x^{-2n}\cos\left(2(a+bx^n)\right)}{4n} + \frac{bx^{-n}\sin\left(2(a+bx^n)\right)}{2n} - \frac{(b^2\cos(2a))\,\text{Subst}\left(\int \frac{\cos(2bx)}{x}\,dx, x, x^n\right)}{n} + \frac{(b^2\sin(2a))\,\text{Subst}\left(\int \frac{\sin(2bx)}{x}\,dx, x, x^n\right)}{n}$$

$$= -\frac{x^{-2n}}{4n} - \frac{x^{-2n}\cos\left(2(a+bx^n)\right)}{4n} - \frac{b^2\cos(2a)\,\text{CosIntegral}\left(2bx^n\right)}{n} + \frac{bx^{-n}\sin\left(2(a+bx^n)\right)}{2n} + \frac{b^2\sin(2a)\text{Si}(2bx^n)}{n}$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.86

$$\int x^{-1-2n}\cos^2\left(a+bx^n\right)\,dx = -\frac{x^{-2n}\left(1+\cos\left(2(a+bx^n)\right)+4b^2x^{2n}\cos(2a)\,\text{CosIntegral}\left(2bx^n\right)-2bx^n\sin\left(2(a+bx^n)\right)-4b^2x^{2n}\sin(2a)\text{S}\right.}{4n}$$

```
[In] Integrate[x^(-1 - 2*n)*Cos[a + b*x^n]^2,x]
```

```
[Out] -1/4*(1 + Cos[2*(a + b*x^n)] + 4*b^2*x^(2*n)*Cos[2*a]*CosIntegral[2*b*x^n]
- 2*b*x^n*Sin[2*(a + b*x^n)] - 4*b^2*x^(2*n)*Sin[2*a]*SinIntegral[2*b*x^n])
/(n*x^(2*n))
```

Maple [A] (verified)

Time = 2.74 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.94

method	result
default	$-\frac{x^{-2n}}{4n}+\frac{2b^2\left(-\frac{\cos(2a+2b\,x^n)x^{-2n}}{8b^2}+\frac{\sin(2a+2b\,x^n)x^{-n}}{4b}+\frac{\text{Si}(2b\,x^n)\sin(2a)}{2}-\frac{\text{Ci}(2b\,x^n)\cos(2a)}{2}\right)}{n}$
risch	$\frac{(-2ib^2\mathrm{e}^{-2ia}\pi\ \text{csgn}(b\,x^n)x^{2n}+4ib^2\mathrm{e}^{-2ia}\ \text{Si}(2b\,x^n)x^{2n}+2b^2\mathrm{e}^{2ia}\ \text{Ei}_1(-2ib\,x^n)x^{2n}+2b^2\mathrm{e}^{-2ia}\ \text{Ei}_1(-2ib\,x^n)x^{2n}+2b\sin(2a+2b\,x^n)x^n-\text{co}}{4n}$

```
[In] int(x^(-1-2*n)*cos(a+b*x^n)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/(x^n)^2/n+2/n*b^2*(-1/8*cos(2*a+2*b*x^n)/(x^n)^2/b^2+1/4*sin(2*a+2*b*x
^n)/(x^n)/b+1/2*Si(2*b*x^n)*sin(2*a)-1/2*Ci(2*b*x^n)*cos(2*a))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.92

$$\int x^{-1-2n}\cos^2\left(a+bx^n\right)\,dx = -\frac{2\,b^2x^{2\,n}\cos\left(2\,a\right)\mathrm{Ci}\left(2\,bx^n\right)-2\,b^2x^{2\,n}\sin\left(2\,a\right)\mathrm{Si}\left(2\,bx^n\right)-2\,bx^n\cos\left(bx^n+a\right)\sin\left(bx^n+a\right)+\cos\left(bx^n+\right.}{2\,nx^{2\,n}}$$

```
[In] integrate(x^(-1-2*n)*cos(a+b*x^n)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*b^2*x^(2*n)*cos(2*a)*cos_integral(2*b*x^n) - 2*b^2*x^(2*n)*sin(2*a)
*sin_integral(2*b*x^n) - 2*b*x^n*cos(b*x^n + a)*sin(b*x^n + a) + cos(b*x^n
+ a)^2)/(n*x^(2*n))
```

Sympy [F]

$$\int x^{-1-2n}\cos^2\left(a+bx^n\right)\,dx = \int x^{-2n-1}\cos^2\left(a+bx^n\right)dx$$

```
[In] integrate(x**(-1-2*n)*cos(a+b*x**n)**2,x)
```

```
[Out] Integral(x**(-2*n - 1)*cos(a + b*x**n)**2, x)
```

Maxima [F]

$$\int x^{-1-2n}\cos^2\left(a+bx^n\right)\,dx = \int x^{-2\,n-1}\cos\left(bx^n+a\right)^2\,dx$$

```
[In] integrate(x^(-1-2*n)*cos(a+b*x^n)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(2*n*x^(2*n)*integrate(cos(2*b*x^n + 2*a)/(x*x^(2*n)), x) - 1)/(n*x^(2*
n))
```

Giac [F]

$$\int x^{-1-2n} \cos^2(a + bx^n) \, dx = \int x^{-2n-1} \cos(bx^n + a)^2 \, dx$$

```
[In] integrate(x^(-1-2*n)*cos(a+b*x^n)^2,x, algorithm="giac")
[Out] integrate(x^(-2*n - 1)*cos(b*x^n + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1-2n} \cos^2(a + bx^n) \, dx = \int \frac{\cos(a + b\,x^n)^2}{x^{2\,n+1}} \, dx$$

```
[In] int(cos(a + b*x^n)^2/x^(2*n + 1),x)
[Out] int(cos(a + b*x^n)^2/x^(2*n + 1), x)
```

3.84 $\int x^{-1-2n}\cos^3(a+bx^n)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 165

$$\int x^{-1-2n}\cos^3(a+bx^n)\,dx=-\frac{3x^{-2n}\cos(a+bx^n)}{8n}-\frac{x^{-2n}\cos(3(a+bx^n))}{8n}-\frac{3b^2\cos(a)\,\text{CosIntegral}(bx^n)}{8n}-\frac{9b^2\cos(3a)\,\text{CosIntegral}(3bx^n)}{8n}+\frac{3bx^{-n}\sin(a+bx^n)}{8n}+\frac{3bx^{-n}\sin(3(a+bx^n))}{8n}+\frac{3b^2\sin(a)\text{Si}(bx^n)}{8n}+\frac{9b^2\sin(3a)\text{Si}(3bx^n)}{8n}$$

```
[Out] -3/8*b^2*Ci(b*x^n)*cos(a)/n-9/8*b^2*Ci(3*b*x^n)*cos(3*a)/n-3/8*cos(a+b*x^n)
/n/(x^(2*n))-1/8*cos(3*a+3*b*x^n)/n/(x^(2*n))+3/8*b^2*Si(b*x^n)*sin(a)/n+9/
8*b^2*Si(3*b*x^n)*sin(3*a)/n+3/8*b*sin(a+b*x^n)/n/(x^n)+3/8*b*sin(3*a+3*b*x
^n)/n/(x^n)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used

$= \{3507, 3461, 3378, 3384, 3380, 3383\}$

$$\int x^{-1-2n}\cos^3\left(a+bx^n\right)\,dx = -\frac{3b^2\cos(a)\,\text{CosIntegral}\left(bx^n\right)}{8n} - \frac{9b^2\cos(3a)\,\text{CosIntegral}\left(3bx^n\right)}{8n} + \frac{3b^2\sin(a)\text{Si}(bx^n)}{8n} + \frac{9b^2\sin(3a)\text{Si}(3bx^n)}{8n} + \frac{3bx^{-n}\sin\left(a+bx^n\right)}{8n} + \frac{3bx^{-n}\sin\left(3(a+bx^n)\right)}{8n} - \frac{3x^{-2n}\cos\left(a+bx^n\right)}{8n} - \frac{x^{-2n}\cos\left(3(a+bx^n)\right)}{8n}$$

```
[In] Int[x^(-1 - 2*n)*Cos[a + b*x^n]^3,x]
```

```
[Out] (-3*Cos[a + b*x^n])/(8*n*x^(2*n)) - Cos[3*(a + b*x^n)]/(8*n*x^(2*n)) - (3*b
^2*Cos[a]*CosIntegral[b*x^n])/(8*n) - (9*b^2*Cos[3*a]*CosIntegral[3*b*x^n])
/(8*n) + (3*b*Sin[a + b*x^n])/(8*n*x^n) + (3*b*Sin[3*(a + b*x^n)])/(8*n*x^n
) + (3*b^2*Sin[a]*SinIntegral[b*x^n])/(8*n) + (9*b^2*Sin[3*a]*SinIntegral[3
*b*x^n])/(8*n)
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
```

```
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3507

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_)*((e_.)*(x_))^(m_.), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Cos[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{3}{4}x^{-1-2n}\cos\left(a+bx^n\right)+\frac{1}{4}x^{-1-2n}\cos\left(3a+3bx^n\right)\right)\,dx \\
&= \frac{1}{4}\int x^{-1-2n}\cos\left(3a+3bx^n\right)\,dx+\frac{3}{4}\int x^{-1-2n}\cos\left(a+bx^n\right)\,dx \\
&= \frac{\text{Subst}\left(\int \frac{\cos(3a+3bx)}{x^3}\,dx,x,x^n\right)}{4n}+\frac{3\text{Subst}\left(\int \frac{\cos(a+bx)}{x^3}\,dx,x,x^n\right)}{4n} \\
&= -\frac{3x^{-2n}\cos\left(a+bx^n\right)}{8n}-\frac{x^{-2n}\cos\left(3(a+bx^n)\right)}{8n} \\
&\quad -\frac{(3b)\text{Subst}\left(\int \frac{\sin(a+bx)}{x^2}\,dx,x,x^n\right)}{8n}-\frac{(3b)\text{Subst}\left(\int \frac{\sin(3a+3bx)}{x^2}\,dx,x,x^n\right)}{8n} \\
&= -\frac{3x^{-2n}\cos\left(a+bx^n\right)}{8n}-\frac{x^{-2n}\cos\left(3(a+bx^n)\right)}{8n} \\
&\quad +\frac{3bx^{-n}\sin\left(a+bx^n\right)}{8n}+\frac{3bx^{-n}\sin\left(3(a+bx^n)\right)}{8n} \\
&\quad -\frac{(3b^2)\text{ Subst}\left(\int \frac{\cos(a+bx)}{x}\,dx,x,x^n\right)}{8n}-\frac{(9b^2)\text{ Subst}\left(\int \frac{\cos(3a+3bx)}{x}\,dx,x,x^n\right)}{8n} \\
&= -\frac{3x^{-2n}\cos\left(a+bx^n\right)}{8n}-\frac{x^{-2n}\cos\left(3(a+bx^n)\right)}{8n}+\frac{3bx^{-n}\sin\left(a+bx^n\right)}{8n} \\
&\quad +\frac{3bx^{-n}\sin\left(3(a+bx^n)\right)}{8n}-\frac{(3b^2\cos(a))\text{ Subst}\left(\int \frac{\cos(bx)}{x}\,dx,x,x^n\right)}{8n} \\
&\quad -\frac{(9b^2\cos(3a))\text{ Subst}\left(\int \frac{\cos(3bx)}{x}\,dx,x,x^n\right)}{8n} \\
&\quad +\frac{(3b^2\sin(a))\text{ Subst}\left(\int \frac{\sin(bx)}{x}\,dx,x,x^n\right)}{8n}+\frac{(9b^2\sin(3a))\text{ Subst}\left(\int \frac{\sin(3bx)}{x}\,dx,x,x^n\right)}{8n}
\end{aligned}$$

$$= -\frac{3x^{-2n}\cos(a+bx^n)}{8n} - \frac{x^{-2n}\cos(3(a+bx^n))}{8n} - \frac{3b^2\cos(a)\,\text{CosIntegral}(bx^n)}{8n} - \frac{9b^2\cos(3a)\,\text{CosIntegral}(3bx^n)}{8n} + \frac{3bx^{-n}\sin(a+bx^n)}{8n} + \frac{3bx^{-n}\sin(3(a+bx^n))}{8n} + \frac{3b^2\sin(a)\text{Si}(bx^n)}{8n} + \frac{9b^2\sin(3a)\text{Si}(3bx^n)}{8n}$$

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.85

$$\int x^{-1-2n}\cos^3(a+bx^n)\,dx = -\frac{x^{-2n}(3\cos(a+bx^n)+\cos(3(a+bx^n))+3b^2x^{2n}\cos(a)\,\text{CosIntegral}(bx^n)+9b^2x^{2n}\cos(3a)\,\text{CosIntegral}(\ldots}{8n}$$

```
[In] Integrate[x^(-1 - 2*n)*Cos[a + b*x^n]^3,x]
```

```
[Out] -1/8*(3*Cos[a + b*x^n] + Cos[3*(a + b*x^n)] + 3*b^2*x^(2*n)*Cos[a]*CosInteg
ral[b*x^n] + 9*b^2*x^(2*n)*Cos[3*a]*CosIntegral[3*b*x^n] - 3*b*x^n*Sin[a +
b*x^n] - 3*b*x^n*Sin[3*(a + b*x^n)] - 3*b^2*x^(2*n)*Sin[a]*SinIntegral[b*x^
n] - 9*b^2*x^(2*n)*Sin[3*a]*SinIntegral[3*b*x^n])/(n*x^(2*n))
```

Maple [A] (verified)

Time = 10.52 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.87

method	result
default	$\frac{3b^2\left(-\frac{\cos(a+b\,x^n)x^{-2n}}{2b^2}+\frac{\sin(a+b\,x^n)x^{-n}}{2b}+\frac{\text{Si}(b\,x^n)\sin(a)}{2}-\frac{\text{Ci}(b\,x^n)\cos(a)}{2}\right)}{4n}+\frac{9b^2\left(-\frac{\cos(3a+3b\,x^n)x^{-2n}}{18b^2}+\frac{\sin(3a+3b\,x^n)x^{-n}}{6b}+\frac{\text{Si}(3b\,x^n \ldots}{\ldots}\right)}{4n}$
risch	$-\frac{(9ib^2\text{e}^{-3ia}\pi\ \text{csgn}(b\,x^n)x^{2n}+3ib^2\text{e}^{-ia}\pi\ \text{csgn}(b\,x^n)x^{2n}-18ib^2\text{e}^{-3ia}\ \text{Si}(3b\,x^n)x^{2n}-6ib^2\text{e}^{-ia}\ \text{Si}(b\,x^n)x^{2n}-9b^2\text{e}^{-3ia}\ \text{Ei}_1(-3ib\,x^n)x^{2n} \ldots}{\ldots}$

```
[In] int(x^(-1-2*n)*cos(a+b*x^n)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 3/4/n*b^2*(-1/2*cos(a+b*x^n)/b^2/(x^n)^2+1/2*sin(a+b*x^n)/b/(x^n)+1/2*Si(b*
x^n)*sin(a)-1/2*Ci(b*x^n)*cos(a))+9/4/n*b^2*(-1/18*cos(3*a+3*b*x^n)/(x^n)^2
/b^2+1/6*sin(3*a+3*b*x^n)/(x^n)/b+1/2*Si(3*b*x^n)*sin(3*a)-1/2*Ci(3*b*x^n)*
cos(3*a))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.77

$$\int x^{-1-2n}\cos^3(a+bx^n)\,dx = -\frac{9\,b^2x^{2\,n}\cos(3\,a)\,\mathrm{Ci}(3\,bx^n)+3\,b^2x^{2\,n}\cos(a)\,\mathrm{Ci}(bx^n)-12\,bx^n\cos(bx^n+a)^2\sin(bx^n+a)-9\,b^2x^{2\,n}\sin(\cdots}{8\,nx^{2\,n}}$$

```
[In] integrate(x^(-1-2*n)*cos(a+b*x^n)^3,x, algorithm="fricas")
```

```
[Out] -1/8*(9*b^2*x^(2*n)*cos(3*a)*cos_integral(3*b*x^n) + 3*b^2*x^(2*n)*cos(a)*c
os_integral(b*x^n) - 12*b*x^n*cos(b*x^n + a)^2*sin(b*x^n + a) - 9*b^2*x^(2*
n)*sin(3*a)*sin_integral(3*b*x^n) - 3*b^2*x^(2*n)*sin(a)*sin_integral(b*x^n
) + 4*cos(b*x^n + a)^3)/(n*x^(2*n))
```

Sympy [F]

$$\int x^{-1-2n}\cos^3(a+bx^n)\,dx = \int x^{-2n-1}\cos^3(a+bx^n)\,dx$$

```
[In] integrate(x**(-1-2*n)*cos(a+b*x**n)**3,x)
```

```
[Out] Integral(x**(-2*n - 1)*cos(a + b*x**n)**3, x)
```

Maxima [F]

$$\int x^{-1-2n}\cos^3(a+bx^n)\,dx = \int x^{-2\,n-1}\cos(bx^n+a)^3\,dx$$

```
[In] integrate(x^(-1-2*n)*cos(a+b*x^n)^3,x, algorithm="maxima")
```

```
[Out] integrate(x^(-2*n - 1)*cos(b*x^n + a)^3, x)
```

Giac [F]

$$\int x^{-1-2n}\cos^3(a+bx^n)\,dx=\int x^{-2\,n-1}\cos\left(bx^n+a\right)^3\,dx$$

```
[In] integrate(x^(-1-2*n)*cos(a+b*x^n)^3,x, algorithm="giac")
[Out] integrate(x^(-2*n - 1)*cos(b*x^n + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1-2n}\cos^3(a+bx^n)\,dx=\int\frac{\cos\left(a+b\,x^n\right)^3}{x^{2\,n+1}}\,dx$$

```
[In] int(cos(a + b*x^n)^3/x^(2*n + 1),x)
[Out] int(cos(a + b*x^n)^3/x^(2*n + 1), x)
```

3.85 $\int x^2 \cos\left((a+bx)^2\right) dx$

Optimal result

Integrand size = 12, antiderivative size = 99

$$\int x^2 \cos\left((a+bx)^2\right) dx = \frac{a^2\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}(a+bx)\right)}{b^3} - \frac{\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}(a+bx)\right)}{2b^3} - \frac{a\sin\left((a+bx)^2\right)}{b^3} + \frac{(a+bx)\sin\left((a+bx)^2\right)}{2b^3}$$

```
[Out] -a*sin((b*x+a)^2)/b^3+1/2*(b*x+a)*sin((b*x+a)^2)/b^3+1/2*a^2*FresnelC((b*x+
a)*2^(1/2)/Pi^(1/2))*2^(1/2)*Pi^(1/2)/b^3-1/4*FresnelS((b*x+a)*2^(1/2)/Pi^(
1/2))*2^(1/2)*Pi^(1/2)/b^3
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3515, 3433, 3461, 2717, 3467, 3432}

$$\int x^2 \cos\left((a+bx)^2\right) dx = \frac{\sqrt{\frac{\pi}{2}}a^2\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}(a+bx)\right)}{b^3} - \frac{\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}(a+bx)\right)}{2b^3} - \frac{a\sin\left((a+bx)^2\right)}{b^3} + \frac{(a+bx)\sin\left((a+bx)^2\right)}{2b^3}$$

```
[In] Int[x^2*Cos[(a + b*x)^2],x]
```

```
[Out] (a^2*Sqrt[Pi/2]*FresnelC[Sqrt[2/Pi]*(a + b*x)])/b^3 - (Sqrt[Pi/2]*FresnelS[
Sqrt[2/Pi]*(a + b*x)])/(2*b^3) - (a*Sin[(a + b*x)^2])/b^3 + ((a + b*x)*Sin[
(a + b*x)^2])/(2*b^3)
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3467

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[e^(n
 - 1)*(e*x)^(m - n + 1)*(Sin[c + d*x^n]/(d*n)), x] - Dist[e^n*((m - n + 1)/
(d*n)), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3515

```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_.)*((g_
.) + (h_.)*(x_))^(m_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Cos[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \left(a^2\cos\left(x^2\right)-2ax\cos\left(x^2\right)+x^2\cos\left(x^2\right)\right)\,dx,x,a+bx\right)}{b^3}\\
&= \frac{\text{Subst}\left(\int x^2\cos\left(x^2\right)\,dx,x,a+bx\right)}{b^3}-\frac{(2a)\text{Subst}\left(\int x\cos\left(x^2\right)\,dx,x,a+bx\right)}{b^3}\\
&\quad+\frac{a^2\text{Subst}\left(\int \cos\left(x^2\right)\,dx,x,a+bx\right)}{b^3}
\end{aligned}$$

$$= \frac{a^2\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}(a+bx)\right)}{b^3} + \frac{(a+bx)\sin\left((a+bx)^2\right)}{2b^3} - \frac{\text{Subst}\left(\int \sin\left(x^2\right)\,dx, x, a+bx\right)}{2b^3} - \frac{a\text{Subst}\left(\int \cos(x)\,dx, x, (a+bx)^2\right)}{b^3}$$

$$= \frac{a^2\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}(a+bx)\right)}{b^3} - \frac{\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}(a+bx)\right)}{2b^3} - \frac{a\sin\left((a+bx)^2\right)}{b^3} + \frac{(a+bx)\sin\left((a+bx)^2\right)}{2b^3}$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.77

$$\int x^2\cos\left((a+bx)^2\right)\,dx = -\frac{-2a^2\sqrt{2\pi}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}(a+bx)\right) + \sqrt{2\pi}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}(a+bx)\right) + 2(a-bx)\sin\left((a+bx)^2\right)}{4b^3}$$

```
[In] Integrate[x^2*Cos[(a + b*x)^2],x]
[Out] -1/4*(-2*a^2*Sqrt[2*Pi]*FresnelC[Sqrt[2/Pi]*(a + b*x)] + Sqrt[2*Pi]*Fresnel
S[Sqrt[2/Pi]*(a + b*x)] + 2*(a - b*x)*Sin[(a + b*x)^2])/b^3
```

Maple [A] (verified)

Time = 1.82 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.32

method	result
default	$\frac{x\sin\left(x^2b^2+2abx+a^2\right)}{2b^2}-\frac{a\left(\frac{\sin\left(x^2b^2+2abx+a^2\right)}{2b^2}-\frac{a\sqrt{2}\,\sqrt{\pi}\,\mathrm{C}\left(\frac{\sqrt{2}\,\left(b^2x+ab\right)}{\sqrt{\pi}\,\sqrt{b^2}}\right)}{2b\sqrt{b^2}}\right)}{b}-\frac{\sqrt{2}\,\sqrt{\pi}\,\mathrm{S}\left(\frac{\sqrt{2}\,\left(b^2x+ab\right)}{\sqrt{\pi}\,\sqrt{b^2}}\right)}{4b^2\sqrt{b^2}}$
risch	$-\frac{a^2(-1)^{\frac{3}{4}}\sqrt{\pi}\,\mathrm{erf}\left(b(-1)^{\frac{1}{4}}x+(-1)^{\frac{1}{4}}a\right)}{4b^3}-\frac{(-1)^{\frac{1}{4}}\sqrt{\pi}\,\mathrm{erf}\left(b(-1)^{\frac{1}{4}}x+(-1)^{\frac{1}{4}}a\right)}{8b^3}-\frac{a^2\sqrt{\pi}\,\mathrm{erf}\left(-b\sqrt{-i}\,x+\frac{ia}{\sqrt{-i}}\right)}{4b^3\sqrt{-i}}-\frac{i\sqrt{\pi}\,\mathrm{erf}\left(-b\sqrt{-i}\right.}{8b^3\sqrt{-}}$
parts	$\frac{\sqrt{2}\,\sqrt{\pi}\,\mathrm{C}\left(\frac{\sqrt{2}\,\left(b^2x+ab\right)}{\sqrt{\pi}\,\sqrt{b^2}}\right)x^2}{2\sqrt{b^2}}-\frac{\sqrt{2}\,\pi^{\frac{3}{2}}\left(-\frac{\mathrm{C}\left(\frac{\sqrt{2}\,b^2x}{\sqrt{\pi}\,\sqrt{b^2}}+\frac{\sqrt{2}\,ab}{\sqrt{\pi}\,\sqrt{b^2}}\right)\mathrm{csgn}(b)\left(-\frac{\sqrt{\pi}\,\mathrm{csgn}(b)\left(\frac{\sqrt{2}\,b^2x}{\sqrt{\pi}\,\sqrt{b^2}}+\frac{\sqrt{2}\,ab}{\sqrt{\pi}\,\sqrt{b^2}}\right)^2}{2}+\left(\frac{\sqrt{2}\,b^2x}{\sqrt{\pi}\,\sqrt{b^2}}+\frac{\sqrt{2}\,ab}{\sqrt{\pi}\,\sqrt{b^2}}\right)\sqrt{2}\,a\right.}{\sqrt{\pi}}\right.}{}$

[In] int(x^2*cos((b*x+a)^2),x,method=_RETURNVERBOSE)

[Out] 1/2/b^2*x*sin(b^2*x^2+2*a*b*x+a^2)-a/b*(1/2/b^2*sin(b^2*x^2+2*a*b*x+a^2)-1/2*a/b*2^(1/2)*Pi^(1/2)/(b^2)^(1/2)*FresnelC(2^(1/2)/Pi^(1/2)/(b^2)^(1/2)*(b^2*x+a*b)))-1/4/b^2*2^(1/2)*Pi^(1/2)/(b^2)^(1/2)*FresnelS(2^(1/2)/Pi^(1/2)/(b^2)^(1/2)*(b^2*x+a*b))

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.13

$$\int x^2\cos\left((a+bx)^2\right)\,dx$$
$$=\frac{2\sqrt{2}\pi a^2\sqrt{\frac{b^2}{\pi}}\,\mathrm{C}\left(\frac{\sqrt{2}(bx+a)\sqrt{\frac{b^2}{\pi}}}{b}\right)-\sqrt{2}\pi\sqrt{\frac{b^2}{\pi}}\,\mathrm{S}\left(\frac{\sqrt{2}(bx+a)\sqrt{\frac{b^2}{\pi}}}{b}\right)+2\left(b^2x-ab\right)\sin\left(b^2x^2+2\,abx+a^2\right)}{4\,b^4}$$

[In] integrate(x^2*cos((b*x+a)^2),x, algorithm="fricas")

[Out] 1/4*(2*sqrt(2)*pi*a^2*sqrt(b^2/pi)*fresnel_cos(sqrt(2)*(b*x + a)*sqrt(b^2/pi)/b) - sqrt(2)*pi*sqrt(b^2/pi)*fresnel_sin(sqrt(2)*(b*x + a)*sqrt(b^2/pi)/b) + 2*(b^2*x - a*b)*sin(b^2*x^2 + 2*a*b*x + a^2))/b^4

Sympy [F]

$$\int x^2 \cos \left((a+bx)^2\right) \, dx = \int x^2 \cos \left(a^2 + 2abx + b^2x^2\right) dx$$

```
[In] integrate(x**2*cos((b*x+a)**2),x)
[Out] Integral(x**2*cos(a**2 + 2*a*b*x + b**2*x**2), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.63 (sec) , antiderivative size = 258, normalized size of antiderivative = 2.61

$$\int x^2 \cos \left((a+bx)^2\right) \, dx = -\frac{4\,abx\left(-i\,e^{\left(i\,b^2x^2+2i\,abx+i\,a^2\right)}+i\,e^{\left(-i\,b^2x^2-2i\,abx-i\,a^2\right)}\right)+4\,a^2\left(-i\,e^{\left(i\,b^2x^2+2i\,abx+i\,a^2\right)}+i\,e^{\left(-i\,b^2x^2-2i\,abx-i\,a^2\right)}\right)-$$

```
[In] integrate(x^2*cos((b*x+a)^2),x, algorithm="maxima")
[Out] -1/8*(4*a*b*x*(-I*e^(I*b^2*x^2 + 2*I*a*b*x + I*a^2) + I*e^(-I*b^2*x^2 - 2*I
*a*b*x - I*a^2)) + 4*a^2*(-I*e^(I*b^2*x^2 + 2*I*a*b*x + I*a^2) + I*e^(-I*b^
2*x^2 - 2*I*a*b*x - I*a^2)) - sqrt(b^2*x^2 + 2*a*b*x + a^2)*((-(I - 1)*sqrt
(2)*sqrt(pi)*(erf(sqrt(I*b^2*x^2 + 2*I*a*b*x + I*a^2)) - 1) + (I + 1)*sqrt(
2)*sqrt(pi)*(erf(sqrt(-I*b^2*x^2 - 2*I*a*b*x - I*a^2)) - 1))*a^2 + (I + 1)*
sqrt(2)*gamma(3/2, I*b^2*x^2 + 2*I*a*b*x + I*a^2) - (I - 1)*sqrt(2)*gamma(3
/2, -I*b^2*x^2 - 2*I*a*b*x - I*a^2)))/(b^4*x + a*b^3)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.61

$$\int x^2 \cos \left((a+bx)^2\right) \, dx = -\frac{\frac{(i+1)\,\sqrt{2}\sqrt{\pi}\left(2\,a^2+i\right)\,\mathrm{erf}\left(\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}\left(x+\frac{a}{b}\right)|b|\right)}{|b|}+\frac{4\left(i\,b\left(x+\frac{a}{b}\right)-2i\,a\right)e^{\left(i\,b^2x^2+2i\,abx+i\,a^2\right)}}{b}}{16\,b^2} - \frac{-\frac{(i-1)\,\sqrt{2}\sqrt{\pi}\left(2\,a^2-i\right)\,\mathrm{erf}\left(-\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}\left(x+\frac{a}{b}\right)|b|\right)}{|b|}+\frac{4\left(-i\,b\left(x+\frac{a}{b}\right)+2i\,a\right)e^{\left(-i\,b^2x^2-2i\,abx-i\,a^2\right)}}{b}}{16\,b^2}$$

```
[In] integrate(x^2*cos((b*x+a)^2),x, algorithm="giac")
```

```
[Out] -1/16*((I + 1)*sqrt(2)*sqrt(pi)*(2*a^2 + I)*erf((1/2*I - 1/2)*sqrt(2)*(x +
a/b)*abs(b))/abs(b) + 4*(I*b*(x + a/b) - 2*I*a)*e^(I*b^2*x^2 + 2*I*a*b*x +
I*a^2)/b)/b^2 - 1/16*(-(I - 1)*sqrt(2)*sqrt(pi)*(2*a^2 - I)*erf(-(1/2*I + 1
/2)*sqrt(2)*(x + a/b)*abs(b))/abs(b) + 4*(-I*b*(x + a/b) + 2*I*a)*e^(-I*b^2
*x^2 - 2*I*a*b*x - I*a^2)/b)/b^2
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.81

$$\int x^2 \cos\left((a+bx)^2\right)\,dx = \frac{x\,\sin\left((a+b\,x)^2\right)}{2\,b^2} - \frac{a\,\sin\left((a+b\,x)^2\right)}{2\,b^3} - \frac{\sqrt{2}\,\sqrt{\pi}\,\mathrm{S}\left(\frac{\sqrt{2}\,(a+b\,x)}{\sqrt{\pi}}\right)}{4\,b^3} + \frac{\sqrt{2}\,a^2\,\sqrt{\pi}\,\mathrm{C}\left(\frac{\sqrt{2}\,(a+b\,x)}{\sqrt{\pi}}\right)}{2\,b^3}$$

```
[In] int(x^2*cos((a + b*x)^2),x)
```

```
[Out] (x*sin((a + b*x)^2))/(2*b^2) - (a*sin((a + b*x)^2))/(2*b^3) - (2^(1/2)*pi^(
1/2)*fresnels((2^(1/2)*(a + b*x))/pi^(1/2)))/(4*b^3) + (2^(1/2)*a^2*pi^(1/2
)*fresnelc((2^(1/2)*(a + b*x))/pi^(1/2)))/(2*b^3)
```

3.86 $\int x \cos\left((a+bx)^2\right) \, dx$

Optimal result

Integrand size = 10, antiderivative size = 47

$$\int x \cos\left((a+bx)^2\right) \, dx = -\frac{a\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}(a+bx)\right)}{b^2} + \frac{\sin\left((a+bx)^2\right)}{2b^2}$$

```
[Out] 1/2*sin((b*x+a)^2)/b^2-1/2*a*FresnelC((b*x+a)*2^(1/2)/Pi^(1/2))*2^(1/2)*Pi^
(1/2)/b^2
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3515, 3433, 3461, 2717}

$$\int x \cos\left((a+bx)^2\right) \, dx = \frac{\sin\left((a+bx)^2\right)}{2b^2} - \frac{\sqrt{\frac{\pi}{2}}a\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}(a+bx)\right)}{b^2}$$

```
[In] Int[x*Cos[(a + b*x)^2],x]
```

```
[Out] -((a*Sqrt[Pi/2]*FresnelC[Sqrt[2/Pi]*(a + b*x)])/b^2) + Sin[(a + b*x)^2]/(2*
b^2)
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3515

```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_.)*((g_
.) + (h_.)*(x_))^(m_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Cos[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \left(-a\cos\left(x^2\right)+x\cos\left(x^2\right)\right)\,dx,x,a+bx\right)}{b^2}\\
&= \frac{\text{Subst}\left(\int x\cos\left(x^2\right)\,dx,x,a+bx\right)}{b^2}-\frac{a\text{Subst}\left(\int \cos\left(x^2\right)\,dx,x,a+bx\right)}{b^2}\\
&= -\frac{a\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}(a+bx)\right)}{b^2}+\frac{\text{Subst}\left(\int \cos(x)\,dx,x,(a+bx)^2\right)}{2b^2}\\
&= -\frac{a\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}(a+bx)\right)}{b^2}+\frac{\sin\left((a+bx)^2\right)}{2b^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.89

$$
\int x\cos\left((a+bx)^2\right)\,dx=\frac{-a\sqrt{2\pi}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}(a+bx)\right)+\sin\left((a+bx)^2\right)}{2b^2}
$$

```
[In] Integrate[x*Cos[(a + b*x)^2],x]
[Out] (-(a*Sqrt[2*Pi]*FresnelC[Sqrt[2/Pi]*(a + b*x)]) + Sin[(a + b*x)^2])/(2*b^2)
```

Maple [A] (verified)

Time = 1.18 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.34

method	result	size
default	$\frac{\sin\left(x^{2} b^{2}+2 a b x+a^{2}\right)}{2 b^{2}}-\frac{a \sqrt{2}\, \sqrt{\pi}\, \mathrm{C}\left(\frac{\sqrt{2}\, \left(b^{2} x+a b\right)}{\sqrt{\pi}\, \sqrt{b^{2}}}\right)}{2 b \sqrt{b^{2}}}$	63
risch	$\frac{\left(-1\right)^{\frac{3}{4}} a \sqrt{\pi}\, \mathrm{erf}\left(b \left(-1\right)^{\frac{1}{4}} x+\left(-1\right)^{\frac{1}{4}} a\right)}{4 b^{2}}+\frac{a \sqrt{\pi}\, \mathrm{erf}\left(-b \sqrt{-i}\, x+\frac{i a}{\sqrt{-i}}\right)}{4 b^{2} \sqrt{-i}}+\frac{\sin\left(\left(b x+a\right)^{2}\right)}{2 b^{2}}$	71
parts	$\frac{\sqrt{2}\, \sqrt{\pi}\, \mathrm{C}\left(\frac{\sqrt{2}\, \left(b^{2} x+a b\right)}{\sqrt{\pi}\, \sqrt{b^{2}}}\right) x}{2 \sqrt{b^{2}}}-\frac{\pi \left(\mathrm{C}\left(\frac{\sqrt{2}\, b^{2} x}{\sqrt{\pi}\, \sqrt{b^{2}}}+\frac{\sqrt{2}\, a b}{\sqrt{\pi}\, \sqrt{b^{2}}}\right) \left(\frac{\sqrt{2}\, b^{2} x}{\sqrt{\pi}\, \sqrt{b^{2}}}+\frac{\sqrt{2}\, a b}{\sqrt{\pi}\, \sqrt{b^{2}}}\right)-\frac{\sin\left(\frac{\pi \left(\frac{\sqrt{2}\, b^{2} x}{\sqrt{\pi}\, \sqrt{b^{2}}}+\frac{\sqrt{2}\, a b}{\sqrt{\pi}\, \sqrt{b^{2}}}\right)^{2}}{2}\right)}{\pi}\right)}{2 b^{2}}$	151

```
[In] int(x*cos((b*x+a)^2),x,method=_RETURNVERBOSE)
[Out] 1/2/b^2*sin(b^2*x^2+2*a*b*x+a^2)-1/2*a/b*2^(1/2)*Pi^(1/2)/(b^2)^(1/2)*Fresn
elC(2^(1/2)/Pi^(1/2)/(b^2)^(1/2)*(b^2*x+a*b))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.34

$$\int x \cos\left((a+bx)^2\right)\,dx=-\frac{\sqrt{2}\pi a\sqrt{\frac{b^2}{\pi}}\,\mathrm{C}\left(\frac{\sqrt{2}(bx+a)\sqrt{\frac{b^2}{\pi}}}{b}\right)-b\sin\left(b^2x^2+2\,abx+a^2\right)}{2\,b^3}$$

```
[In] integrate(x*cos((b*x+a)^2),x, algorithm="fricas")
[Out] -1/2*(sqrt(2)*pi*a*sqrt(b^2/pi)*fresnel_cos(sqrt(2)*(b*x + a)*sqrt(b^2/pi)/
b) - b*sin(b^2*x^2 + 2*a*b*x + a^2))/b^3
```

Sympy [F]

$$\int x \cos\left((a+bx)^2\right)\,dx=\int x\cos\left(a^2+2abx+b^2x^2\right)dx$$

```
[In] integrate(x*cos((b*x+a)**2),x)
[Out] Integral(x*cos(a**2 + 2*a*b*x + b**2*x**2), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.56 (sec) , antiderivative size = 199, normalized size of antiderivative = 4.23

$$\int x\cos\left((a+bx)^2\right)\,dx = \frac{2\,bx\left(-i\,e^{\left(i\,b^2x^2+2i\,abx+i\,a^2\right)}+i\,e^{\left(-i\,b^2x^2-2i\,abx-i\,a^2\right)}\right)-\sqrt{b^2x^2+2\,abx+a^2}\left(-(i-1)\,\sqrt{2}\sqrt{\pi}\left(\operatorname{erf}\left(\sqrt{i\,b^2x^2+2i\right.\right.\right.}{}$$

```
[In] integrate(x*cos((b*x+a)^2),x, algorithm="maxima")
```

```
[Out] 1/8*(2*b*x*(-I*e^(I*b^2*x^2 + 2*I*a*b*x + I*a^2) + I*e^(-I*b^2*x^2 - 2*I*a*
b*x - I*a^2)) - sqrt(b^2*x^2 + 2*a*b*x + a^2)*(-(I - 1)*sqrt(2)*sqrt(pi)*(e
rf(sqrt(I*b^2*x^2 + 2*I*a*b*x + I*a^2)) - 1) + (I + 1)*sqrt(2)*sqrt(pi)*(er
f(sqrt(-I*b^2*x^2 - 2*I*a*b*x - I*a^2)) - 1))*a + 2*a*(-I*e^(I*b^2*x^2 + 2*
I*a*b*x + I*a^2) + I*e^(-I*b^2*x^2 - 2*I*a*b*x - I*a^2)))/(b^3*x + a*b^2)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.39 (sec) , antiderivative size = 119, normalized size of antiderivative = 2.53

$$\int x\cos\left((a+bx)^2\right)\,dx = -\frac{-\frac{(i+1)\,\sqrt{2}\sqrt{\pi}a\operatorname{erf}\left(\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}\left(x+\frac{a}{b}\right)|b|\right)}{|b|}+\frac{2i\,e^{\left(i\,b^2x^2+2i\,abx+i\,a^2\right)}}{b}}{8\,b} - \frac{\frac{(i-1)\,\sqrt{2}\sqrt{\pi}a\operatorname{erf}\left(-\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}\left(x+\frac{a}{b}\right)|b|\right)}{|b|}-\frac{2i\,e^{\left(-i\,b^2x^2-2i\,abx-i\,a^2\right)}}{b}}{8\,b}$$

```
[In] integrate(x*cos((b*x+a)^2),x, algorithm="giac")
```

```
[Out] -1/8*(-(I + 1)*sqrt(2)*sqrt(pi)*a*erf((1/2*I - 1/2)*sqrt(2)*(x + a/b)*abs(b
))/abs(b) + 2*I*e^(I*b^2*x^2 + 2*I*a*b*x + I*a^2)/b)/b - 1/8*((I - 1)*sqrt(
2)*sqrt(pi)*a*erf(-(1/2*I + 1/2)*sqrt(2)*(x + a/b)*abs(b))/abs(b) - 2*I*e^(
-I*b^2*x^2 - 2*I*a*b*x - I*a^2)/b)/b
```

Mupad [B] (verification not implemented)

Time = 13.29 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.83

$$\int x\cos \left ((a+bx)^2\right) \, dx=\frac {\sin \left ({\left (a+b\,x\right)}^2\right)}{2\,b^2}-\frac {\sqrt {2}\,a\,\sqrt {\pi }\,\mathrm {C}\left (\frac {\sqrt {2}\,\left (a+b\,x\right)}{\sqrt {\pi }}\right)}{2\,b^2}$$

```
[In] int(x*cos((a + b*x)^2),x)
```

```
[Out] sin((a + b*x)^2)/(2*b^2) - (2^(1/2)*a*pi^(1/2)*fresnelc((2^(1/2)*(a + b*x))
/pi^(1/2)))/(2*b^2)
```

3.87 $\int \cos\left((a+bx)^2\right) dx$

Optimal result

Integrand size = 8, antiderivative size = 29

$$\int \cos\left((a+bx)^2\right) dx = \frac{\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}(a+bx)\right)}{b}$$

```
[Out] 1/2*FresnelC((b*x+a)*2^(1/2)/Pi^(1/2))*2^(1/2)*Pi^(1/2)/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3433}

$$\int \cos\left((a+bx)^2\right) dx = \frac{\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}(a+bx)\right)}{b}$$

```
[In] Int[Cos[(a + b*x)^2],x]
```

```
[Out] (Sqrt[Pi/2]*FresnelC[Sqrt[2/Pi]*(a + b*x)])/b
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}(a+bx)\right)}{b}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \cos\left((a+bx)^2\right)\,dx=\frac{\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}(a+bx)\right)}{b}$$

```
[In] Integrate[Cos[(a + b*x)^2],x]
[Out] (Sqrt[Pi/2]*FresnelC[Sqrt[2/Pi]*(a + b*x)])/b
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.24

method	result	size
default	$\frac{\sqrt{2}\,\sqrt{\pi}\,\mathrm{C}\left(\frac{\sqrt{2}\left(b^{2}x+ab\right)}{\sqrt{\pi}\,\sqrt{b^{2}}}\right)}{2\sqrt{b^{2}}}$	36
risch	$-\frac{\sqrt{\pi}\,(-1)^{\frac{3}{4}}\,\mathrm{erf}\left(b(-1)^{\frac{1}{4}}x+(-1)^{\frac{1}{4}}a\right)}{4b}-\frac{\sqrt{\pi}\,\mathrm{erf}\left(-b\sqrt{-i}\,x+\frac{ia}{\sqrt{-i}}\right)}{4b\sqrt{-i}}$	56

```
[In] int(cos((b*x+a)^2),x,method=_RETURNVERBOSE)
[Out] 1/2*2^(1/2)*Pi^(1/2)/(b^2)^(1/2)*FresnelC(2^(1/2)/Pi^(1/2)/(b^2)^(1/2)*(b^2
*x+a*b))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.38

$$\int \cos\left((a+bx)^2\right)\,dx=\frac{\sqrt{2}\pi\sqrt{\frac{b^2}{\pi}}\,\mathrm{C}\left(\frac{\sqrt{2}(bx+a)\sqrt{\frac{b^2}{\pi}}}{b}\right)}{2\,b^2}$$

```
[In] integrate(cos((b*x+a)^2),x, algorithm="fricas")
[Out] 1/2*sqrt(2)*pi*sqrt(b^2/pi)*fresnel_cos(sqrt(2)*(b*x + a)*sqrt(b^2/pi)/b)/b
^2
```

Sympy [F]

$$\int \cos \left((a+bx)^2\right) \, dx = \int \cos \left((a+bx)^2\right) dx$$

```
[In] integrate(cos((b*x+a)**2),x)
```

```
[Out] Integral(cos((a + b*x)**2), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 84, normalized size of antiderivative = 2.90

$$\int \cos \left((a+bx)^2\right) \, dx =$$
$$-\frac{\sqrt{\pi}\left((i-1)\sqrt{2}\,\mathrm{erf}\left(-(-1)^{\frac{3}{4}}(i\,bx+i\,a)\right)-(i+1)\sqrt{2}\,\mathrm{erf}\left(\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}(-i\,bx-i\,a)\right)+(i-1)\sqrt{2}\,\mathrm{erf}\,(}{16\,b}$$

```
[In] integrate(cos((b*x+a)^2),x, algorithm="maxima")
```

```
[Out] -1/16*sqrt(pi)*((I - 1)*sqrt(2)*erf(-(-1)^(3/4)*(I*b*x + I*a)) - (I + 1)*sq
rt(2)*erf((1/2*I + 1/2)*sqrt(2)*(-I*b*x - I*a)) + (I - 1)*sqrt(2)*erf((1/2*
I - 1/2)*sqrt(2)*(-I*b*x - I*a)) + (I + 1)*sqrt(2)*erf((I*b*x + I*a)/sqrt(-
I)))/b
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.90

$$\int \cos \left((a+bx)^2\right) \, dx = -\frac{(i+1)\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}\left(x+\frac{a}{b}\right)|b|\right)}{8\,|b|} + \frac{(i-1)\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(-\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}\left(x+\frac{a}{b}\right)|b|\right)}{8\,|b|}$$

```
[In] integrate(cos((b*x+a)^2),x, algorithm="giac")
```

```
[Out] -(1/8*I + 1/8)*sqrt(2)*sqrt(pi)*erf((1/2*I - 1/2)*sqrt(2)*(x + a/b)*abs(b))
/abs(b) + (1/8*I - 1/8)*sqrt(2)*sqrt(pi)*erf(-(1/2*I + 1/2)*sqrt(2)*(x + a/
b)*abs(b))/abs(b)
```

Mupad [B] (verification not implemented)

Time = 13.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.10

$$\int \cos \left((a+bx)^2\right) \, dx = \frac{\sqrt{2}\,\sqrt{\pi}\,\mathrm{C}\left(\frac{\sqrt{2}\,b\,\sqrt{\frac{1}{b^2}}\,(a+b\,x)}{\sqrt{\pi}}\right)\,\sqrt{\frac{1}{b^2}}}{2}$$

```
[In] int(cos((a + b*x)^2),x)
```

```
[Out] (2^(1/2)*pi^(1/2)*fresnelc((2^(1/2)*b*(1/b^2)^(1/2)*(a + b*x))/pi^(1/2))*(1
/b^2)^(1/2))/2
```

3.88 $\int \frac{\cos((a+bx)^2)}{x}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{\cos((a+bx)^2)}{x}\,dx = \text{Int}\left(\frac{\cos((a+bx)^2)}{x},x\right)$$

```
[Out] Unintegrable(cos((b*x+a)^2)/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\cos((a+bx)^2)}{x}\,dx = \int \frac{\cos((a+bx)^2)}{x}\,dx$$

```
[In] Int[Cos[(a + b*x)^2]/x,x]
```

```
[Out] Defer[Int][Cos[(a + b*x)^2]/x, x]
```

Rubi steps

$$\text{integral} = \int \frac{\cos((a+bx)^2)}{x}\,dx$$

Mathematica [N/A]

Not integrable

Time = 2.30 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\cos \left((a+bx)^2\right)}{x} \, dx=\int \frac{\cos \left((a+bx)^2\right)}{x} \, dx$$

```
[In] Integrate[Cos[(a + b*x)^2]/x,x]
[Out] Integrate[Cos[(a + b*x)^2]/x, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.15 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\cos \left(\left(bx+a\right)^{2}\right)}{x}dx$$

```
[In] int(cos((b*x+a)^2)/x,x)
[Out] int(cos((b*x+a)^2)/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.92

$$\int \frac{\cos \left((a+bx)^2\right)}{x} \, dx=\int \frac{\cos \left({\left(bx+a\right)}^{2}\right)}{x} \, dx$$

```
[In] integrate(cos((b*x+a)^2)/x,x, algorithm="fricas")
[Out] integral(cos(b^2*x^2 + 2*a*b*x + a^2)/x, x)
```

Sympy [N/A]

Not integrable

Time = 0.99 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.67

$$\int \frac{\cos \left((a+bx)^2\right)}{x} \, dx=\int \frac{\cos \left(a^{2}+2abx+b^{2}x^{2}\right)}{x}\, dx$$

```
[In] integrate(cos((b*x+a)**2)/x,x)
[Out] Integral(cos(a**2 + 2*a*b*x + b**2*x**2)/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\cos \left((a+bx)^2\right)}{x}\,dx=\int \frac{\cos \left((bx+a)^2\right)}{x}\,dx$$

```
[In] integrate(cos((b*x+a)^2)/x,x, algorithm="maxima")
[Out] integrate(cos((b*x + a)^2)/x, x)
```

Giac [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\cos \left((a+bx)^2\right)}{x}\,dx=\int \frac{\cos \left((bx+a)^2\right)}{x}\,dx$$

```
[In] integrate(cos((b*x+a)^2)/x,x, algorithm="giac")
[Out] integrate(cos((b*x + a)^2)/x, x)
```

Mupad [N/A]

Not integrable

Time = 13.15 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\cos \left((a+bx)^2\right)}{x}\,dx=\int \frac{\cos \left((a+b\,x)^2\right)}{x}\,dx$$

```
[In] int(cos((a + b*x)^2)/x,x)
[Out] int(cos((a + b*x)^2)/x, x)
```

3.89 $\int \frac{\cos\left((a+bx)^2\right)}{x^2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{\cos\left((a+bx)^2\right)}{x^2}\,dx = \text{Int}\left(\frac{\cos\left((a+bx)^2\right)}{x^2},x\right)$$

```
[Out] Unintegrable(cos((b*x+a)^2)/x^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\cos\left((a+bx)^2\right)}{x^2}\,dx = \int \frac{\cos\left((a+bx)^2\right)}{x^2}\,dx$$

```
[In] Int[Cos[(a + b*x)^2]/x^2,x]
[Out] Defer[Int][Cos[(a + b*x)^2]/x^2, x]
```

Rubi steps

$$\text{integral} = \int \frac{\cos\left((a+bx)^2\right)}{x^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 3.93 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\cos \left((a+bx)^2\right)}{x^2} \, dx = \int \frac{\cos \left((a+bx)^2\right)}{x^2} \, dx$$

```
[In] Integrate[Cos[(a + b*x)^2]/x^2,x]
[Out] Integrate[Cos[(a + b*x)^2]/x^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.16 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\cos \left((bx+a)^2\right)}{x^2}dx$$

```
[In] int(cos((b*x+a)^2)/x^2,x)
[Out] int(cos((b*x+a)^2)/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.92

$$\int \frac{\cos \left((a+bx)^2\right)}{x^2} \, dx = \int \frac{\cos \left((bx+a)^2\right)}{x^2} \, dx$$

```
[In] integrate(cos((b*x+a)^2)/x^2,x, algorithm="fricas")
[Out] integral(cos(b^2*x^2 + 2*a*b*x + a^2)/x^2, x)
```

Sympy [N/A]

Not integrable

Time = 0.95 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.83

$$\int \frac{\cos \left((a+bx)^2\right)}{x^2} \, dx = \int \frac{\cos \left(a^2+2abx+b^2x^2\right)}{x^2} \, dx$$

```
[In] integrate(cos((b*x+a)**2)/x**2,x)
[Out] Integral(cos(a**2 + 2*a*b*x + b**2*x**2)/x**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\cos \left((a+bx)^2\right)}{x^2} \, dx = \int \frac{\cos \left((bx+a)^2\right)}{x^2} \, dx$$

```
[In] integrate(cos((b*x+a)^2)/x^2,x, algorithm="maxima")
[Out] integrate(cos((b*x + a)^2)/x^2, x)
```

Giac [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\cos \left((a+bx)^2\right)}{x^2} \, dx = \int \frac{\cos \left((bx+a)^2\right)}{x^2} \, dx$$

```
[In] integrate(cos((b*x+a)^2)/x^2,x, algorithm="giac")
[Out] integrate(cos((b*x + a)^2)/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 13.09 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\cos \left((a+bx)^2\right)}{x^2} \, dx = \int \frac{\cos \left((a+b\,x)^2\right)}{x^2} \, dx$$

```
[In] int(cos((a + b*x)^2)/x^2,x)
[Out] int(cos((a + b*x)^2)/x^2, x)
```

3.90 $\int x^2 \cos \left(a+b\sqrt{c+dx}\right) \, dx$

Optimal result

Integrand size = 18, antiderivative size = 346

$$\begin{aligned}\int x^2 \cos \left(a+b\sqrt{c+dx}\right) \, dx &= \frac{240 \cos \left(a+b\sqrt{c+dx}\right)}{b^6 d^3}+\frac{24 c \cos \left(a+b\sqrt{c+dx}\right)}{b^4 d^3} \\ &+\frac{2 c^2 \cos \left(a+b\sqrt{c+dx}\right)}{b^2 d^3}-\frac{120 (c+dx) \cos \left(a+b\sqrt{c+dx}\right)}{b^4 d^3} \\ &-\frac{12 c (c+dx) \cos \left(a+b\sqrt{c+dx}\right)}{b^2 d^3} \\ &+\frac{10 (c+dx)^2 \cos \left(a+b\sqrt{c+dx}\right)}{b^2 d^3} \\ &+\frac{240 \sqrt{c+dx} \sin \left(a+b\sqrt{c+dx}\right)}{b^5 d^3} \\ &+\frac{24 c \sqrt{c+dx} \sin \left(a+b\sqrt{c+dx}\right)}{b^3 d^3} \\ &+\frac{2 c^2 \sqrt{c+dx} \sin \left(a+b\sqrt{c+dx}\right)}{b d^3} \\ &-\frac{40 (c+dx)^{3/2} \sin \left(a+b\sqrt{c+dx}\right)}{b^3 d^3} \\ &-\frac{4 c (c+dx)^{3/2} \sin \left(a+b\sqrt{c+dx}\right)}{b d^3} \\ &+\frac{2 (c+dx)^{5/2} \sin \left(a+b\sqrt{c+dx}\right)}{b d^3}\end{aligned}$$

```
[Out] 240*cos(a+b*(d*x+c)^(1/2))/b^6/d^3+24*c*cos(a+b*(d*x+c)^(1/2))/b^4/d^3+2*c^
2*cos(a+b*(d*x+c)^(1/2))/b^2/d^3-120*(d*x+c)*cos(a+b*(d*x+c)^(1/2))/b^4/d^3
-12*c*(d*x+c)*cos(a+b*(d*x+c)^(1/2))/b^2/d^3+10*(d*x+c)^2*cos(a+b*(d*x+c)^(
1/2))/b^2/d^3-40*(d*x+c)^(3/2)*sin(a+b*(d*x+c)^(1/2))/b^3/d^3-4*c*(d*x+c)^(
3/2)*sin(a+b*(d*x+c)^(1/2))/b/d^3+2*(d*x+c)^(5/2)*sin(a+b*(d*x+c)^(1/2))/b/
```

```
d^3+240*sin(a+b*(d*x+c)^(1/2))*(d*x+c)^(1/2)/b^5/d^3+24*c*sin(a+b*(d*x+c)^(
1/2))*(d*x+c)^(1/2)/b^3/d^3+2*c^2*sin(a+b*(d*x+c)^(1/2))*(d*x+c)^(1/2)/b/d^
3
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 346, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3513, 3377, 2718}

$$\begin{aligned}\int x^2\cos\left(a+b\sqrt{c+dx}\right)dx = &\frac{240\cos\left(a+b\sqrt{c+dx}\right)}{b^6d^3}+\frac{240\sqrt{c+dx}\sin\left(a+b\sqrt{c+dx}\right)}{b^5d^3}\\&-\frac{120(c+dx)\cos\left(a+b\sqrt{c+dx}\right)}{b^4d^3}+\frac{24c\cos\left(a+b\sqrt{c+dx}\right)}{b^4d^3}\\&-\frac{40(c+dx)^{3/2}\sin\left(a+b\sqrt{c+dx}\right)}{b^3d^3}\\&+\frac{24c\sqrt{c+dx}\sin\left(a+b\sqrt{c+dx}\right)}{b^3d^3}\\&+\frac{2c^2\cos\left(a+b\sqrt{c+dx}\right)}{b^2d^3}+\frac{10(c+dx)^2\cos\left(a+b\sqrt{c+dx}\right)}{b^2d^3}\\&-\frac{12c(c+dx)\cos\left(a+b\sqrt{c+dx}\right)}{b^2d^3}\\&+\frac{2c^2\sqrt{c+dx}\sin\left(a+b\sqrt{c+dx}\right)}{bd^3}\\&+\frac{2(c+dx)^{5/2}\sin\left(a+b\sqrt{c+dx}\right)}{bd^3}\\&-\frac{4c(c+dx)^{3/2}\sin\left(a+b\sqrt{c+dx}\right)}{bd^3}\end{aligned}$$

```
[In] Int[x^2*Cos[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (240*Cos[a + b*Sqrt[c + d*x]])/(b^6*d^3) + (24*c*Cos[a + b*Sqrt[c + d*x]])/
(b^4*d^3) + (2*c^2*Cos[a + b*Sqrt[c + d*x]])/(b^2*d^3) - (120*(c + d*x)*Cos
[a + b*Sqrt[c + d*x]])/(b^4*d^3) - (12*c*(c + d*x)*Cos[a + b*Sqrt[c + d*x]]
)/(b^2*d^3) + (10*(c + d*x)^2*Cos[a + b*Sqrt[c + d*x]])/(b^2*d^3) + (240*Sq
rt[c + d*x]*Sin[a + b*Sqrt[c + d*x]])/(b^5*d^3) + (24*c*Sqrt[c + d*x]*Sin[a
 + b*Sqrt[c + d*x]])/(b^3*d^3) + (2*c^2*Sqrt[c + d*x]*Sin[a + b*Sqrt[c + d*
x]])/(b*d^3) - (40*(c + d*x)^(3/2)*Sin[a + b*Sqrt[c + d*x]])/(b^3*d^3) - (4
*c*(c + d*x)^(3/2)*Sin[a + b*Sqrt[c + d*x]])/(b*d^3) + (2*(c + d*x)^(5/2)*S
in[a + b*Sqrt[c + d*x]])/(b*d^3)
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3513

```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_.)*((g_
.) + (h_.)*(x_))^(m_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Cos[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rubi steps

$$\text{integral} = \frac{2\text{Subst}\left(\int \left(\frac{c^2x\cos(a+bx)}{d^2} - \frac{2cx^3\cos(a+bx)}{d^2} + \frac{x^5\cos(a+bx)}{d^2}\right) dx, x, \sqrt{c+dx}\right)}{d}$$

$$= \frac{2\text{Subst}\left(\int x^5\cos(a+bx)\,dx, x, \sqrt{c+dx}\right)}{d^3} - \frac{(4c)\text{Subst}\left(\int x^3\cos(a+bx)\,dx, x, \sqrt{c+dx}\right)}{d^3} + \frac{(2c^2)\,\text{Subst}\left(\int x\cos(a+bx)\,dx, x, \sqrt{c+dx}\right)}{d^3}$$

$$= \frac{2c^2\sqrt{c+dx}\sin\left(a+b\sqrt{c+dx}\right)}{bd^3} - \frac{4c(c+dx)^{3/2}\sin\left(a+b\sqrt{c+dx}\right)}{bd^3} + \frac{2(c+dx)^{5/2}\sin\left(a+b\sqrt{c+dx}\right)}{bd^3} - \frac{10\text{Subst}\left(\int x^4\sin(a+bx)\,dx, x, \sqrt{c+dx}\right)}{bd^3} + \frac{(12c)\text{Subst}\left(\int x^2\sin(a+bx)\,dx, x, \sqrt{c+dx}\right)}{bd^3} - \frac{(2c^2)\,\text{Subst}\left(\int \sin(a+bx)\,dx, x, \sqrt{c+dx}\right)}{bd^3}$$

$$= \frac{2c^2\cos\left(a+b\sqrt{c+dx}\right)}{b^2d^3} - \frac{12c(c+dx)\cos\left(a+b\sqrt{c+dx}\right)}{b^2d^3} + \frac{10(c+dx)^2\cos\left(a+b\sqrt{c+dx}\right)}{b^2d^3} + \frac{2c^2\sqrt{c+dx}\sin\left(a+b\sqrt{c+dx}\right)}{bd^3} - \frac{4c(c+dx)^{3/2}\sin\left(a+b\sqrt{c+dx}\right)}{bd^3} + \frac{2(c+dx)^{5/2}\sin\left(a+b\sqrt{c+dx}\right)}{bd^3} - \frac{40\text{Subst}\left(\int x^3\cos(a+bx)\,dx, x, \sqrt{c+dx}\right)}{b^2d^3} + \frac{(24c)\text{Subst}\left(\int x\cos(a+bx)\,dx, x, \sqrt{c+dx}\right)}{b^2d^3}$$

$$
\begin{aligned}
&= \frac{2c^2\cos\left(a+b\sqrt{c+dx}\right)}{b^2d^3} - \frac{12c(c+dx)\cos\left(a+b\sqrt{c+dx}\right)}{b^2d^3} \\
&\quad + \frac{10(c+dx)^2\cos\left(a+b\sqrt{c+dx}\right)}{b^2d^3} + \frac{24c\sqrt{c+dx}\sin\left(a+b\sqrt{c+dx}\right)}{b^3d^3} \\
&\quad + \frac{2c^2\sqrt{c+dx}\sin\left(a+b\sqrt{c+dx}\right)}{bd^3} - \frac{40(c+dx)^{3/2}\sin\left(a+b\sqrt{c+dx}\right)}{b^3d^3} \\
&\quad - \frac{4c(c+dx)^{3/2}\sin\left(a+b\sqrt{c+dx}\right)}{bd^3} + \frac{2(c+dx)^{5/2}\sin\left(a+b\sqrt{c+dx}\right)}{bd^3} \\
&\quad + \frac{120\text{Subst}\left(\int x^2\sin(a+bx)\,dx, x, \sqrt{c+dx}\right)}{b^3d^3} \\
&\quad - \frac{(24c)\text{Subst}\left(\int \sin(a+bx)\,dx, x, \sqrt{c+dx}\right)}{b^3d^3}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{24c\cos\left(a+b\sqrt{c+dx}\right)}{b^4d^3} + \frac{2c^2\cos\left(a+b\sqrt{c+dx}\right)}{b^2d^3} - \frac{120(c+dx)\cos\left(a+b\sqrt{c+dx}\right)}{b^4d^3} \\
&\quad - \frac{12c(c+dx)\cos\left(a+b\sqrt{c+dx}\right)}{b^2d^3} + \frac{10(c+dx)^2\cos\left(a+b\sqrt{c+dx}\right)}{b^2d^3} \\
&\quad + \frac{24c\sqrt{c+dx}\sin\left(a+b\sqrt{c+dx}\right)}{b^3d^3} + \frac{2c^2\sqrt{c+dx}\sin\left(a+b\sqrt{c+dx}\right)}{bd^3} \\
&\quad - \frac{40(c+dx)^{3/2}\sin\left(a+b\sqrt{c+dx}\right)}{b^3d^3} - \frac{4c(c+dx)^{3/2}\sin\left(a+b\sqrt{c+dx}\right)}{bd^3} \\
&\quad + \frac{2(c+dx)^{5/2}\sin\left(a+b\sqrt{c+dx}\right)}{bd^3} + \frac{240\text{Subst}\left(\int x\cos(a+bx)\,dx, x, \sqrt{c+dx}\right)}{b^4d^3}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{24c\cos\left(a+b\sqrt{c+dx}\right)}{b^4d^3} + \frac{2c^2\cos\left(a+b\sqrt{c+dx}\right)}{b^2d^3} \\
&\quad - \frac{120(c+dx)\cos\left(a+b\sqrt{c+dx}\right)}{b^4d^3} - \frac{12c(c+dx)\cos\left(a+b\sqrt{c+dx}\right)}{b^2d^3} \\
&\quad + \frac{10(c+dx)^2\cos\left(a+b\sqrt{c+dx}\right)}{b^2d^3} + \frac{240\sqrt{c+dx}\sin\left(a+b\sqrt{c+dx}\right)}{b^5d^3} \\
&\quad + \frac{24c\sqrt{c+dx}\sin\left(a+b\sqrt{c+dx}\right)}{b^3d^3} + \frac{2c^2\sqrt{c+dx}\sin\left(a+b\sqrt{c+dx}\right)}{bd^3} \\
&\quad - \frac{40(c+dx)^{3/2}\sin\left(a+b\sqrt{c+dx}\right)}{b^3d^3} - \frac{4c(c+dx)^{3/2}\sin\left(a+b\sqrt{c+dx}\right)}{bd^3} \\
&\quad + \frac{2(c+dx)^{5/2}\sin\left(a+b\sqrt{c+dx}\right)}{bd^3} - \frac{240\text{Subst}\left(\int \sin(a+bx)\,dx, x, \sqrt{c+dx}\right)}{b^5d^3}
\end{aligned}
$$

$$= \frac{240\cos\left(a+b\sqrt{c+dx}\right)}{b^6d^3} + \frac{24c\cos\left(a+b\sqrt{c+dx}\right)}{b^4d^3} + \frac{2c^2\cos\left(a+b\sqrt{c+dx}\right)}{b^2d^3} - \frac{120(c+dx)\cos\left(a+b\sqrt{c+dx}\right)}{b^4d^3} - \frac{12c(c+dx)\cos\left(a+b\sqrt{c+dx}\right)}{b^2d^3} + \frac{10(c+dx)^2\cos\left(a+b\sqrt{c+dx}\right)}{b^2d^3} + \frac{240\sqrt{c+dx}\sin\left(a+b\sqrt{c+dx}\right)}{b^5d^3} + \frac{24c\sqrt{c+dx}\sin\left(a+b\sqrt{c+dx}\right)}{b^3d^3} + \frac{2c^2\sqrt{c+dx}\sin\left(a+b\sqrt{c+dx}\right)}{bd^3} - \frac{40(c+dx)^{3/2}\sin\left(a+b\sqrt{c+dx}\right)}{b^3d^3} - \frac{4c(c+dx)^{3/2}\sin\left(a+b\sqrt{c+dx}\right)}{bd^3} + \frac{2(c+dx)^{5/2}\sin\left(a+b\sqrt{c+dx}\right)}{bd^3}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.93 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.65

$$\int x^2\cos\left(a+b\sqrt{c+dx}\right)\,dx = \frac{e^{-i\left(a+b\sqrt{c+dx}\right)}\left(120+120ib\sqrt{c+dx}+ib^5d^2x^2\sqrt{c+dx}-4ib^3\sqrt{c+dx}(2c+5dx)-12b^2(4c+5dx)+b^4dx(4\right.}{}$$

```
[In] Integrate[x^2*Cos[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (120 + (120*I)*b*Sqrt[c + d*x] + I*b^5*d^2*x^2*Sqrt[c + d*x] - (4*I)*b^3*Sq
rt[c + d*x]*(2*c + 5*d*x) - 12*b^2*(4*c + 5*d*x) + b^4*d*x*(4*c + 5*d*x) +
E^((2*I)*(a + b*Sqrt[c + d*x]))*(120 - (120*I)*b*Sqrt[c + d*x] - I*b^5*d^2*
x^2*Sqrt[c + d*x] + (4*I)*b^3*Sqrt[c + d*x]*(2*c + 5*d*x) - 12*b^2*(4*c + 5
*d*x) + b^4*d*x*(4*c + 5*d*x)))/(b^6*d^3*E^(I*(a + b*Sqrt[c + d*x])))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 824 vs. 2(310) = 620.

Time = 1.51 (sec) , antiderivative size = 825, normalized size of antiderivative = 2.38

method	result
derivativedivides	$\frac{-2a\,c^2\sin(a+b\sqrt{dx+c})+2c^2(\cos(a+b\sqrt{dx+c})+(a+b\sqrt{dx+c})\sin(a+b\sqrt{dx+c}))+\frac{4a^3c\sin(a+b\sqrt{dx+c})}{b^2}-\frac{12a^2c(\cos(a+b\sqrt{}}{}}{}$
default	$\frac{-2a\,c^2\sin(a+b\sqrt{dx+c})+2c^2(\cos(a+b\sqrt{dx+c})+(a+b\sqrt{dx+c})\sin(a+b\sqrt{dx+c}))+\frac{4a^3c\sin(a+b\sqrt{dx+c})}{b^2}-\frac{12a^2c(\cos(a+b\sqrt{}}{}}{}$
parts	$\frac{2x^2\sqrt{dx+c}\,\sin(a+b\sqrt{dx+c})}{db}+\frac{2x^2\cos(a+b\sqrt{dx+c})}{d\,b^2}-\frac{8\left(2ac(\sin(a+b\sqrt{dx+c})-(a+b\sqrt{dx+c})\cos(a+b\sqrt{dx+c}))+a\right.}{}$

```
[In] int(x^2*cos(a+b*(d*x+c)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d^3/b^2*(-a*c^2*sin(a+b*(d*x+c)^(1/2))+c^2*(cos(a+b*(d*x+c)^(1/2))+(a+b*(
d*x+c)^(1/2))*sin(a+b*(d*x+c)^(1/2)))+2/b^2*a^3*c*sin(a+b*(d*x+c)^(1/2))-6/
b^2*a^2*c*(cos(a+b*(d*x+c)^(1/2))+(a+b*(d*x+c)^(1/2))*sin(a+b*(d*x+c)^(1/2)
))+6/b^2*a*c*((a+b*(d*x+c)^(1/2))^2*sin(a+b*(d*x+c)^(1/2))-2*sin(a+b*(d*x+c
)^(1/2))+2*(a+b*(d*x+c)^(1/2))*cos(a+b*(d*x+c)^(1/2)))-2/b^2*c*((a+b*(d*x+c
)^(1/2))^3*sin(a+b*(d*x+c)^(1/2))+3*(a+b*(d*x+c)^(1/2))^2*cos(a+b*(d*x+c)^(
1/2))-6*cos(a+b*(d*x+c)^(1/2))-6*(a+b*(d*x+c)^(1/2))*sin(a+b*(d*x+c)^(1/2))
)-1/b^4*a^5*sin(a+b*(d*x+c)^(1/2))+5/b^4*a^4*(cos(a+b*(d*x+c)^(1/2))+(a+b*(
d*x+c)^(1/2))*sin(a+b*(d*x+c)^(1/2)))-10/b^4*a^3*((a+b*(d*x+c)^(1/2))^2*sin
(a+b*(d*x+c)^(1/2))-2*sin(a+b*(d*x+c)^(1/2))+2*(a+b*(d*x+c)^(1/2))*cos(a+b*
(d*x+c)^(1/2)))+10/b^4*a^2*((a+b*(d*x+c)^(1/2))^3*sin(a+b*(d*x+c)^(1/2))+3*
(a+b*(d*x+c)^(1/2))^2*cos(a+b*(d*x+c)^(1/2))-6*cos(a+b*(d*x+c)^(1/2))-6*(a+
b*(d*x+c)^(1/2))*sin(a+b*(d*x+c)^(1/2)))-5/b^4*a*((a+b*(d*x+c)^(1/2))^4*sin
(a+b*(d*x+c)^(1/2))+4*(a+b*(d*x+c)^(1/2))^3*cos(a+b*(d*x+c)^(1/2))-12*(a+b*
(d*x+c)^(1/2))^2*sin(a+b*(d*x+c)^(1/2))+24*sin(a+b*(d*x+c)^(1/2))-24*(a+b*(
d*x+c)^(1/2))*cos(a+b*(d*x+c)^(1/2)))+1/b^4*((a+b*(d*x+c)^(1/2))^5*sin(a+b*
(d*x+c)^(1/2))+5*(a+b*(d*x+c)^(1/2))^4*cos(a+b*(d*x+c)^(1/2))-20*(a+b*(d*x+
c)^(1/2))^3*sin(a+b*(d*x+c)^(1/2))-60*(a+b*(d*x+c)^(1/2))^2*cos(a+b*(d*x+c)
^(1/2))+120*cos(a+b*(d*x+c)^(1/2))+120*(a+b*(d*x+c)^(1/2))*sin(a+b*(d*x+c)^
(1/2))))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.30

$$\int x^2\cos\left(a+b\sqrt{c+dx}\right)\,dx$$
$$=\frac{2\left(\left(b^5d^2x^2-20\,b^3dx-8\,b^3c+120\,b\right)\sqrt{dx+c}\sin\left(\sqrt{dx+c}b+a\right)+\left(5\,b^4d^2x^2-48\,b^2c+4\left(b^4c-15\,b^2\right)dx\right.}{b^6d^3}$$

```
[In] integrate(x^2*cos(a+b*(d*x+c)^(1/2)),x, algorithm="fricas")
```

```
[Out] 2*((b^5*d^2*x^2 - 20*b^3*d*x - 8*b^3*c + 120*b)*sqrt(d*x + c)*sin(sqrt(d*x
+ c)*b + a) + (5*b^4*d^2*x^2 - 48*b^2*c + 4*(b^4*c - 15*b^2)*d*x + 120)*cos
(sqrt(d*x + c)*b + a))/(b^6*d^3)
```

Sympy [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 269, normalized size of antiderivative = 0.78

$$\int x^2\cos\left(a+b\sqrt{c+dx}\right)\,dx$$
$$=\begin{cases}\frac{x^3\cos(a)}{3} \\ \frac{x^3\cos(a+b\sqrt{c})}{3} \\ \frac{2x^2\sqrt{c+dx}\sin(a+b\sqrt{c+dx})}{bd}+\frac{8cx\cos(a+b\sqrt{c+dx})}{b^2d^2}+\frac{10x^2\cos(a+b\sqrt{c+dx})}{b^2d}-\frac{16c\sqrt{c+dx}\sin(a+b\sqrt{c+dx})}{b^3d^3}-\frac{40x\sqrt{c+dx}\sin(a+b\sqrt{c+d}}{b^3d^2}\end{cases}$$

```
[In] integrate(x**2*cos(a+b*(d*x+c)**(1/2)),x)
```

```
[Out] Piecewise((x**3*cos(a)/3, Eq(b, 0) & (Eq(b, 0) | Eq(d, 0))), (x**3*cos(a +
b*sqrt(c))/3, Eq(d, 0)), (2*x**2*sqrt(c + d*x)*sin(a + b*sqrt(c + d*x))/(b*
d) + 8*c*x*cos(a + b*sqrt(c + d*x))/(b**2*d**2) + 10*x**2*cos(a + b*sqrt(c
+ d*x))/(b**2*d) - 16*c*sqrt(c + d*x)*sin(a + b*sqrt(c + d*x))/(b**3*d**3)
- 40*x*sqrt(c + d*x)*sin(a + b*sqrt(c + d*x))/(b**3*d**2) - 96*c*cos(a + b*
sqrt(c + d*x))/(b**4*d**3) - 120*x*cos(a + b*sqrt(c + d*x))/(b**4*d**2) + 2
40*sqrt(c + d*x)*sin(a + b*sqrt(c + d*x))/(b**5*d**3) + 240*cos(a + b*sqrt(
c + d*x))/(b**6*d**3), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 672 vs. 2(310) = 620.

Time = 0.29 (sec) , antiderivative size = 672, normalized size of antiderivative = 1.94

$$\int x^2\cos\left(a+b\sqrt{c+dx}\right)\,dx =$$
$$-\frac{2\left(ac^2\sin\left(\sqrt{dx+c}b+a\right)-\left(\left(\sqrt{dx+c}b+a\right)\sin\left(\sqrt{dx+c}b+a\right)+\cos\left(\sqrt{dx+c}b+a\right)\right)c^2-\frac{2a^3c\sin(\sqrt{dx}}{b^2}\right.}{}$$

```
[In] integrate(x^2*cos(a+b*(d*x+c)^(1/2)),x, algorithm="maxima")
```

```
[Out] -2*(a*c^2*sin(sqrt(d*x + c)*b + a) - ((sqrt(d*x + c)*b + a)*sin(sqrt(d*x +
c)*b + a) + cos(sqrt(d*x + c)*b + a))*c^2 - 2*a^3*c*sin(sqrt(d*x + c)*b + a
)/b^2 + 6*((sqrt(d*x + c)*b + a)*sin(sqrt(d*x + c)*b + a) + cos(sqrt(d*x +
c)*b + a))*a^2*c/b^2 + a^5*sin(sqrt(d*x + c)*b + a)/b^4 - 5*((sqrt(d*x + c)
```

```
*b + a)*sin(sqrt(d*x + c)*b + a) + cos(sqrt(d*x + c)*b + a))*a^4/b^4 - 6*(2
*(sqrt(d*x + c)*b + a)*cos(sqrt(d*x + c)*b + a) + ((sqrt(d*x + c)*b + a)^2
- 2)*sin(sqrt(d*x + c)*b + a))*a*c/b^2 + 10*(2*(sqrt(d*x + c)*b + a)*cos(sq
rt(d*x + c)*b + a) + ((sqrt(d*x + c)*b + a)^2 - 2)*sin(sqrt(d*x + c)*b + a)
)*a^3/b^4 + 2*(3*((sqrt(d*x + c)*b + a)^2 - 2)*cos(sqrt(d*x + c)*b + a) + (
(sqrt(d*x + c)*b + a)^3 - 6*sqrt(d*x + c)*b - 6*a)*sin(sqrt(d*x + c)*b + a)
)*c/b^2 - 10*(3*((sqrt(d*x + c)*b + a)^2 - 2)*cos(sqrt(d*x + c)*b + a) + ((
sqrt(d*x + c)*b + a)^3 - 6*sqrt(d*x + c)*b - 6*a)*sin(sqrt(d*x + c)*b + a))
*a^2/b^4 + 5*(4*((sqrt(d*x + c)*b + a)^3 - 6*sqrt(d*x + c)*b - 6*a)*cos(sqr
t(d*x + c)*b + a) + ((sqrt(d*x + c)*b + a)^4 - 12*(sqrt(d*x + c)*b + a)^2 +
 24)*sin(sqrt(d*x + c)*b + a))*a/b^4 - (5*((sqrt(d*x + c)*b + a)^4 - 12*(sq
rt(d*x + c)*b + a)^2 + 24)*cos(sqrt(d*x + c)*b + a) + ((sqrt(d*x + c)*b + a
)^5 - 20*(sqrt(d*x + c)*b + a)^3 + 120*sqrt(d*x + c)*b + 120*a)*sin(sqrt(d*
x + c)*b + a))/b^4)/(b^2*d^3)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 480, normalized size of antiderivative = 1.39

$$\int x^2 \cos\left(a + b\sqrt{c + dx}\right)\,dx$$
$$= \frac{2\left(\frac{\left(b^4c^2 - 6\left(\sqrt{dx+c}b+a\right)^2b^2c + 12\left(\sqrt{dx+c}b+a\right)ab^2c - 6a^2b^2c + 5\left(\sqrt{dx+c}b+a\right)^4 - 20\left(\sqrt{dx+c}b+a\right)^3a + 30\left(\sqrt{dx+c}b+a\right)^2a^2 - 20\left(\sqrt{dx+c}b+a\right)a^3}{b^4d^2}\right.}{}$$

```
[In] integrate(x^2*cos(a+b*(d*x+c)^(1/2)),x, algorithm="giac")
```

```
[Out] 2*((b^4*c^2 - 6*(sqrt(d*x + c)*b + a)^2*b^2*c + 12*(sqrt(d*x + c)*b + a)*a*
b^2*c - 6*a^2*b^2*c + 5*(sqrt(d*x + c)*b + a)^4 - 20*(sqrt(d*x + c)*b + a)^
3*a + 30*(sqrt(d*x + c)*b + a)^2*a^2 - 20*(sqrt(d*x + c)*b + a)*a^3 + 5*a^4
 + 12*b^2*c - 60*(sqrt(d*x + c)*b + a)^2 + 120*(sqrt(d*x + c)*b + a)*a - 60
*a^2 + 120)*cos(sqrt(d*x + c)*b + a)/(b^4*d^2) + ((sqrt(d*x + c)*b + a)*b^4
*c^2 - a*b^4*c^2 - 2*(sqrt(d*x + c)*b + a)^3*b^2*c + 6*(sqrt(d*x + c)*b + a
)^2*a*b^2*c - 6*(sqrt(d*x + c)*b + a)*a^2*b^2*c + 2*a^3*b^2*c + (sqrt(d*x +
 c)*b + a)^5 - 5*(sqrt(d*x + c)*b + a)^4*a + 10*(sqrt(d*x + c)*b + a)^3*a^2
 - 10*(sqrt(d*x + c)*b + a)^2*a^3 + 5*(sqrt(d*x + c)*b + a)*a^4 - a^5 + 12*
(sqrt(d*x + c)*b + a)*b^2*c - 12*a*b^2*c - 20*(sqrt(d*x + c)*b + a)^3 + 60*
(sqrt(d*x + c)*b + a)^2*a - 60*(sqrt(d*x + c)*b + a)*a^2 + 20*a^3 + 120*sqr
t(d*x + c)*b)*sin(sqrt(d*x + c)*b + a)/(b^4*d^2))/(b^2*d)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \cos \left(a+b\sqrt{c+dx}\right) \, dx = \int x^2 \, \cos \left(a+b\,\sqrt{c+d\,x}\right) \, dx$$

```
[In] int(x^2*cos(a + b*(c + d*x)^(1/2)),x)
[Out] int(x^2*cos(a + b*(c + d*x)^(1/2)), x)
```

3.91 $\int x\cos\left(a+b\sqrt{c+dx}\right)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 167

$$\int x\cos\left(a+b\sqrt{c+dx}\right)\,dx=-\frac{12\cos\left(a+b\sqrt{c+dx}\right)}{b^4d^2}-\frac{2c\cos\left(a+b\sqrt{c+dx}\right)}{b^2d^2}+\frac{6(c+dx)\cos\left(a+b\sqrt{c+dx}\right)}{b^2d^2}-\frac{12\sqrt{c+dx}\sin\left(a+b\sqrt{c+dx}\right)}{b^3d^2}-\frac{2c\sqrt{c+dx}\sin\left(a+b\sqrt{c+dx}\right)}{bd^2}+\frac{2(c+dx)^{3/2}\sin\left(a+b\sqrt{c+dx}\right)}{bd^2}$$

```
[Out] -12*cos(a+b*(d*x+c)^(1/2))/b^4/d^2-2*c*cos(a+b*(d*x+c)^(1/2))/b^2/d^2+6*(d*
x+c)*cos(a+b*(d*x+c)^(1/2))/b^2/d^2+2*(d*x+c)^(3/2)*sin(a+b*(d*x+c)^(1/2))/
b/d^2-12*sin(a+b*(d*x+c)^(1/2))*(d*x+c)^(1/2)/b^3/d^2-2*c*sin(a+b*(d*x+c)^(
1/2))*(d*x+c)^(1/2)/b/d^2
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used

= {3513, 3377, 2718}

$$\int x\cos\left(a+b\sqrt{c+dx}\right)\,dx = -\frac{12\cos\left(a+b\sqrt{c+dx}\right)}{b^4d^2} - \frac{12\sqrt{c+dx}\sin\left(a+b\sqrt{c+dx}\right)}{b^3d^2} + \frac{6(c+dx)\cos\left(a+b\sqrt{c+dx}\right)}{b^2d^2} - \frac{2c\cos\left(a+b\sqrt{c+dx}\right)}{b^2d^2} + \frac{2(c+dx)^{3/2}\sin\left(a+b\sqrt{c+dx}\right)}{bd^2} - \frac{2c\sqrt{c+dx}\sin\left(a+b\sqrt{c+dx}\right)}{bd^2}$$

```
[In] Int[x*Cos[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (-12*Cos[a + b*Sqrt[c + d*x]])/(b^4*d^2) - (2*c*Cos[a + b*Sqrt[c + d*x]])/(
b^2*d^2) + (6*(c + d*x)*Cos[a + b*Sqrt[c + d*x]])/(b^2*d^2) - (12*Sqrt[c +
d*x]*Sin[a + b*Sqrt[c + d*x]])/(b^3*d^2) - (2*c*Sqrt[c + d*x]*Sin[a + b*Sqr
t[c + d*x]])/(b*d^2) + (2*(c + d*x)^(3/2)*Sin[a + b*Sqrt[c + d*x]])/(b*d^2)
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3513

```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_.)*((g_
.) + (h_.)*(x_))^(m_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Cos[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rubi steps

$$\text{integral} = \frac{2\text{Subst}\left(\int\left(-\frac{cx\cos(a+bx)}{d}+\frac{x^3\cos(a+bx)}{d}\right)\,dx,x,\sqrt{c+dx}\right)}{d}$$

$$= \frac{2\text{Subst}\left(\int x^3\cos(a+bx)\,dx,x,\sqrt{c+dx}\right)}{d^2} - \frac{(2c)\text{Subst}\left(\int x\cos(a+bx)\,dx,x,\sqrt{c+dx}\right)}{d^2}$$

$$= -\frac{2c\sqrt{c+dx}\sin\left(a+b\sqrt{c+dx}\right)}{bd^2}+\frac{2(c+dx)^{3/2}\sin\left(a+b\sqrt{c+dx}\right)}{bd^2} - \frac{6\text{Subst}\left(\int x^2\sin(a+bx)\,dx,x,\sqrt{c+dx}\right)}{bd^2} + \frac{(2c)\text{Subst}\left(\int \sin(a+bx)\,dx,x,\sqrt{c+dx}\right)}{bd^2}$$

$$= -\frac{2c\cos\left(a+b\sqrt{c+dx}\right)}{b^2d^2}+\frac{6(c+dx)\cos\left(a+b\sqrt{c+dx}\right)}{b^2d^2}-\frac{2c\sqrt{c+dx}\sin\left(a+b\sqrt{c+dx}\right)}{bd^2} + \frac{2(c+dx)^{3/2}\sin\left(a+b\sqrt{c+dx}\right)}{bd^2}-\frac{12\text{Subst}\left(\int x\cos(a+bx)\,dx,x,\sqrt{c+dx}\right)}{b^2d^2}$$

$$= -\frac{2c\cos\left(a+b\sqrt{c+dx}\right)}{b^2d^2}+\frac{6(c+dx)\cos\left(a+b\sqrt{c+dx}\right)}{b^2d^2} - \frac{12\sqrt{c+dx}\sin\left(a+b\sqrt{c+dx}\right)}{b^3d^2}-\frac{2c\sqrt{c+dx}\sin\left(a+b\sqrt{c+dx}\right)}{bd^2} + \frac{2(c+dx)^{3/2}\sin\left(a+b\sqrt{c+dx}\right)}{bd^2}+\frac{12\text{Subst}\left(\int \sin(a+bx)\,dx,x,\sqrt{c+dx}\right)}{b^3d^2}$$

$$= -\frac{12\cos\left(a+b\sqrt{c+dx}\right)}{b^4d^2}-\frac{2c\cos\left(a+b\sqrt{c+dx}\right)}{b^2d^2} + \frac{6(c+dx)\cos\left(a+b\sqrt{c+dx}\right)}{b^2d^2}-\frac{12\sqrt{c+dx}\sin\left(a+b\sqrt{c+dx}\right)}{b^3d^2} - \frac{2c\sqrt{c+dx}\sin\left(a+b\sqrt{c+dx}\right)}{bd^2}+\frac{2(c+dx)^{3/2}\sin\left(a+b\sqrt{c+dx}\right)}{bd^2}$$

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.43

$$\int x\cos\left(a+b\sqrt{c+dx}\right)\,dx = \frac{2\left((-6+b^2(2c+3dx))\cos\left(a+b\sqrt{c+dx}\right)+b\sqrt{c+dx}(-6+b^2dx)\sin\left(a+b\sqrt{c+dx}\right)\right)}{b^4d^2}$$

```
[In] Integrate[x*Cos[a + b*Sqrt[c + d*x]],x]
[Out] (2*((-6 + b^2*(2*c + 3*d*x))*Cos[a + b*Sqrt[c + d*x]] + b*Sqrt[c + d*x]*(-6
 + b^2*d*x)*Sin[a + b*Sqrt[c + d*x]]))/(b^4*d^2)
```

Maple [A] (verified)

Time = 1.59 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.78

method	result
parts	$\frac{2x\sqrt{dx+c}\,\sin\left(a+b\sqrt{dx+c}\right)}{db}+\frac{2x\cos\left(a+b\sqrt{dx+c}\right)}{d\,b^2}-\frac{2\left(\frac{-2\left(a+b\sqrt{dx+c}\right)^2\cos\left(a+b\sqrt{dx+c}\right)+4\cos\left(a+b\sqrt{dx+c}\right)+4\left(a+b\sqrt{dx+c}\right)}{b}\right.}{}$
derivativedivides	$\frac{2ac\sin\left(a+b\sqrt{dx+c}\right)-2c\left(\cos\left(a+b\sqrt{dx+c}\right)+\left(a+b\sqrt{dx+c}\right)\sin\left(a+b\sqrt{dx+c}\right)\right)-\frac{2a^3\sin\left(a+b\sqrt{dx+c}\right)}{b^2}+\frac{6a^2\left(\cos\left(a+b\sqrt{dx+c}\right)+(a-\right.}{}}{}$
default	$\frac{2ac\sin\left(a+b\sqrt{dx+c}\right)-2c\left(\cos\left(a+b\sqrt{dx+c}\right)+\left(a+b\sqrt{dx+c}\right)\sin\left(a+b\sqrt{dx+c}\right)\right)-\frac{2a^3\sin\left(a+b\sqrt{dx+c}\right)}{b^2}+\frac{6a^2\left(\cos\left(a+b\sqrt{dx+c}\right)+(a-\right.}{}}{}$

```
[In] int(x*cos(a+b*(d*x+c)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/b*x*(d*x+c)^(1/2)*sin(a+b*(d*x+c)^(1/2))+2/d/b^2*x*cos(a+b*(d*x+c)^(1/2
))-2/d/b^2*(2/d/b^2*(-(a+b*(d*x+c)^(1/2))^2*cos(a+b*(d*x+c)^(1/2))+2*cos(a+
b*(d*x+c)^(1/2))+2*(a+b*(d*x+c)^(1/2))*sin(a+b*(d*x+c)^(1/2))-a*(sin(a+b*(d
*x+c)^(1/2))-(a+b*(d*x+c)^(1/2))*cos(a+b*(d*x+c)^(1/2))))-2*a/d/b^2*(sin(a+
b*(d*x+c)^(1/2))-(a+b*(d*x+c)^(1/2))*cos(a+b*(d*x+c)^(1/2))+a*cos(a+b*(d*x+
c)^(1/2)))+2/d/b^2*(cos(a+b*(d*x+c)^(1/2))+(a+b*(d*x+c)^(1/2))*sin(a+b*(d*x
+c)^(1/2))-a*sin(a+b*(d*x+c)^(1/2))))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.40

$$\int x\cos\left(a+b\sqrt{c+dx}\right)\,dx$$
$$=\frac{2\left(\left(b^3dx-6\,b\right)\sqrt{dx+c}\sin\left(\sqrt{dx+c}b+a\right)+\left(3\,b^2dx+2\,b^2c-6\right)\cos\left(\sqrt{dx+c}b+a\right)\right)}{b^4d^2}$$

```
[In] integrate(x*cos(a+b*(d*x+c)^(1/2)),x, algorithm="fricas")
```

```
[Out] 2*((b^3*d*x - 6*b)*sqrt(d*x + c)*sin(sqrt(d*x + c)*b + a) + (3*b^2*d*x + 2*
b^2*c - 6)*cos(sqrt(d*x + c)*b + a))/(b^4*d^2)
```

Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.90

$$\int x\cos\left(a+b\sqrt{c+dx}\right)\,dx$$
$$=\begin{cases}\frac{x^2\cos(a)}{2} & \text{for } b\\ \frac{x^2\cos(a+b\sqrt{c})}{2} & \text{for } d\\ \frac{2x\sqrt{c+dx}\sin(a+b\sqrt{c+dx})}{bd}+\frac{4c\cos(a+b\sqrt{c+dx})}{b^2d^2}+\frac{6x\cos(a+b\sqrt{c+dx})}{b^2d}-\frac{12\sqrt{c+dx}\sin(a+b\sqrt{c+dx})}{b^3d^2}-\frac{12\cos(a+b\sqrt{c+dx})}{b^4d^2} & \text{othe}\end{cases}$$

```
[In] integrate(x*cos(a+b*(d*x+c)**(1/2)),x)
```

```
[Out] Piecewise((x**2*cos(a)/2, Eq(b, 0) & (Eq(b, 0) | Eq(d, 0))), (x**2*cos(a +
b*sqrt(c))/2, Eq(d, 0)), (2*x*sqrt(c + d*x)*sin(a + b*sqrt(c + d*x))/(b*d)
+ 4*c*cos(a + b*sqrt(c + d*x))/(b**2*d**2) + 6*x*cos(a + b*sqrt(c + d*x))/(
b**2*d) - 12*sqrt(c + d*x)*sin(a + b*sqrt(c + d*x))/(b**3*d**2) - 12*cos(a
+ b*sqrt(c + d*x))/(b**4*d**2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.57

$$\int x\cos\left(a+b\sqrt{c+dx}\right)\,dx$$
$$=\frac{2\left(ac\sin\left(\sqrt{dx+c}b+a\right)-\left(\left(\sqrt{dx+c}b+a\right)\sin\left(\sqrt{dx+c}b+a\right)+\cos\left(\sqrt{dx+c}b+a\right)\right)c-\frac{a^3\sin(\sqrt{dx+c}b+a}{b^2}\right.}{}$$

```
[In] integrate(x*cos(a+b*(d*x+c)^(1/2)),x, algorithm="maxima")
```

```
[Out] 2*(a*c*sin(sqrt(d*x + c)*b + a) - ((sqrt(d*x + c)*b + a)*sin(sqrt(d*x + c)*
b + a) + cos(sqrt(d*x + c)*b + a))*c - a^3*sin(sqrt(d*x + c)*b + a)/b^2 + 3
*((sqrt(d*x + c)*b + a)*sin(sqrt(d*x + c)*b + a) + cos(sqrt(d*x + c)*b + a)
)*a^2/b^2 - 3*(2*(sqrt(d*x + c)*b + a)*cos(sqrt(d*x + c)*b + a) + ((sqrt(d*
x + c)*b + a)^2 - 2)*sin(sqrt(d*x + c)*b + a))*a/b^2 + (3*((sqrt(d*x + c)*b
 + a)^2 - 2)*cos(sqrt(d*x + c)*b + a) + ((sqrt(d*x + c)*b + a)^3 - 6*sqrt(d
*x + c)*b - 6*a)*sin(sqrt(d*x + c)*b + a))/b^2)/(b^2*d^2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.99

$$\int x\cos\left(a+b\sqrt{c+dx}\right)\,dx = -\frac{2\left(\frac{\left(b^2c-3\left(\sqrt{dx+c}b+a\right)^2+6\left(\sqrt{dx+c}b+a\right)a-3a^2+6\right)\cos\left(\sqrt{dx+c}b+a\right)}{b^2}+\frac{\left(\left(\sqrt{dx+c}b+a\right)b^2c-ab^2c-\left(\sqrt{dx+c}b+a\right)^3+3\left(\sqrt{dx+c}b+a\right)^2a-3\left(\right.}{b^2}\right.}{b^2d^2}$$

```
[In] integrate(x*cos(a+b*(d*x+c)^(1/2)),x, algorithm="giac")
```

```
[Out] -2*((b^2*c - 3*(sqrt(d*x + c)*b + a)^2 + 6*(sqrt(d*x + c)*b + a)*a - 3*a^2
+ 6)*cos(sqrt(d*x + c)*b + a)/b^2 + ((sqrt(d*x + c)*b + a)*b^2*c - a*b^2*c
- (sqrt(d*x + c)*b + a)^3 + 3*(sqrt(d*x + c)*b + a)^2*a - 3*(sqrt(d*x + c)*
b + a)*a^2 + a^3 + 6*sqrt(d*x + c)*b)*sin(sqrt(d*x + c)*b + a)/b^2)/(b^2*d^
2)
```

Mupad [F(-1)]

Timed out.

$$\int x\cos\left(a+b\sqrt{c+dx}\right)\,dx = \int x\,\cos\left(a+b\,\sqrt{c+d\,x}\right)\,dx$$

```
[In] int(x*cos(a + b*(c + d*x)^(1/2)),x)
```

```
[Out] int(x*cos(a + b*(c + d*x)^(1/2)), x)
```

3.92 $\int \cos \left(a+b\sqrt{c+dx}\right) \, dx$

Optimal result

Integrand size = 14, antiderivative size = 54

$$\int \cos \left(a+b\sqrt{c+dx}\right) \, dx=\frac{2\cos \left(a+b\sqrt{c+dx}\right)}{b^2d}+\frac{2\sqrt{c+dx}\sin \left(a+b\sqrt{c+dx}\right)}{bd}$$

```
[Out] 2*cos(a+b*(d*x+c)^(1/2))/b^2/d+2*sin(a+b*(d*x+c)^(1/2))*(d*x+c)^(1/2)/b/d
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3443, 3377, 2718}

$$\int \cos \left(a+b\sqrt{c+dx}\right) \, dx=\frac{2\cos \left(a+b\sqrt{c+dx}\right)}{b^2d}+\frac{2\sqrt{c+dx}\sin \left(a+b\sqrt{c+dx}\right)}{bd}$$

```
[In] Int[Cos[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (2*Cos[a + b*Sqrt[c + d*x]])/(b^2*d) + (2*Sqrt[c + d*x]*Sin[a + b*Sqrt[c + d*x]])/(b*d)
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
```

```
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3443

```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_.), x_S
ymbol] :> Dist[1/(n*f), Subst[Int[x^(1/n - 1)*(a + b*Cos[c + d*x])^p, x], x
, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && Integer
Q[1/n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{2\text{Subst}\left(\int x\cos(a+bx)\,dx, x, \sqrt{c+dx}\right)}{d}\\ &= \frac{2\sqrt{c+dx}\sin\left(a+b\sqrt{c+dx}\right)}{bd} - \frac{2\text{Subst}\left(\int \sin(a+bx)\,dx, x, \sqrt{c+dx}\right)}{bd}\\ &= \frac{2\cos\left(a+b\sqrt{c+dx}\right)}{b^2d} + \frac{2\sqrt{c+dx}\sin\left(a+b\sqrt{c+dx}\right)}{bd}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.89

$$\int \cos\left(a+b\sqrt{c+dx}\right)\,dx = \frac{2\left(\cos\left(a+b\sqrt{c+dx}\right)+b\sqrt{c+dx}\sin\left(a+b\sqrt{c+dx}\right)\right)}{b^2d}$$

```
[In] Integrate[Cos[a + b*Sqrt[c + d*x]],x]
[Out] (2*(Cos[a + b*Sqrt[c + d*x]] + b*Sqrt[c + d*x]*Sin[a + b*Sqrt[c + d*x]]))/(
b^2*d)
```

Maple [A] (verified)

Time = 0.99 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.13

method	result	size
derivativedivides	$\frac{2\cos\left(a+b\sqrt{dx+c}\right)+2\left(a+b\sqrt{dx+c}\right)\sin\left(a+b\sqrt{dx+c}\right)-2a\sin\left(a+b\sqrt{dx+c}\right)}{b^2d}$	61
default	$\frac{2\cos\left(a+b\sqrt{dx+c}\right)+2\left(a+b\sqrt{dx+c}\right)\sin\left(a+b\sqrt{dx+c}\right)-2a\sin\left(a+b\sqrt{dx+c}\right)}{b^2d}$	61

```
[In] int(cos(a+b*(d*x+c)^(1/2)),x,method=_RETURNVERBOSE)
[Out] 2/d/b^2*(cos(a+b*(d*x+c)^(1/2))+(a+b*(d*x+c)^(1/2))*sin(a+b*(d*x+c)^(1/2))-
a*sin(a+b*(d*x+c)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.78

$$\int \cos \left(a+b\sqrt{c+dx}\right) \, dx = \frac{2\left(\sqrt{dx+c}b\sin\left(\sqrt{dx+c}b+a\right)+\cos\left(\sqrt{dx+c}b+a\right)\right)}{b^2d}$$

```
[In] integrate(cos(a+b*(d*x+c)^(1/2)),x, algorithm="fricas")
```

```
[Out] 2*(sqrt(d*x + c)*b*sin(sqrt(d*x + c)*b + a) + cos(sqrt(d*x + c)*b + a))/(b^
2*d)
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.20

$$\int \cos \left(a+b\sqrt{c+dx}\right) \, dx = \begin{cases} x\cos\left(a\right) & \text{for } b=0 \wedge (b=0 \vee d=0) \\ x\cos\left(a+b\sqrt{c}\right) & \text{for } d=0 \\ \frac{2\sqrt{c+dx}\sin\left(a+b\sqrt{c+dx}\right)}{bd}+\frac{2\cos\left(a+b\sqrt{c+dx}\right)}{b^2d} & \text{otherwise} \end{cases}$$

```
[In] integrate(cos(a+b*(d*x+c)**(1/2)),x)
```

```
[Out] Piecewise((x*cos(a), Eq(b, 0) & (Eq(b, 0) | Eq(d, 0))), (x*cos(a + b*sqrt(c
)), Eq(d, 0)), (2*sqrt(c + d*x)*sin(a + b*sqrt(c + d*x))/(b*d) + 2*cos(a +
b*sqrt(c + d*x))/(b**2*d), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.11

$$\int \cos \left(a+b\sqrt{c+dx}\right) \, dx = \frac{2\left(\left(\sqrt{dx+c}b+a\right)\sin\left(\sqrt{dx+c}b+a\right)-a\sin\left(\sqrt{dx+c}b+a\right)+\cos\left(\sqrt{dx+c}b+a\right)\right)}{b^2d}$$

```
[In] integrate(cos(a+b*(d*x+c)^(1/2)),x, algorithm="maxima")
```

```
[Out] 2*((sqrt(d*x + c)*b + a)*sin(sqrt(d*x + c)*b + a) - a*sin(sqrt(d*x + c)*b +
 a) + cos(sqrt(d*x + c)*b + a))/(b^2*d)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.78

$$\int \cos \left(a+b\sqrt{c+dx}\right) \, dx = \frac{2 \left(\sqrt{dx+c}b \sin \left(\sqrt{dx+c}b+a\right)+\cos \left(\sqrt{dx+c}b+a\right)\right)}{b^2 d}$$

```
[In] integrate(cos(a+b*(d*x+c)^(1/2)),x, algorithm="giac")
```

```
[Out] 2*(sqrt(d*x + c)*b*sin(sqrt(d*x + c)*b + a) + cos(sqrt(d*x + c)*b + a))/(b^
2*d)
```

Mupad [B] (verification not implemented)

Time = 13.13 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.78

$$\int \cos \left(a+b\sqrt{c+dx}\right) \, dx = \frac{2 \left(\cos \left(a+b\sqrt{c+d\,x}\right)+b\,\sin \left(a+b\sqrt{c+d\,x}\right)\,\sqrt{c+d\,x}\right)}{b^2\,d}$$

```
[In] int(cos(a + b*(c + d*x)^(1/2)),x)
```

```
[Out] (2*(cos(a + b*(c + d*x)^(1/2)) + b*sin(a + b*(c + d*x)^(1/2))*(c + d*x)^(1/
2)))/(b^2*d)
```

3.93 $\int \frac{\cos\left(a+b\sqrt{c+dx}\right)}{x}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 126

$$\int \frac{\cos\left(a+b\sqrt{c+dx}\right)}{x}\,dx = \cos\left(a-b\sqrt{c}\right)\text{CosIntegral}\left(b\left(\sqrt{c}+\sqrt{c+dx}\right)\right) + \cos\left(a+b\sqrt{c}\right)\text{CosIntegral}\left(b\sqrt{c}-b\sqrt{c+dx}\right) - \sin\left(a-b\sqrt{c}\right)\text{Si}\left(b\left(\sqrt{c}+\sqrt{c+dx}\right)\right) + \sin\left(a+b\sqrt{c}\right)\text{Si}\left(b\sqrt{c}-b\sqrt{c+dx}\right)$$

```
[Out] Ci(b*(c^(1/2)+(d*x+c)^(1/2)))*cos(a-b*c^(1/2))+Ci(b*c^(1/2)-b*(d*x+c)^(1/2)
)*cos(a+b*c^(1/2))-Si(b*(c^(1/2)+(d*x+c)^(1/2)))*sin(a-b*c^(1/2))+Si(b*c^(1
/2)-b*(d*x+c)^(1/2))*sin(a+b*c^(1/2))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3513, 3384, 3380, 3383}

$$\int \frac{\cos\left(a+b\sqrt{c+dx}\right)}{x}\,dx = \cos\left(a-b\sqrt{c}\right)\text{CosIntegral}\left(b\left(\sqrt{c}+\sqrt{c+dx}\right)\right) + \cos\left(a+b\sqrt{c}\right)\text{CosIntegral}\left(b\sqrt{c}-b\sqrt{c+dx}\right) - \sin\left(a-b\sqrt{c}\right)\text{Si}\left(b\left(\sqrt{c}+\sqrt{c+dx}\right)\right) + \sin\left(a+b\sqrt{c}\right)\text{Si}\left(b\sqrt{c}-b\sqrt{c+dx}\right)$$

```
[In] Int[Cos[a + b*Sqrt[c + d*x]]/x,x]
```

```
[Out] Cos[a - b*Sqrt[c]]*CosIntegral[b*(Sqrt[c] + Sqrt[c + d*x])] + Cos[a + b*Sqr
t[c]]*CosIntegral[b*Sqrt[c] - b*Sqrt[c + d*x]] - Sin[a - b*Sqrt[c]]*SinInte
gral[b*(Sqrt[c] + Sqrt[c + d*x])] + Sin[a + b*Sqrt[c]]*SinIntegral[b*Sqrt[c
] - b*Sqrt[c + d*x]]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3513

```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_.)*((g_
.) + (h_.)*(x_))^(m_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Cos[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{2\text{Subst}\left(\int\left(-\frac{d\cos(a+bx)}{2(\sqrt{c}-x)}+\frac{d\cos(a+bx)}{2(\sqrt{c}+x)}\right)dx,x,\sqrt{c+dx}\right)}{d}\\ &= -\text{Subst}\left(\int\frac{\cos(a+bx)}{\sqrt{c}-x}\,dx,x,\sqrt{c+dx}\right)+\text{Subst}\left(\int\frac{\cos(a+bx)}{\sqrt{c}+x}\,dx,x,\sqrt{c+dx}\right)\end{aligned}$$

$$= \cos\left(a-b\sqrt{c}\right)\text{Subst}\left(\int\frac{\cos\left(b\sqrt{c}+bx\right)}{\sqrt{c}+x}\,dx,x,\sqrt{c+dx}\right)$$
$$-\cos\left(a+b\sqrt{c}\right)\text{Subst}\left(\int\frac{\cos\left(b\sqrt{c}-bx\right)}{\sqrt{c}-x}\,dx,x,\sqrt{c+dx}\right)$$
$$-\sin\left(a-b\sqrt{c}\right)\text{Subst}\left(\int\frac{\sin\left(b\sqrt{c}+bx\right)}{\sqrt{c}+x}\,dx,x,\sqrt{c+dx}\right)$$
$$-\sin\left(a+b\sqrt{c}\right)\text{Subst}\left(\int\frac{\sin\left(b\sqrt{c}-bx\right)}{\sqrt{c}-x}\,dx,x,\sqrt{c+dx}\right)$$
$$= \cos\left(a-b\sqrt{c}\right)\text{CosIntegral}\left(b\left(\sqrt{c}+\sqrt{c+dx}\right)\right)$$
$$+\cos\left(a+b\sqrt{c}\right)\text{CosIntegral}\left(b\sqrt{c}-b\sqrt{c+dx}\right)$$
$$-\sin\left(a-b\sqrt{c}\right)\text{Si}\left(b\left(\sqrt{c}+\sqrt{c+dx}\right)\right)+\sin\left(a+b\sqrt{c}\right)\text{Si}\left(b\sqrt{c}-b\sqrt{c+dx}\right)$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.27 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.15

$$\int\frac{\cos\left(a+b\sqrt{c+dx}\right)}{x}\,dx = \frac{1}{2}e^{-i\left(a+b\sqrt{c}\right)}\left(\text{ExpIntegralEi}\left(-ib\left(-\sqrt{c}+\sqrt{c+dx}\right)\right)\right.$$
$$+e^{2i\left(a+b\sqrt{c}\right)}\text{ExpIntegralEi}\left(ib\left(-\sqrt{c}+\sqrt{c+dx}\right)\right)$$
$$+e^{2ib\sqrt{c}}\text{ExpIntegralEi}\left(-ib\left(\sqrt{c}+\sqrt{c+dx}\right)\right)$$
$$\left.+e^{2ia}\text{ExpIntegralEi}\left(ib\left(\sqrt{c}+\sqrt{c+dx}\right)\right)\right)$$

```
[In] Integrate[Cos[a + b*Sqrt[c + d*x]]/x,x]
```

```
[Out] (ExpIntegralEi[(-I)*b*(-Sqrt[c] + Sqrt[c + d*x])] + E^((2*I)*(a + b*Sqrt[c]
))*ExpIntegralEi[I*b*(-Sqrt[c] + Sqrt[c + d*x])] + E^((2*I)*b*Sqrt[c])*ExpI
ntegralEi[(-I)*b*(Sqrt[c] + Sqrt[c + d*x])] + E^((2*I)*a)*ExpIntegralEi[I*b
*(Sqrt[c] + Sqrt[c + d*x])])/(2*E^(I*(a + b*Sqrt[c])))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 270 vs. 2(102) = 204.

Time = 1.08 (sec) , antiderivative size = 271, normalized size of antiderivative = 2.15

method	result
derivativedivides	$\frac{\frac{b(a+b\sqrt{c})(\mathrm{Si}(b\sqrt{c}-b\sqrt{dx+c})\sin(a+b\sqrt{c})+\mathrm{Ci}(b\sqrt{dx+c}-b\sqrt{c})\cos(a+b\sqrt{c}))}{\sqrt{c}}-\frac{b(a-b\sqrt{c})(-\mathrm{Si}(b\sqrt{dx+c}+b\sqrt{c})\sin(a-b\sqrt{c})+\mathrm{Ci}(b\sqrt{dx+c}+}{\sqrt{c}}}{}$
default	$\frac{\frac{b(a+b\sqrt{c})(\mathrm{Si}(b\sqrt{c}-b\sqrt{dx+c})\sin(a+b\sqrt{c})+\mathrm{Ci}(b\sqrt{dx+c}-b\sqrt{c})\cos(a+b\sqrt{c}))}{\sqrt{c}}-\frac{b(a-b\sqrt{c})(-\mathrm{Si}(b\sqrt{dx+c}+b\sqrt{c})\sin(a-b\sqrt{c})+\mathrm{Ci}(b\sqrt{dx+c}+}{\sqrt{c}}}{}$

```
[In] int(cos(a+b*(d*x+c)^(1/2))/x,x,method=_RETURNVERBOSE)
```

```
[Out] 2/b^2*(1/2*b*(a+b*c^(1/2))/c^(1/2)*(Si(b*c^(1/2)-b*(d*x+c)^(1/2))*sin(a+b*c
^(1/2))+Ci(b*(d*x+c)^(1/2)-b*c^(1/2))*cos(a+b*c^(1/2)))-1/2*b*(a-b*c^(1/2))
/c^(1/2)*(-Si(b*(d*x+c)^(1/2)+b*c^(1/2))*sin(a-b*c^(1/2))+Ci(b*(d*x+c)^(1/2
)+b*c^(1/2))*cos(a-b*c^(1/2)))-b^2*a*(1/2/b/c^(1/2)*(Si(b*c^(1/2)-b*(d*x+c)
^(1/2))*sin(a+b*c^(1/2))+Ci(b*(d*x+c)^(1/2)-b*c^(1/2))*cos(a+b*c^(1/2)))-1/
2/b/c^(1/2)*(-Si(b*(d*x+c)^(1/2)+b*c^(1/2))*sin(a-b*c^(1/2))+Ci(b*(d*x+c)^(
1/2)+b*c^(1/2))*cos(a-b*c^(1/2)))))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.18

$$
\begin{aligned}
\int \frac{\cos\left(a+b\sqrt{c+dx}\right)}{x}\,dx = {} & \frac{1}{2}\,\mathrm{Ei}\left(i\,\sqrt{dx+c}b-\sqrt{-b^2c}\right)e^{\left(ia+\sqrt{-b^2c}\right)} \\
& + \frac{1}{2}\,\mathrm{Ei}\left(i\,\sqrt{dx+c}b+\sqrt{-b^2c}\right)e^{\left(ia-\sqrt{-b^2c}\right)} \\
& + \frac{1}{2}\,\mathrm{Ei}\left(-i\,\sqrt{dx+c}b-\sqrt{-b^2c}\right)e^{\left(-ia+\sqrt{-b^2c}\right)} \\
& + \frac{1}{2}\,\mathrm{Ei}\left(-i\,\sqrt{dx+c}b+\sqrt{-b^2c}\right)e^{\left(-ia-\sqrt{-b^2c}\right)}
\end{aligned}
$$

```
[In] integrate(cos(a+b*(d*x+c)^(1/2))/x,x, algorithm="fricas")
```

```
[Out] 1/2*Ei(I*sqrt(d*x + c)*b - sqrt(-b^2*c))*e^(I*a + sqrt(-b^2*c)) + 1/2*Ei(I*
sqrt(d*x + c)*b + sqrt(-b^2*c))*e^(I*a - sqrt(-b^2*c)) + 1/2*Ei(-I*sqrt(d*x
 + c)*b - sqrt(-b^2*c))*e^(-I*a + sqrt(-b^2*c)) + 1/2*Ei(-I*sqrt(d*x + c)*b
 + sqrt(-b^2*c))*e^(-I*a - sqrt(-b^2*c))
```

Sympy [F]

$$\int \frac{\cos \left(a+b\sqrt{c+dx}\right)}{x}\,dx=\int \frac{\cos \left(a+b\sqrt{c+dx}\right)}{x}\,dx$$

```
[In] integrate(cos(a+b*(d*x+c)**(1/2))/x,x)
[Out] Integral(cos(a + b*sqrt(c + d*x))/x, x)
```

Maxima [F]

$$\int \frac{\cos \left(a+b\sqrt{c+dx}\right)}{x}\,dx=\int \frac{\cos \left(\sqrt{dx+c}b+a\right)}{x}\,dx$$

```
[In] integrate(cos(a+b*(d*x+c)^(1/2))/x,x, algorithm="maxima")
[Out] integrate(cos(sqrt(d*x + c)*b + a)/x, x)
```

Giac [F]

$$\int \frac{\cos \left(a+b\sqrt{c+dx}\right)}{x}\,dx=\int \frac{\cos \left(\sqrt{dx+c}b+a\right)}{x}\,dx$$

```
[In] integrate(cos(a+b*(d*x+c)^(1/2))/x,x, algorithm="giac")
[Out] integrate(cos(sqrt(d*x + c)*b + a)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos \left(a+b\sqrt{c+dx}\right)}{x}\,dx=\int \frac{\cos \left(a+b\,\sqrt{c+d\,x}\right)}{x}\,dx$$

```
[In] int(cos(a + b*(c + d*x)^(1/2))/x,x)
[Out] int(cos(a + b*(c + d*x)^(1/2))/x, x)
```

3.94 $\int \frac{\cos\left(a+b\sqrt{c+dx}\right)}{x^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 184

$$\int \frac{\cos\left(a+b\sqrt{c+dx}\right)}{x^2}\,dx = -\frac{\cos\left(a+b\sqrt{c+dx}\right)}{x} + \frac{bd\,\text{CosIntegral}\left(b\left(\sqrt{c}+\sqrt{c+dx}\right)\right)\sin\left(a-b\sqrt{c}\right)}{2\sqrt{c}} - \frac{bd\,\text{CosIntegral}\left(b\sqrt{c}-b\sqrt{c+dx}\right)\sin\left(a+b\sqrt{c}\right)}{2\sqrt{c}} + \frac{bd\cos\left(a-b\sqrt{c}\right)\text{Si}\left(b\left(\sqrt{c}+\sqrt{c+dx}\right)\right)}{2\sqrt{c}} + \frac{bd\cos\left(a+b\sqrt{c}\right)\text{Si}\left(b\sqrt{c}-b\sqrt{c+dx}\right)}{2\sqrt{c}}$$

```
[Out] -cos(a+b*(d*x+c)^(1/2))/x+1/2*b*d*cos(a-b*c^(1/2))*Si(b*(c^(1/2)+(d*x+c)^(1
/2)))/c^(1/2)+1/2*b*d*cos(a+b*c^(1/2))*Si(b*c^(1/2)-b*(d*x+c)^(1/2))/c^(1/2
)+1/2*b*d*Ci(b*(c^(1/2)+(d*x+c)^(1/2)))*sin(a-b*c^(1/2))/c^(1/2)-1/2*b*d*Ci
(b*c^(1/2)-b*(d*x+c)^(1/2))*sin(a+b*c^(1/2))/c^(1/2)
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used

= {3513, 3423, 3414, 3384, 3380, 3383}

$$\int \frac{\cos\left(a+b\sqrt{c+dx}\right)}{x^2}\,dx = \frac{bd\sin\left(a-b\sqrt{c}\right)\operatorname{CosIntegral}\left(b\left(\sqrt{c}+\sqrt{c+dx}\right)\right)}{2\sqrt{c}} - \frac{bd\sin\left(a+b\sqrt{c}\right)\operatorname{CosIntegral}\left(b\sqrt{c}-b\sqrt{c+dx}\right)}{2\sqrt{c}} + \frac{bd\cos\left(a-b\sqrt{c}\right)\operatorname{Si}\left(b\left(\sqrt{c}+\sqrt{c+dx}\right)\right)}{2\sqrt{c}} + \frac{bd\cos\left(a+b\sqrt{c}\right)\operatorname{Si}\left(b\sqrt{c}-b\sqrt{c+dx}\right)}{2\sqrt{c}} - \frac{\cos\left(a+b\sqrt{c+dx}\right)}{x}$$

```
[In] Int[Cos[a + b*Sqrt[c + d*x]]/x^2,x]
```

```
[Out] -(Cos[a + b*Sqrt[c + d*x]]/x) + (b*d*CosIntegral[b*(Sqrt[c] + Sqrt[c + d*x]
)]*Sin[a - b*Sqrt[c]])/(2*Sqrt[c]) - (b*d*CosIntegral[b*Sqrt[c] - b*Sqrt[c
+ d*x]]*Sin[a + b*Sqrt[c]])/(2*Sqrt[c]) + (b*d*Cos[a - b*Sqrt[c]]*SinIntegr
al[b*(Sqrt[c] + Sqrt[c + d*x])])/(2*Sqrt[c]) + (b*d*Cos[a + b*Sqrt[c]]*SinI
ntegral[b*Sqrt[c] - b*Sqrt[c + d*x]])/(2*Sqrt[c])
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Int
[ExpandIntegrand[Sin[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 3423

```
Int[Cos[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_
), x_Symbol] :> Simp[e^m*(a + b*x^n)^(p + 1)*(Cos[c + d*x]/(b*n*(p + 1))),
```

```
x] + Dist[d*(e^m/(b*n*(p + 1))), Int[(a + b*x^n)^(p + 1)*Sin[c + d*x], x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && ILtQ[p, -1] && EqQ[m, n - 1] && (I
ntegerQ[n] || GtQ[e, 0])
```

Rule 3513

```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_.)*((g_
.) + (h_.)*(x_))^(m_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Cos[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rubi steps

$$\text{integral} = \frac{2\text{Subst}\left(\int \frac{x\cos(a+bx)}{\left(-\frac{c}{d}+\frac{x^2}{d}\right)^2}\,dx, x, \sqrt{c+dx}\right)}{d}$$

$$= -\frac{\cos\left(a+b\sqrt{c+dx}\right)}{x} - b\text{Subst}\left(\int \frac{\sin(a+bx)}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx}\right)$$

$$= -\frac{\cos\left(a+b\sqrt{c+dx}\right)}{x} - b\text{Subst}\left(\int \left(-\frac{d\sin(a+bx)}{2\sqrt{c}\left(\sqrt{c}-x\right)} - \frac{d\sin(a+bx)}{2\sqrt{c}\left(\sqrt{c}+x\right)}\right)\,dx, x, \sqrt{c+dx}\right)$$

$$= -\frac{\cos\left(a+b\sqrt{c+dx}\right)}{x} + \frac{(bd)\text{Subst}\left(\int \frac{\sin(a+bx)}{\sqrt{c}-x}\,dx, x, \sqrt{c+dx}\right)}{2\sqrt{c}} + \frac{(bd)\text{Subst}\left(\int \frac{\sin(a+bx)}{\sqrt{c}+x}\,dx, x, \sqrt{c+dx}\right)}{2\sqrt{c}}$$

$$= -\frac{\cos\left(a+b\sqrt{c+dx}\right)}{x} + \frac{\left(bd\cos\left(a-b\sqrt{c}\right)\right)\text{Subst}\left(\int \frac{\sin\left(b\sqrt{c}+bx\right)}{\sqrt{c}+x}\,dx, x, \sqrt{c+dx}\right)}{2\sqrt{c}} - \frac{\left(bd\cos\left(a+b\sqrt{c}\right)\right)\text{Subst}\left(\int \frac{\sin\left(b\sqrt{c}-bx\right)}{\sqrt{c}-x}\,dx, x, \sqrt{c+dx}\right)}{2\sqrt{c}} + \frac{\left(bd\sin\left(a-b\sqrt{c}\right)\right)\text{Subst}\left(\int \frac{\cos\left(b\sqrt{c}+bx\right)}{\sqrt{c}+x}\,dx, x, \sqrt{c+dx}\right)}{2\sqrt{c}} + \frac{\left(bd\sin\left(a+b\sqrt{c}\right)\right)\text{Subst}\left(\int \frac{\cos\left(b\sqrt{c}-bx\right)}{\sqrt{c}-x}\,dx, x, \sqrt{c+dx}\right)}{2\sqrt{c}}$$

$$= -\frac{\cos\left(a+b\sqrt{c+dx}\right)}{x} + \frac{bd\,\text{CosIntegral}\left(b\left(\sqrt{c}+\sqrt{c+dx}\right)\right)\sin\left(a-b\sqrt{c}\right)}{2\sqrt{c}}$$
$$- \frac{bd\,\text{CosIntegral}\left(b\sqrt{c}-b\sqrt{c+dx}\right)\sin\left(a+b\sqrt{c}\right)}{2\sqrt{c}}$$
$$+ \frac{bd\cos\left(a-b\sqrt{c}\right)\text{Si}\left(b\left(\sqrt{c}+\sqrt{c+dx}\right)\right)}{2\sqrt{c}} + \frac{bd\cos\left(a+b\sqrt{c}\right)\text{Si}\left(b\sqrt{c}-b\sqrt{c+dx}\right)}{2\sqrt{c}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.38 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.30

$$\int \frac{\cos\left(a+b\sqrt{c+dx}\right)}{x^2}\,dx$$
$$= \frac{i\left(e^{-ia}\left(2i\sqrt{c}e^{-ib\sqrt{c+dx}} - bde^{-ib\sqrt{c}}x\,\text{ExpIntegralEi}\left(-ib\left(-\sqrt{c}+\sqrt{c+dx}\right)\right) + bde^{ib\sqrt{c}}x\,\text{ExpIntegralEi}\left(-ib\right.\right.\right.}{}$$

```
[In] Integrate[Cos[a + b*Sqrt[c + d*x]]/x^2,x]
```

```
[Out] ((I/4)*((((2*I)*Sqrt[c])/E^(I*b*Sqrt[c + d*x]) - (b*d*x*ExpIntegralEi[(-I)*
b*(-Sqrt[c] + Sqrt[c + d*x])])/E^(I*b*Sqrt[c]) + b*d*E^(I*b*Sqrt[c])*x*ExpI
ntegralEi[(-I)*b*(Sqrt[c] + Sqrt[c + d*x])])/E^(I*a) + E^(I*(a - b*Sqrt[c])
)*((2*I)*Sqrt[c]*E^(I*b*(Sqrt[c] + Sqrt[c + d*x])) + b*d*E^((2*I)*b*Sqrt[c]
)*x*ExpIntegralEi[I*b*(-Sqrt[c] + Sqrt[c + d*x])] - b*d*x*ExpIntegralEi[I*b
*(Sqrt[c] + Sqrt[c + d*x])])))/(Sqrt[c]*x)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 713 vs. $2(142) = 284$.

Time = 1.34 (sec) , antiderivative size = 714, normalized size of antiderivative = 3.88

method	result
derivativedivides	$\frac{2d\left(\frac{\cos\left(a+b\sqrt{dx+c}\right)\left(-\frac{a\,b^2\left(a+b\sqrt{dx+c}\right)}{2c}+\frac{b^2\left(-b^2c+a^2\right)}{2c}\right)}{-b^2c+a^2-2\left(a+b\sqrt{dx+c}\right)a+\left(a+b\sqrt{dx+c}\right)^2}-\frac{ab\left(\text{Si}\left(b\sqrt{c}-b\sqrt{dx+c}\right)\sin\left(a+b\sqrt{c}\right)+\text{Ci}\left(b\sqrt{dx+c}-b\sqrt{c}\right)\cos\left(a+b\sqrt{c}\right)\right)}{4c^{\frac{3}{2}}}+\right.}{}$
default	$\frac{2d\left(\frac{\cos\left(a+b\sqrt{dx+c}\right)\left(-\frac{a\,b^2\left(a+b\sqrt{dx+c}\right)}{2c}+\frac{b^2\left(-b^2c+a^2\right)}{2c}\right)}{-b^2c+a^2-2\left(a+b\sqrt{dx+c}\right)a+\left(a+b\sqrt{dx+c}\right)^2}-\frac{ab\left(\text{Si}\left(b\sqrt{c}-b\sqrt{dx+c}\right)\sin\left(a+b\sqrt{c}\right)+\text{Ci}\left(b\sqrt{dx+c}-b\sqrt{c}\right)\cos\left(a+b\sqrt{c}\right)\right)}{4c^{\frac{3}{2}}}+\right.}{}$

```
[In] int(cos(a+b*(d*x+c)^(1/2))/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2*d/b^2*(cos(a+b*(d*x+c)^(1/2))*(-1/2*a*b^2/c*(a+b*(d*x+c)^(1/2))+1/2*b^2*(
-b^2*c+a^2)/c)/(-b^2*c+a^2-2*(a+b*(d*x+c)^(1/2))*a+(a+b*(d*x+c)^(1/2))^2)-1
/4*a*b/c^(3/2)*(Si(b*c^(1/2)-b*(d*x+c)^(1/2))*sin(a+b*c^(1/2))+Ci(b*(d*x+c)
^(1/2)-b*c^(1/2))*cos(a+b*c^(1/2)))+1/4*a*b/c^(3/2)*(-Si(b*(d*x+c)^(1/2)+b*
c^(1/2))*sin(a-b*c^(1/2))+Ci(b*(d*x+c)^(1/2)+b*c^(1/2))*cos(a-b*c^(1/2)))+1
/4*b*(-b^2*c+a^2-(a+b*c^(1/2))*a)/c^(3/2)*(-Si(b*c^(1/2)-b*(d*x+c)^(1/2))*c
os(a+b*c^(1/2))+Ci(b*(d*x+c)^(1/2)-b*c^(1/2))*sin(a+b*c^(1/2)))-1/4*b*(-b^2
*c+a^2-(a-b*c^(1/2))*a)/c^(3/2)*(Si(b*(d*x+c)^(1/2)+b*c^(1/2))*cos(a-b*c^(1
/2))+Ci(b*(d*x+c)^(1/2)+b*c^(1/2))*sin(a-b*c^(1/2)))-a*b^4*(cos(a+b*(d*x+c)
^(1/2))*(-1/2/c/b^2*(a+b*(d*x+c)^(1/2))+1/2*a/c/b^2)/(-b^2*c+a^2-2*(a+b*(d*
x+c)^(1/2))*a+(a+b*(d*x+c)^(1/2))^2)-1/4/c^(3/2)/b^3*(Si(b*c^(1/2)-b*(d*x+c
)^(1/2))*sin(a+b*c^(1/2))+Ci(b*(d*x+c)^(1/2)-b*c^(1/2))*cos(a+b*c^(1/2)))+1
/4/c^(3/2)/b^3*(-Si(b*(d*x+c)^(1/2)+b*c^(1/2))*sin(a-b*c^(1/2))+Ci(b*(d*x+c
)^(1/2)+b*c^(1/2))*cos(a-b*c^(1/2)))-1/4/c/b^2*(-Si(b*c^(1/2)-b*(d*x+c)^(1/
2))*cos(a+b*c^(1/2))+Ci(b*(d*x+c)^(1/2)-b*c^(1/2))*sin(a+b*c^(1/2)))-1/4/c/
b^2*(Si(b*(d*x+c)^(1/2)+b*c^(1/2))*cos(a-b*c^(1/2))+Ci(b*(d*x+c)^(1/2)+b*c^
(1/2))*sin(a-b*c^(1/2)))))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.33 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.14

$$\int \frac{\cos\left(a+b\sqrt{c+dx}\right)}{x^2}\,dx$$
$$= \frac{\sqrt{-b^2c}dx\mathrm{Ei}\left(i\sqrt{dx+c}b-\sqrt{-b^2c}\right)e^{\left(ia+\sqrt{-b^2c}\right)} - \sqrt{-b^2c}dx\mathrm{Ei}\left(i\sqrt{dx+c}b+\sqrt{-b^2c}\right)e^{\left(ia-\sqrt{-b^2c}\right)} + \sqrt{-b^2c}d}{}$$

```
[In] integrate(cos(a+b*(d*x+c)^(1/2))/x^2,x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(-b^2*c)*d*x*Ei(I*sqrt(d*x + c)*b - sqrt(-b^2*c))*e^(I*a + sqrt(-b
^2*c)) - sqrt(-b^2*c)*d*x*Ei(I*sqrt(d*x + c)*b + sqrt(-b^2*c))*e^(I*a - sqr
t(-b^2*c)) + sqrt(-b^2*c)*d*x*Ei(-I*sqrt(d*x + c)*b - sqrt(-b^2*c))*e^(-I*a
 + sqrt(-b^2*c)) - sqrt(-b^2*c)*d*x*Ei(-I*sqrt(d*x + c)*b + sqrt(-b^2*c))*e
^(-I*a - sqrt(-b^2*c)) - 4*c*cos(sqrt(d*x + c)*b + a))/(c*x)
```

Sympy [F]

$$\int \frac{\cos\left(a+b\sqrt{c+dx}\right)}{x^2}\,dx = \int \frac{\cos\left(a+b\sqrt{c+dx}\right)}{x^2}\,dx$$

```
[In] integrate(cos(a+b*(d*x+c)**(1/2))/x**2,x)
[Out] Integral(cos(a + b*sqrt(c + d*x))/x**2, x)
```

Maxima [F]

$$\int \frac{\cos\left(a+b\sqrt{c+dx}\right)}{x^2}\,dx = \int \frac{\cos\left(\sqrt{dx+c}b+a\right)}{x^2}\,dx$$

```
[In] integrate(cos(a+b*(d*x+c)^(1/2))/x^2,x, algorithm="maxima")
[Out] integrate(cos(sqrt(d*x + c)*b + a)/x^2, x)
```

Giac [F]

$$\int \frac{\cos\left(a+b\sqrt{c+dx}\right)}{x^2}\,dx = \int \frac{\cos\left(\sqrt{dx+c}b+a\right)}{x^2}\,dx$$

```
[In] integrate(cos(a+b*(d*x+c)^(1/2))/x^2,x, algorithm="giac")
[Out] integrate(cos(sqrt(d*x + c)*b + a)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos\left(a+b\sqrt{c+dx}\right)}{x^2}\,dx = \int \frac{\cos\left(a+b\,\sqrt{c+d\,x}\right)}{x^2}\,dx$$

```
[In] int(cos(a + b*(c + d*x)^(1/2))/x^2,x)
[Out] int(cos(a + b*(c + d*x)^(1/2))/x^2, x)
```

3.95 $\int x^2 \cos \left(a+b\sqrt[3]{c+dx}\right) \, dx$

Optimal result

Integrand size = 18, antiderivative size = 537

$$\int x^2 \cos\left(a+b\sqrt[3]{c+dx}\right) dx = -\frac{720c\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^6d^3} - \frac{120960\sqrt[3]{c+dx}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^8d^3} + \frac{6c^2\sqrt[3]{c+dx}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^3} + \frac{360c(c+dx)^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^4d^3} + \frac{20160(c+dx)\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^6d^3} - \frac{30c(c+dx)^{4/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^3} - \frac{1008(c+dx)^{5/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^4d^3} + \frac{24(c+dx)^{7/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^3} + \frac{120960\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^9d^3} - \frac{6c^2\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^3} - \frac{720c\sqrt[3]{c+dx}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^5d^3} - \frac{60480(c+dx)^{2/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^7d^3} + \frac{3c^2(c+dx)^{2/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{bd^3} + \frac{120c(c+dx)\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^3} + \frac{5040(c+dx)^{4/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^5d^3} - \frac{6c(c+dx)^{5/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{bd^3} - \frac{168(c+dx)^2\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^3} + \frac{3(c+dx)^{8/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{bd^3}$$

```
[Out] -720*c*cos(a+b*(d*x+c)^(1/3))/b^6/d^3-120960*(d*x+c)^(1/3)*cos(a+b*(d*x+c)^
(1/3))/b^8/d^3+6*c^2*(d*x+c)^(1/3)*cos(a+b*(d*x+c)^(1/3))/b^2/d^3+360*c*(d*
x+c)^(2/3)*cos(a+b*(d*x+c)^(1/3))/b^4/d^3+20160*(d*x+c)*cos(a+b*(d*x+c)^(1/
3))/b^6/d^3-30*c*(d*x+c)^(4/3)*cos(a+b*(d*x+c)^(1/3))/b^2/d^3-1008*(d*x+c)^
(5/3)*cos(a+b*(d*x+c)^(1/3))/b^4/d^3+24*(d*x+c)^(7/3)*cos(a+b*(d*x+c)^(1/3)
)/b^2/d^3+120960*sin(a+b*(d*x+c)^(1/3))/b^9/d^3-6*c^2*sin(a+b*(d*x+c)^(1/3)
)/b^3/d^3-720*c*(d*x+c)^(1/3)*sin(a+b*(d*x+c)^(1/3))/b^5/d^3-60480*(d*x+c)^
(2/3)*sin(a+b*(d*x+c)^(1/3))/b^7/d^3+3*c^2*(d*x+c)^(2/3)*sin(a+b*(d*x+c)^(1
/3))/b/d^3+120*c*(d*x+c)*sin(a+b*(d*x+c)^(1/3))/b^3/d^3+5040*(d*x+c)^(4/3)*
sin(a+b*(d*x+c)^(1/3))/b^5/d^3-6*c*(d*x+c)^(5/3)*sin(a+b*(d*x+c)^(1/3))/b/d
^3-168*(d*x+c)^2*sin(a+b*(d*x+c)^(1/3))/b^3/d^3+3*(d*x+c)^(8/3)*sin(a+b*(d*
x+c)^(1/3))/b/d^3
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 537, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used

$= \{3513, 3377, 2717, 2718\}$

$$\int x^2 \cos\left(a+b\sqrt[3]{c+dx}\right)\,dx = \frac{120960\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^9d^3} - \frac{120960\sqrt[3]{c+dx}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^8d^3} - \frac{60480(c+dx)^{2/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^7d^3} + \frac{20160(c+dx)\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^6d^3} - \frac{720c\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^6d^3} + \frac{5040(c+dx)^{4/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^5d^3} - \frac{720c\sqrt[3]{c+dx}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^5d^3} - \frac{1008(c+dx)^{5/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^4d^3} + \frac{360c(c+dx)^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^4d^3} - \frac{6c^2\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^3} - \frac{168(c+dx)^2\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^3} + \frac{120c(c+dx)\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^3} + \frac{6c^2\sqrt[3]{c+dx}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^3} + \frac{24(c+dx)^{7/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^3} - \frac{30c(c+dx)^{4/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^3} + \frac{3c^2(c+dx)^{2/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{bd^3} + \frac{3(c+dx)^{8/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{bd^3} - \frac{6c(c+dx)^{5/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{bd^3}$$

```
[In] Int[x^2*Cos[a + b*(c + d*x)^(1/3)],x]
```

```
[Out] (-720*c*Cos[a + b*(c + d*x)^(1/3)])/(b^6*d^3) - (120960*(c + d*x)^(1/3)*Cos
[a + b*(c + d*x)^(1/3)])/(b^8*d^3) + (6*c^2*(c + d*x)^(1/3)*Cos[a + b*(c +
d*x)^(1/3)])/(b^2*d^3) + (360*c*(c + d*x)^(2/3)*Cos[a + b*(c + d*x)^(1/3)])
/(b^4*d^3) + (20160*(c + d*x)*Cos[a + b*(c + d*x)^(1/3)])/(b^6*d^3) - (30*c
*(c + d*x)^(4/3)*Cos[a + b*(c + d*x)^(1/3)])/(b^2*d^3) - (1008*(c + d*x)^(5
/3)*Cos[a + b*(c + d*x)^(1/3)])/(b^4*d^3) + (24*(c + d*x)^(7/3)*Cos[a + b*(
c + d*x)^(1/3)])/(b^2*d^3) + (120960*Sin[a + b*(c + d*x)^(1/3)])/(b^9*d^3)
- (6*c^2*Sin[a + b*(c + d*x)^(1/3)])/(b^3*d^3) - (720*c*(c + d*x)^(1/3)*Sin
[a + b*(c + d*x)^(1/3)])/(b^5*d^3) - (60480*(c + d*x)^(2/3)*Sin[a + b*(c +
d*x)^(1/3)])/(b^7*d^3) + (3*c^2*(c + d*x)^(2/3)*Sin[a + b*(c + d*x)^(1/3)])
/(b*d^3) + (120*c*(c + d*x)*Sin[a + b*(c + d*x)^(1/3)])/(b^3*d^3) + (5040*(
c + d*x)^(4/3)*Sin[a + b*(c + d*x)^(1/3)])/(b^5*d^3) - (6*c*(c + d*x)^(5/3)
*Sin[a + b*(c + d*x)^(1/3)])/(b*d^3) - (168*(c + d*x)^2*Sin[a + b*(c + d*x)
^(1/3)])/(b^3*d^3) + (3*(c + d*x)^(8/3)*Sin[a + b*(c + d*x)^(1/3)])/(b*d^3)
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3513

```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_.)*((g_
.) + (h_.)*(x_))^(m_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Cos[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rubi steps

$$\text{integral} = \frac{3\text{Subst}\left(\int \left(\frac{c^2x^2\cos(a+bx)}{d^2} - \frac{2cx^5\cos(a+bx)}{d^2} + \frac{x^8\cos(a+bx)}{d^2}\right)\,dx, x, \sqrt[3]{c+dx}\right)}{d}$$

$$= \frac{3\text{Subst}\left(\int x^8 \cos(a+bx)\,dx, x, \sqrt[3]{c+dx}\right)}{d^3} - \frac{(6c)\text{Subst}\left(\int x^5 \cos(a+bx)\,dx, x, \sqrt[3]{c+dx}\right)}{d^3} + \frac{(3c^2)\,\text{Subst}\left(\int x^2 \cos(a+bx)\,dx, x, \sqrt[3]{c+dx}\right)}{d^3}$$

$$= \frac{3c^2(c+dx)^{2/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{bd^3} - \frac{6c(c+dx)^{5/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{bd^3} + \frac{3(c+dx)^{8/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{bd^3} - \frac{24\text{Subst}\left(\int x^7 \sin(a+bx)\,dx, x, \sqrt[3]{c+dx}\right)}{bd^3} + \frac{(30c)\text{Subst}\left(\int x^4 \sin(a+bx)\,dx, x, \sqrt[3]{c+dx}\right)}{bd^3} - \frac{(6c^2)\,\text{Subst}\left(\int x \sin(a+bx)\,dx, x, \sqrt[3]{c+dx}\right)}{bd^3}$$

$$= \frac{6c^2\sqrt[3]{c+dx}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^3} - \frac{30c(c+dx)^{4/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^3} + \frac{24(c+dx)^{7/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^3} + \frac{3c^2(c+dx)^{2/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{bd^3} - \frac{6c(c+dx)^{5/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{bd^3} + \frac{3(c+dx)^{8/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{bd^3} - \frac{168\text{Subst}\left(\int x^6 \cos(a+bx)\,dx, x, \sqrt[3]{c+dx}\right)}{b^2d^3} + \frac{(120c)\text{Subst}\left(\int x^3 \cos(a+bx)\,dx, x, \sqrt[3]{c+dx}\right)}{b^2d^3} - \frac{(6c^2)\,\text{Subst}\left(\int \cos(a+bx)\,dx, x, \sqrt[3]{c+dx}\right)}{b^2d^3}$$

$$= \frac{6c^2 \sqrt[3]{c+dx} \cos\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^3} - \frac{30c(c+dx)^{4/3} \cos\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^3} + \frac{24(c+dx)^{7/3} \cos\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^3} - \frac{6c^2 \sin\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^3} + \frac{3c^2(c+dx)^{2/3} \sin\left(a+b\sqrt[3]{c+dx}\right)}{bd^3} + \frac{120c(c+dx) \sin\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^3} - \frac{6c(c+dx)^{5/3} \sin\left(a+b\sqrt[3]{c+dx}\right)}{bd^3} - \frac{168(c+dx)^2 \sin\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^3} + \frac{3(c+dx)^{8/3} \sin\left(a+b\sqrt[3]{c+dx}\right)}{bd^3} + \frac{1008\text{Subst}\left(\int x^5 \sin(a+bx)\,dx, x, \sqrt[3]{c+dx}\right)}{b^3d^3} - \frac{(360c)\text{Subst}\left(\int x^2 \sin(a+bx)\,dx, x, \sqrt[3]{c+dx}\right)}{b^3d^3}$$

$$= \frac{6c^2 \sqrt[3]{c+dx} \cos\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^3} + \frac{360c(c+dx)^{2/3} \cos\left(a+b\sqrt[3]{c+dx}\right)}{b^4d^3} - \frac{30c(c+dx)^{4/3} \cos\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^3} - \frac{1008(c+dx)^{5/3} \cos\left(a+b\sqrt[3]{c+dx}\right)}{b^4d^3} + \frac{24(c+dx)^{7/3} \cos\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^3} - \frac{6c^2 \sin\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^3} + \frac{3c^2(c+dx)^{2/3} \sin\left(a+b\sqrt[3]{c+dx}\right)}{bd^3} + \frac{120c(c+dx) \sin\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^3} - \frac{6c(c+dx)^{5/3} \sin\left(a+b\sqrt[3]{c+dx}\right)}{bd^3} - \frac{168(c+dx)^2 \sin\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^3} + \frac{3(c+dx)^{8/3} \sin\left(a+b\sqrt[3]{c+dx}\right)}{bd^3} + \frac{5040\text{Subst}\left(\int x^4 \cos(a+bx)\,dx, x, \sqrt[3]{c+dx}\right)}{b^4d^3} - \frac{(720c)\text{Subst}\left(\int x \cos(a+bx)\,dx, x, \sqrt[3]{c+dx}\right)}{b^4d^3}$$

$$= \frac{6c^2\sqrt[3]{c+dx}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^3} + \frac{360c(c+dx)^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^4d^3} - \frac{30c(c+dx)^{4/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^3} - \frac{1008(c+dx)^{5/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^4d^3} + \frac{24(c+dx)^{7/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^3} - \frac{6c^2\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^3} - \frac{720c\sqrt[3]{c+dx}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^5d^3} + \frac{3c^2(c+dx)^{2/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{bd^3} + \frac{120c(c+dx)\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^3} + \frac{5040(c+dx)^{4/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^5d^3} - \frac{6c(c+dx)^{5/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{bd^3} - \frac{168(c+dx)^2\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^3} + \frac{3(c+dx)^{8/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{bd^3} - \frac{20160\text{Subst}\left(\int x^3\sin(a+bx)\,dx, x, \sqrt[3]{c+dx}\right)}{b^5d^3} + \frac{(720c)\text{Subst}\left(\int \sin(a+bx)\,dx, x, \sqrt[3]{c+dx}\right)}{b^5d^3}$$

$$
\begin{aligned}
&= -\frac{720c\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^6d^3} + \frac{6c^2\sqrt[3]{c+dx}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^3} \\
&+ \frac{360c(c+dx)^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^4d^3} \\
&+ \frac{20160(c+dx)\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^6d^3} - \frac{30c(c+dx)^{4/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^3} \\
&- \frac{1008(c+dx)^{5/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^4d^3} + \frac{24(c+dx)^{7/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^3} \\
&- \frac{6c^2\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^3} - \frac{720c\sqrt[3]{c+dx}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^5d^3} \\
&+ \frac{3c^2(c+dx)^{2/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{bd^3} + \frac{120c(c+dx)\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^3} \\
&+ \frac{5040(c+dx)^{4/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^5d^3} - \frac{6c(c+dx)^{5/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{bd^3} \\
&- \frac{168(c+dx)^2\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^3} + \frac{3(c+dx)^{8/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{bd^3} \\
&- \frac{60480\text{Subst}\left(\int x^2\cos(a+bx)\,dx, x, \sqrt[3]{c+dx}\right)}{b^6d^3}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{720c\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^6d^3} + \frac{6c^2\sqrt[3]{c+dx}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^3} \\
&+ \frac{360c(c+dx)^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^4d^3} + \frac{20160(c+dx)\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^6d^3} \\
&- \frac{30c(c+dx)^{4/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^3} - \frac{1008(c+dx)^{5/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^4d^3} \\
&+ \frac{24(c+dx)^{7/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^3} - \frac{6c^2\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^3} \\
&- \frac{720c\sqrt[3]{c+dx}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^5d^3} - \frac{60480(c+dx)^{2/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^7d^3} \\
&+ \frac{3c^2(c+dx)^{2/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{bd^3} + \frac{120c(c+dx)\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^3} \\
&+ \frac{5040(c+dx)^{4/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^5d^3} - \frac{6c(c+dx)^{5/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{bd^3} \\
&- \frac{168(c+dx)^2\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^3} + \frac{3(c+dx)^{8/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{bd^3} \\
&+ \frac{120960\text{Subst}\left(\int x\sin(a+bx)\,dx, x, \sqrt[3]{c+dx}\right)}{b^7d^3}
\end{aligned}
$$

$$= -\frac{720c\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^6d^3} - \frac{120960\sqrt[3]{c+dx}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^8d^3}$$
$$+ \frac{6c^2\sqrt[3]{c+dx}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^3} + \frac{360c(c+dx)^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^4d^3}$$
$$+ \frac{20160(c+dx)\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^6d^3} - \frac{30c(c+dx)^{4/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^3}$$
$$- \frac{1008(c+dx)^{5/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^4d^3} + \frac{24(c+dx)^{7/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^3}$$
$$- \frac{6c^2\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^3} - \frac{720c\sqrt[3]{c+dx}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^5d^3}$$
$$- \frac{60480(c+dx)^{2/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^7d^3} + \frac{3c^2(c+dx)^{2/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{bd^3}$$
$$+ \frac{120c(c+dx)\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^3} + \frac{5040(c+dx)^{4/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^5d^3}$$
$$- \frac{6c(c+dx)^{5/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{bd^3} - \frac{168(c+dx)^2\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^3}$$
$$+ \frac{3(c+dx)^{8/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{bd^3} + \frac{120960\text{Subst}\left(\int\cos(a+bx)\,dx, x, \sqrt[3]{c+dx}\right)}{b^8d^3}$$

$$= -\frac{720c\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^6d^3} - \frac{120960\sqrt[3]{c+dx}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^8d^3} + \frac{6c^2\sqrt[3]{c+dx}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^3} + \frac{360c(c+dx)^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^4d^3} + \frac{20160(c+dx)\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^6d^3} - \frac{30c(c+dx)^{4/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^3} - \frac{1008(c+dx)^{5/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^4d^3} + \frac{24(c+dx)^{7/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^3} + \frac{120960\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^9d^3} - \frac{6c^2\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^3} - \frac{720c\sqrt[3]{c+dx}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^5d^3} - \frac{60480(c+dx)^{2/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^7d^3} + \frac{3c^2(c+dx)^{2/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{bd^3} + \frac{120c(c+dx)\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^3} + \frac{5040(c+dx)^{4/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^5d^3} - \frac{6c(c+dx)^{5/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{bd^3} - \frac{168(c+dx)^2\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^3} + \frac{3(c+dx)^{8/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{bd^3}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.21 (sec) , antiderivative size = 382, normalized size of antiderivative = 0.71

$$\int x^2\cos\left(a+b\sqrt[3]{c+dx}\right)\,dx$$

$$= \frac{3e^{-i\left(a+b\sqrt[3]{c+dx}\right)}\left(-40320i\left(-1+e^{2i\left(a+b\sqrt[3]{c+dx}\right)}\right)-40320b\left(1+e^{2i\left(a+b\sqrt[3]{c+dx}\right)}\right)\sqrt[3]{c+dx}+20160ib^2\left(-1\right.\right.}{}$$

```
[In] Integrate[x^2*Cos[a + b*(c + d*x)^(1/3)],x]
```

```
[Out] (3*((-40320*I)*(-1 + E^((2*I)*(a + b*(c + d*x)^(1/3)))) - 40320*b*(1 + E^((
2*I)*(a + b*(c + d*x)^(1/3))))*(c + d*x)^(1/3) + (20160*I)*b^2*(-1 + E^((2*
I)*(a + b*(c + d*x)^(1/3))))*(c + d*x)^(2/3) - I*b^8*d^2*(-1 + E^((2*I)*(a
+ b*(c + d*x)^(1/3))))*x^2*(c + d*x)^(2/3) + 2*b^7*d*(1 + E^((2*I)*(a + b*(
c + d*x)^(1/3))))*x*(c + d*x)^(1/3)*(3*c + 4*d*x) - (240*I)*b^4*(-1 + E^((2
*I)*(a + b*(c + d*x)^(1/3))))*(c + d*x)^(1/3)*(6*c + 7*d*x) - 24*b^5*(1 + E
^((2*I)*(a + b*(c + d*x)^(1/3))))*(c + d*x)^(2/3)*(9*c + 14*d*x) + 240*b^3*
(1 + E^((2*I)*(a + b*(c + d*x)^(1/3))))*(27*c + 28*d*x) + (2*I)*b^6*(-1 + E
```

```
^((2*I)*(a + b*(c + d*x)^(1/3))))*(9*c^2 + 36*c*d*x + 28*d^2*x^2)))/(2*b^9*
d^3*E^(I*(a + b*(c + d*x)^(1/3))))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1808 vs. $2(477) = 954$.

Time = 1.62 (sec) , antiderivative size = 1809, normalized size of antiderivative = 3.37

method	result	size
derivativedivides	Expression too large to display	1809
default	Expression too large to display	1809
parts	Expression too large to display	2944

```
[In] int(x^2*cos(a+b*(d*x+c)^(1/3)),x,method=_RETURNVERBOSE)
```

```
[Out] 3/d^3/b^3*(a^2*c^2*sin(a+b*(d*x+c)^(1/3))-2*a*c^2*(cos(a+b*(d*x+c)^(1/3))+(
a+b*(d*x+c)^(1/3))*sin(a+b*(d*x+c)^(1/3)))+c^2*((a+b*(d*x+c)^(1/3))^2*sin(a
+b*(d*x+c)^(1/3))-2*sin(a+b*(d*x+c)^(1/3))+2*(a+b*(d*x+c)^(1/3))*cos(a+b*(d
*x+c)^(1/3)))+2/b^3*a^5*c*sin(a+b*(d*x+c)^(1/3))-10/b^3*a^4*c*(cos(a+b*(d*x
+c)^(1/3))+(a+b*(d*x+c)^(1/3))*sin(a+b*(d*x+c)^(1/3)))+20/b^3*a^3*c*((a+b*(
d*x+c)^(1/3))^2*sin(a+b*(d*x+c)^(1/3))-2*sin(a+b*(d*x+c)^(1/3))+2*(a+b*(d*x
+c)^(1/3))*cos(a+b*(d*x+c)^(1/3)))-20/b^3*a^2*c*((a+b*(d*x+c)^(1/3))^3*sin(
a+b*(d*x+c)^(1/3))+3*(a+b*(d*x+c)^(1/3))^2*cos(a+b*(d*x+c)^(1/3))-6*cos(a+b
*(d*x+c)^(1/3))-6*(a+b*(d*x+c)^(1/3))*sin(a+b*(d*x+c)^(1/3)))+10/b^3*a*c*((
a+b*(d*x+c)^(1/3))^4*sin(a+b*(d*x+c)^(1/3))+4*(a+b*(d*x+c)^(1/3))^3*cos(a+b
*(d*x+c)^(1/3))-12*(a+b*(d*x+c)^(1/3))^2*sin(a+b*(d*x+c)^(1/3))+24*sin(a+b*
(d*x+c)^(1/3))-24*(a+b*(d*x+c)^(1/3))*cos(a+b*(d*x+c)^(1/3)))-2/b^3*c*((a+b
*(d*x+c)^(1/3))^5*sin(a+b*(d*x+c)^(1/3))+5*(a+b*(d*x+c)^(1/3))^4*cos(a+b*(d
*x+c)^(1/3))-20*(a+b*(d*x+c)^(1/3))^3*sin(a+b*(d*x+c)^(1/3))-60*(a+b*(d*x+c
)^(1/3))^2*cos(a+b*(d*x+c)^(1/3))+120*cos(a+b*(d*x+c)^(1/3))+120*(a+b*(d*x+
c)^(1/3))*sin(a+b*(d*x+c)^(1/3)))+1/b^6*a^8*sin(a+b*(d*x+c)^(1/3))-8/b^6*a^
7*(cos(a+b*(d*x+c)^(1/3))+(a+b*(d*x+c)^(1/3))*sin(a+b*(d*x+c)^(1/3)))+28/b^
6*a^6*((a+b*(d*x+c)^(1/3))^2*sin(a+b*(d*x+c)^(1/3))-2*sin(a+b*(d*x+c)^(1/3)
)+2*(a+b*(d*x+c)^(1/3))*cos(a+b*(d*x+c)^(1/3)))-56/b^6*a^5*((a+b*(d*x+c)^(1
/3))^3*sin(a+b*(d*x+c)^(1/3))+3*(a+b*(d*x+c)^(1/3))^2*cos(a+b*(d*x+c)^(1/3)
)-6*cos(a+b*(d*x+c)^(1/3))-6*(a+b*(d*x+c)^(1/3))*sin(a+b*(d*x+c)^(1/3)))+70
/b^6*a^4*((a+b*(d*x+c)^(1/3))^4*sin(a+b*(d*x+c)^(1/3))+4*(a+b*(d*x+c)^(1/3)
)^3*cos(a+b*(d*x+c)^(1/3))-12*(a+b*(d*x+c)^(1/3))^2*sin(a+b*(d*x+c)^(1/3))+
24*sin(a+b*(d*x+c)^(1/3))-24*(a+b*(d*x+c)^(1/3))*cos(a+b*(d*x+c)^(1/3)))-56
/b^6*a^3*((a+b*(d*x+c)^(1/3))^5*sin(a+b*(d*x+c)^(1/3))+5*(a+b*(d*x+c)^(1/3)
)^4*cos(a+b*(d*x+c)^(1/3))-20*(a+b*(d*x+c)^(1/3))^3*sin(a+b*(d*x+c)^(1/3))-
60*(a+b*(d*x+c)^(1/3))^2*cos(a+b*(d*x+c)^(1/3))+120*cos(a+b*(d*x+c)^(1/3))+
120*(a+b*(d*x+c)^(1/3))*sin(a+b*(d*x+c)^(1/3)))+28/b^6*a^2*((a+b*(d*x+c)^(1
/3))^6*sin(a+b*(d*x+c)^(1/3))+6*(a+b*(d*x+c)^(1/3))^5*cos(a+b*(d*x+c)^(1/3)
```

```
)-30*(a+b*(d*x+c)^(1/3))^4*sin(a+b*(d*x+c)^(1/3))-120*(a+b*(d*x+c)^(1/3))^3
*cos(a+b*(d*x+c)^(1/3))+360*(a+b*(d*x+c)^(1/3))^2*sin(a+b*(d*x+c)^(1/3))-72
0*sin(a+b*(d*x+c)^(1/3))+720*(a+b*(d*x+c)^(1/3))*cos(a+b*(d*x+c)^(1/3)))-8/
b^6*a*((a+b*(d*x+c)^(1/3))^7*sin(a+b*(d*x+c)^(1/3))+7*(a+b*(d*x+c)^(1/3))^6
*cos(a+b*(d*x+c)^(1/3))-42*(a+b*(d*x+c)^(1/3))^5*sin(a+b*(d*x+c)^(1/3))-210
*(a+b*(d*x+c)^(1/3))^4*cos(a+b*(d*x+c)^(1/3))+840*(a+b*(d*x+c)^(1/3))^3*sin
(a+b*(d*x+c)^(1/3))+2520*(a+b*(d*x+c)^(1/3))^2*cos(a+b*(d*x+c)^(1/3))-5040*
cos(a+b*(d*x+c)^(1/3))-5040*(a+b*(d*x+c)^(1/3))*sin(a+b*(d*x+c)^(1/3)))+1/b
^6*((a+b*(d*x+c)^(1/3))^8*sin(a+b*(d*x+c)^(1/3))+8*(a+b*(d*x+c)^(1/3))^7*co
s(a+b*(d*x+c)^(1/3))-56*(a+b*(d*x+c)^(1/3))^6*sin(a+b*(d*x+c)^(1/3))-336*(a
+b*(d*x+c)^(1/3))^5*cos(a+b*(d*x+c)^(1/3))+1680*(a+b*(d*x+c)^(1/3))^4*sin(a
+b*(d*x+c)^(1/3))+6720*(a+b*(d*x+c)^(1/3))^3*cos(a+b*(d*x+c)^(1/3))-20160*(
a+b*(d*x+c)^(1/3))^2*sin(a+b*(d*x+c)^(1/3))+40320*sin(a+b*(d*x+c)^(1/3))-40
320*(a+b*(d*x+c)^(1/3))*cos(a+b*(d*x+c)^(1/3))))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.34

$$\int x^2 \cos\left(a+b\sqrt[3]{c+dx}\right)\,dx = \frac{3\left(2\left(3360\,b^3dx+3240\,b^3c-12\,(14\,b^5dx+9\,b^5c)(dx+c)^{\frac{2}{3}}+(4\,b^7d^2x^2+3\,b^7cdx-20160\,b)(dx+c)^{\frac{1}{3}}\right)\cos\right.}{}$$

```
[In] integrate(x^2*cos(a+b*(d*x+c)^(1/3)),x, algorithm="fricas")
```

```
[Out] 3*(2*(3360*b^3*d*x + 3240*b^3*c - 12*(14*b^5*d*x + 9*b^5*c)*(d*x + c)^(2/3)
 + (4*b^7*d^2*x^2 + 3*b^7*c*d*x - 20160*b)*(d*x + c)^(1/3))*cos((d*x + c)^(
1/3)*b + a) - (56*b^6*d^2*x^2 + 72*b^6*c*d*x + 18*b^6*c^2 - (b^8*d^2*x^2 -
20160*b^2)*(d*x + c)^(2/3) - 240*(7*b^4*d*x + 6*b^4*c)*(d*x + c)^(1/3) - 40
320)*sin((d*x + c)^(1/3)*b + a))/(b^9*d^3)
```

Sympy [F]

$$\int x^2 \cos\left(a+b\sqrt[3]{c+dx}\right)\,dx = \int x^2 \cos\left(a+b\sqrt[3]{c+dx}\right)\,dx$$

```
[In] integrate(x**2*cos(a+b*(d*x+c)**(1/3)),x)
```

```
[Out] Integral(x**2*cos(a + b*(c + d*x)**(1/3)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1349 vs. $2(477) = 954$.

Time = 0.40 (sec) , antiderivative size = 1349, normalized size of antiderivative = 2.51

$$\int x^2 \cos\left(a + b\sqrt[3]{c + dx}\right) dx = \text{Too large to display}$$

```
[In] integrate(x^2*cos(a+b*(d*x+c)^(1/3)),x, algorithm="maxima")
```

```
[Out] 3*(a^2*c^2*sin((d*x + c)^(1/3)*b + a) - 2*(((d*x + c)^(1/3)*b + a)*sin((d*x
 + c)^(1/3)*b + a) + cos((d*x + c)^(1/3)*b + a))*a*c^2 + 2*a^5*c*sin((d*x +
 c)^(1/3)*b + a)/b^3 - 10*(((d*x + c)^(1/3)*b + a)*sin((d*x + c)^(1/3)*b +
a) + cos((d*x + c)^(1/3)*b + a))*a^4*c/b^3 + (2*((d*x + c)^(1/3)*b + a)*cos
((d*x + c)^(1/3)*b + a) + (((d*x + c)^(1/3)*b + a)^2 - 2)*sin((d*x + c)^(1/
3)*b + a))*c^2 + a^8*sin((d*x + c)^(1/3)*b + a)/b^6 - 8*(((d*x + c)^(1/3)*b
 + a)*sin((d*x + c)^(1/3)*b + a) + cos((d*x + c)^(1/3)*b + a))*a^7/b^6 + 20
*(2*((d*x + c)^(1/3)*b + a)*cos((d*x + c)^(1/3)*b + a) + (((d*x + c)^(1/3)*
b + a)^2 - 2)*sin((d*x + c)^(1/3)*b + a))*a^3*c/b^3 + 28*(2*((d*x + c)^(1/3
)*b + a)*cos((d*x + c)^(1/3)*b + a) + (((d*x + c)^(1/3)*b + a)^2 - 2)*sin((
d*x + c)^(1/3)*b + a))*a^6/b^6 - 20*(3*(((d*x + c)^(1/3)*b + a)^2 - 2)*cos(
(d*x + c)^(1/3)*b + a) + (((d*x + c)^(1/3)*b + a)^3 - 6*(d*x + c)^(1/3)*b -
 6*a)*sin((d*x + c)^(1/3)*b + a))*a^2*c/b^3 - 56*(3*(((d*x + c)^(1/3)*b + a
)^2 - 2)*cos((d*x + c)^(1/3)*b + a) + (((d*x + c)^(1/3)*b + a)^3 - 6*(d*x +
 c)^(1/3)*b - 6*a)*sin((d*x + c)^(1/3)*b + a))*a^5/b^6 + 10*(4*(((d*x + c)^
(1/3)*b + a)^3 - 6*(d*x + c)^(1/3)*b - 6*a)*cos((d*x + c)^(1/3)*b + a) + ((
(d*x + c)^(1/3)*b + a)^4 - 12*((d*x + c)^(1/3)*b + a)^2 + 24)*sin((d*x + c)
^(1/3)*b + a))*a*c/b^3 + 70*(4*(((d*x + c)^(1/3)*b + a)^3 - 6*(d*x + c)^(1/
3)*b - 6*a)*cos((d*x + c)^(1/3)*b + a) + (((d*x + c)^(1/3)*b + a)^4 - 12*((
d*x + c)^(1/3)*b + a)^2 + 24)*sin((d*x + c)^(1/3)*b + a))*a^4/b^6 - 2*(5*((
(d*x + c)^(1/3)*b + a)^4 - 12*((d*x + c)^(1/3)*b + a)^2 + 24)*cos((d*x + c)
^(1/3)*b + a) + (((d*x + c)^(1/3)*b + a)^5 - 20*((d*x + c)^(1/3)*b + a)^3 +
 120*(d*x + c)^(1/3)*b + 120*a)*sin((d*x + c)^(1/3)*b + a))*c/b^3 - 56*(5*(
((d*x + c)^(1/3)*b + a)^4 - 12*((d*x + c)^(1/3)*b + a)^2 + 24)*cos((d*x + c
)^(1/3)*b + a) + (((d*x + c)^(1/3)*b + a)^5 - 20*((d*x + c)^(1/3)*b + a)^3
+ 120*(d*x + c)^(1/3)*b + 120*a)*sin((d*x + c)^(1/3)*b + a))*a^3/b^6 + 28*(
6*(((d*x + c)^(1/3)*b + a)^5 - 20*((d*x + c)^(1/3)*b + a)^3 + 120*(d*x + c)
^(1/3)*b + 120*a)*cos((d*x + c)^(1/3)*b + a) + (((d*x + c)^(1/3)*b + a)^6 -
 30*((d*x + c)^(1/3)*b + a)^4 + 360*((d*x + c)^(1/3)*b + a)^2 - 720)*sin((d
*x + c)^(1/3)*b + a))*a^2/b^6 - 8*(7*(((d*x + c)^(1/3)*b + a)^6 - 30*((d*x
+ c)^(1/3)*b + a)^4 + 360*((d*x + c)^(1/3)*b + a)^2 - 720)*cos((d*x + c)^(1
/3)*b + a) + (((d*x + c)^(1/3)*b + a)^7 - 42*((d*x + c)^(1/3)*b + a)^5 + 84
0*((d*x + c)^(1/3)*b + a)^3 - 5040*(d*x + c)^(1/3)*b - 5040*a)*sin((d*x + c
)^(1/3)*b + a))*a/b^6 + (8*(((d*x + c)^(1/3)*b + a)^7 - 42*((d*x + c)^(1/3)
*b + a)^5 + 840*((d*x + c)^(1/3)*b + a)^3 - 5040*(d*x + c)^(1/3)*b - 5040*a
```

```
)*cos((d*x + c)^(1/3)*b + a) + (((d*x + c)^(1/3)*b + a)^8 - 56*((d*x + c)^(
1/3)*b + a)^6 + 1680*((d*x + c)^(1/3)*b + a)^4 - 20160*((d*x + c)^(1/3)*b +
 a)^2 + 40320)*sin((d*x + c)^(1/3)*b + a))/b^6)/(b^3*d^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1104 vs. $2(477) = 954$.

Time = 0.28 (sec) , antiderivative size = 1104, normalized size of antiderivative = 2.06

$$\int x^2 \cos\left(a + b\sqrt[3]{c + dx}\right) dx = \text{Too large to display}$$

```
[In] integrate(x^2*cos(a+b*(d*x+c)^(1/3)),x, algorithm="giac")
```

```
[Out] 3*(2*(((d*x + c)^(1/3)*b + a)*b^6*c^2 - a*b^6*c^2 - 5*((d*x + c)^(1/3)*b +
a)^4*b^3*c + 20*((d*x + c)^(1/3)*b + a)^3*a*b^3*c - 30*((d*x + c)^(1/3)*b +
 a)^2*a^2*b^3*c + 20*((d*x + c)^(1/3)*b + a)*a^3*b^3*c - 5*a^4*b^3*c + 4*((
d*x + c)^(1/3)*b + a)^7 - 28*((d*x + c)^(1/3)*b + a)^6*a + 84*((d*x + c)^(1
/3)*b + a)^5*a^2 - 140*((d*x + c)^(1/3)*b + a)^4*a^3 + 140*((d*x + c)^(1/3)
*b + a)^3*a^4 - 84*((d*x + c)^(1/3)*b + a)^2*a^5 + 28*((d*x + c)^(1/3)*b +
a)*a^6 - 4*a^7 + 60*((d*x + c)^(1/3)*b + a)^2*b^3*c - 120*((d*x + c)^(1/3)*
b + a)*a*b^3*c + 60*a^2*b^3*c - 168*((d*x + c)^(1/3)*b + a)^5 + 840*((d*x +
 c)^(1/3)*b + a)^4*a - 1680*((d*x + c)^(1/3)*b + a)^3*a^2 + 1680*((d*x + c)
^(1/3)*b + a)^2*a^3 - 840*((d*x + c)^(1/3)*b + a)*a^4 + 168*a^5 - 120*b^3*c
 + 3360*((d*x + c)^(1/3)*b + a)^3 - 10080*((d*x + c)^(1/3)*b + a)^2*a + 100
80*((d*x + c)^(1/3)*b + a)*a^2 - 3360*a^3 - 20160*(d*x + c)^(1/3)*b)*cos((d
*x + c)^(1/3)*b + a)/(b^8*d^2) + (((d*x + c)^(1/3)*b + a)^2*b^6*c^2 - 2*((d
*x + c)^(1/3)*b + a)*a*b^6*c^2 + a^2*b^6*c^2 - 2*((d*x + c)^(1/3)*b + a)^5*
b^3*c + 10*((d*x + c)^(1/3)*b + a)^4*a*b^3*c - 20*((d*x + c)^(1/3)*b + a)^3
*a^2*b^3*c + 20*((d*x + c)^(1/3)*b + a)^2*a^3*b^3*c - 10*((d*x + c)^(1/3)*b
 + a)*a^4*b^3*c + 2*a^5*b^3*c + ((d*x + c)^(1/3)*b + a)^8 - 8*((d*x + c)^(1
/3)*b + a)^7*a + 28*((d*x + c)^(1/3)*b + a)^6*a^2 - 56*((d*x + c)^(1/3)*b +
 a)^5*a^3 + 70*((d*x + c)^(1/3)*b + a)^4*a^4 - 56*((d*x + c)^(1/3)*b + a)^3
*a^5 + 28*((d*x + c)^(1/3)*b + a)^2*a^6 - 8*((d*x + c)^(1/3)*b + a)*a^7 + a
^8 - 2*b^6*c^2 + 40*((d*x + c)^(1/3)*b + a)^3*b^3*c - 120*((d*x + c)^(1/3)*
b + a)^2*a*b^3*c + 120*((d*x + c)^(1/3)*b + a)*a^2*b^3*c - 40*a^3*b^3*c - 5
6*((d*x + c)^(1/3)*b + a)^6 + 336*((d*x + c)^(1/3)*b + a)^5*a - 840*((d*x +
 c)^(1/3)*b + a)^4*a^2 + 1120*((d*x + c)^(1/3)*b + a)^3*a^3 - 840*((d*x + c
)^(1/3)*b + a)^2*a^4 + 336*((d*x + c)^(1/3)*b + a)*a^5 - 56*a^6 - 240*((d*x
 + c)^(1/3)*b + a)*b^3*c + 240*a*b^3*c + 1680*((d*x + c)^(1/3)*b + a)^4 - 6
720*((d*x + c)^(1/3)*b + a)^3*a + 10080*((d*x + c)^(1/3)*b + a)^2*a^2 - 672
0*((d*x + c)^(1/3)*b + a)*a^3 + 1680*a^4 - 20160*((d*x + c)^(1/3)*b + a)^2
+ 40320*((d*x + c)^(1/3)*b + a)*a - 20160*a^2 + 40320)*sin((d*x + c)^(1/3)*
b + a)/(b^8*d^2))/(b*d)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \cos\left(a+b\sqrt[3]{c+dx}\right)\,dx=\int x^2\,\cos\left(a+b\,{\left(c+d\,x\right)}^{1/3}\right)\,dx$$

```
[In] int(x^2*cos(a + b*(c + d*x)^(1/3)),x)
[Out] int(x^2*cos(a + b*(c + d*x)^(1/3)), x)
```

3.96 $\int x \cos\left(a+b\sqrt[3]{c+dx}\right)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 261

$$\int x\cos\left(a+b\sqrt[3]{c+dx}\right)dx=\frac{360\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^6d^2}-\frac{6c\sqrt[3]{c+dx}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^2}-\frac{180(c+dx)^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^4d^2}+\frac{15(c+dx)^{4/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^2}+\frac{6c\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^2}+\frac{360\sqrt[3]{c+dx}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^5d^2}-\frac{3c(c+dx)^{2/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{bd^2}-\frac{60(c+dx)\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^2}+\frac{3(c+dx)^{5/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{bd^2}$$

```
[Out] 360*cos(a+b*(d*x+c)^(1/3))/b^6/d^2-6*c*(d*x+c)^(1/3)*cos(a+b*(d*x+c)^(1/3))
/b^2/d^2-180*(d*x+c)^(2/3)*cos(a+b*(d*x+c)^(1/3))/b^4/d^2+15*(d*x+c)^(4/3)*
cos(a+b*(d*x+c)^(1/3))/b^2/d^2+6*c*sin(a+b*(d*x+c)^(1/3))/b^3/d^2+360*(d*x+
c)^(1/3)*sin(a+b*(d*x+c)^(1/3))/b^5/d^2-3*c*(d*x+c)^(2/3)*sin(a+b*(d*x+c)^(
1/3))/b/d^2-60*(d*x+c)*sin(a+b*(d*x+c)^(1/3))/b^3/d^2+3*(d*x+c)^(5/3)*sin(a
+b*(d*x+c)^(1/3))/b/d^2
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3513, 3377, 2717, 2718}

$$\int x\cos\left(a+b\sqrt[3]{c+dx}\right)\,dx=\frac{360\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^6d^2}+\frac{360\sqrt[3]{c+dx}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^5d^2}-\frac{180(c+dx)^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^4d^2}-\frac{60(c+dx)\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^2}+\frac{6c\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^2}+\frac{15(c+dx)^{4/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^2}-\frac{6c\sqrt[3]{c+dx}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^2}+\frac{3(c+dx)^{5/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{bd^2}-\frac{3c(c+dx)^{2/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{bd^2}$$

```
[In] Int[x*Cos[a + b*(c + d*x)^(1/3)],x]
```

```
[Out] (360*Cos[a + b*(c + d*x)^(1/3)])/(b^6*d^2) - (6*c*(c + d*x)^(1/3)*Cos[a + b
*(c + d*x)^(1/3)])/(b^2*d^2) - (180*(c + d*x)^(2/3)*Cos[a + b*(c + d*x)^(1/
3)])/(b^4*d^2) + (15*(c + d*x)^(4/3)*Cos[a + b*(c + d*x)^(1/3)])/(b^2*d^2)
+ (6*c*Sin[a + b*(c + d*x)^(1/3)])/(b^3*d^2) + (360*(c + d*x)^(1/3)*Sin[a +
 b*(c + d*x)^(1/3)])/(b^5*d^2) - (3*c*(c + d*x)^(2/3)*Sin[a + b*(c + d*x)^(
1/3)])/(b*d^2) - (60*(c + d*x)*Sin[a + b*(c + d*x)^(1/3)])/(b^3*d^2) + (3*(
c + d*x)^(5/3)*Sin[a + b*(c + d*x)^(1/3)])/(b*d^2)
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3513

```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_.)*((g_
.) + (h_.)*(x_))^(m_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Cos[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{3\text{Subst}\left(\int \left(-\frac{cx^2\cos(a+bx)}{d}+\frac{x^5\cos(a+bx)}{d}\right)\,dx,x,\sqrt[3]{c+dx}\right)}{d}\\
&= \frac{3\text{Subst}\left(\int x^5\cos(a+bx)\,dx,x,\sqrt[3]{c+dx}\right)}{d^2}-\frac{(3c)\text{Subst}\left(\int x^2\cos(a+bx)\,dx,x,\sqrt[3]{c+dx}\right)}{d^2}\\
&= -\frac{3c(c+dx)^{2/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{bd^2}+\frac{3(c+dx)^{5/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{bd^2}\\
&\quad-\frac{15\text{Subst}\left(\int x^4\sin(a+bx)\,dx,x,\sqrt[3]{c+dx}\right)}{bd^2}\\
&\quad+\frac{(6c)\text{Subst}\left(\int x\sin(a+bx)\,dx,x,\sqrt[3]{c+dx}\right)}{bd^2}\\
&= -\frac{6c\sqrt[3]{c+dx}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^2}+\frac{15(c+dx)^{4/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^2}\\
&\quad-\frac{3c(c+dx)^{2/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{bd^2}+\frac{3(c+dx)^{5/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{bd^2}\\
&\quad-\frac{60\text{Subst}\left(\int x^3\cos(a+bx)\,dx,x,\sqrt[3]{c+dx}\right)}{b^2d^2}\\
&\quad+\frac{(6c)\text{Subst}\left(\int \cos(a+bx)\,dx,x,\sqrt[3]{c+dx}\right)}{b^2d^2}
\end{aligned}$$

$$= -\frac{6c\sqrt[3]{c+dx}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^2} + \frac{15(c+dx)^{4/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^2} + \frac{6c\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^2} - \frac{3c(c+dx)^{2/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{bd^2} - \frac{60(c+dx)\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^2} + \frac{3(c+dx)^{5/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{bd^2} + \frac{180\text{Subst}\left(\int x^2\sin(a+bx)\,dx, x, \sqrt[3]{c+dx}\right)}{b^3d^2}$$

$$= -\frac{6c\sqrt[3]{c+dx}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^2} - \frac{180(c+dx)^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^4d^2} + \frac{15(c+dx)^{4/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^2} + \frac{6c\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^2} - \frac{3c(c+dx)^{2/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{bd^2} - \frac{60(c+dx)\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^2} + \frac{3(c+dx)^{5/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{bd^2} + \frac{360\text{Subst}\left(\int x\cos(a+bx)\,dx, x, \sqrt[3]{c+dx}\right)}{b^4d^2}$$

$$= -\frac{6c\sqrt[3]{c+dx}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^2} - \frac{180(c+dx)^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^4d^2} + \frac{15(c+dx)^{4/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^2} + \frac{6c\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^2} + \frac{360\sqrt[3]{c+dx}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^5d^2} - \frac{3c(c+dx)^{2/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{bd^2} - \frac{60(c+dx)\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^2} + \frac{3(c+dx)^{5/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{bd^2} - \frac{360\text{Subst}\left(\int \sin(a+bx)\,dx, x, \sqrt[3]{c+dx}\right)}{b^5d^2}$$

$$= \frac{360\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^6d^2} - \frac{6c\sqrt[3]{c+dx}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^2} - \frac{180(c+dx)^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^4d^2} + \frac{15(c+dx)^{4/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^2} + \frac{6c\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^2} + \frac{360\sqrt[3]{c+dx}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^5d^2} - \frac{3c(c+dx)^{2/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{bd^2} - \frac{60(c+dx)\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^2} + \frac{3(c+dx)^{5/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{bd^2}$$

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.45

$$\int x\cos\left(a+b\sqrt[3]{c+dx}\right)\,dx = \frac{3\left(\left(120-60b^2(c+dx)^{2/3}+b^4\sqrt[3]{c+dx}(3c+5dx)\right)\cos\left(a+b\sqrt[3]{c+dx}\right)+b\left(120\sqrt[3]{c+dx}+b^4dx(c+dx)\right.\right.}{b^6d^2}$$

```
[In] Integrate[x*Cos[a + b*(c + d*x)^(1/3)],x]
[Out] (3*((120 - 60*b^2*(c + d*x)^(2/3) + b^4*(c + d*x)^(1/3)*(3*c + 5*d*x))*Cos[
a + b*(c + d*x)^(1/3)] + b*(120*(c + d*x)^(1/3) + b^4*d*x*(c + d*x)^(2/3) -
 2*b^2*(9*c + 10*d*x))*Sin[a + b*(c + d*x)^(1/3)]))/(b^6*d^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 654 vs. 2(231) = 462.

Time = 2.10 (sec) , antiderivative size = 655, normalized size of antiderivative = 2.51

method	result
derivativedivides	$\frac{-3a^2c\sin\left(a+b(dx+c)^{\frac{1}{3}}\right)+6ac\left(\cos\left(a+b(dx+c)^{\frac{1}{3}}\right)+\left(a+b(dx+c)^{\frac{1}{3}}\right)\sin\left(a+b(dx+c)^{\frac{1}{3}}\right)\right)-3c\left(\left(a+b(dx+c)^{\frac{1}{3}}\right)^2\sin\left(a\right.\right.}{}$
default	$\frac{-3a^2c\sin\left(a+b(dx+c)^{\frac{1}{3}}\right)+6ac\left(\cos\left(a+b(dx+c)^{\frac{1}{3}}\right)+\left(a+b(dx+c)^{\frac{1}{3}}\right)\sin\left(a+b(dx+c)^{\frac{1}{3}}\right)\right)-3c\left(\left(a+b(dx+c)^{\frac{1}{3}}\right)^2\sin\left(a\right.\right.}{}$
parts	Expression too large to display

```
[In] int(x*cos(a+b*(d*x+c)^(1/3)),x,method=_RETURNVERBOSE)
```

```
[Out] 3/d^2/b^3*(-a^2*c*sin(a+b*(d*x+c)^(1/3))+2*a*c*(cos(a+b*(d*x+c)^(1/3))+(a+b
*(d*x+c)^(1/3))*sin(a+b*(d*x+c)^(1/3)))-c*((a+b*(d*x+c)^(1/3))^2*sin(a+b*(d
*x+c)^(1/3))-2*sin(a+b*(d*x+c)^(1/3))+2*(a+b*(d*x+c)^(1/3))*cos(a+b*(d*x+c)
^(1/3)))-1/b^3*a^5*sin(a+b*(d*x+c)^(1/3))+5/b^3*a^4*(cos(a+b*(d*x+c)^(1/3))
+(a+b*(d*x+c)^(1/3))*sin(a+b*(d*x+c)^(1/3)))-10/b^3*a^3*((a+b*(d*x+c)^(1/3)
)^2*sin(a+b*(d*x+c)^(1/3))-2*sin(a+b*(d*x+c)^(1/3))+2*(a+b*(d*x+c)^(1/3))*c
os(a+b*(d*x+c)^(1/3)))+10/b^3*a^2*((a+b*(d*x+c)^(1/3))^3*sin(a+b*(d*x+c)^(1
/3))+3*(a+b*(d*x+c)^(1/3))^2*cos(a+b*(d*x+c)^(1/3))-6*cos(a+b*(d*x+c)^(1/3)
)-6*(a+b*(d*x+c)^(1/3))*sin(a+b*(d*x+c)^(1/3)))-5/b^3*a*((a+b*(d*x+c)^(1/3)
)^4*sin(a+b*(d*x+c)^(1/3))+4*(a+b*(d*x+c)^(1/3))^3*cos(a+b*(d*x+c)^(1/3))-1
2*(a+b*(d*x+c)^(1/3))^2*sin(a+b*(d*x+c)^(1/3))+24*sin(a+b*(d*x+c)^(1/3))-24
*(a+b*(d*x+c)^(1/3))*cos(a+b*(d*x+c)^(1/3)))+1/b^3*((a+b*(d*x+c)^(1/3))^5*s
in(a+b*(d*x+c)^(1/3))+5*(a+b*(d*x+c)^(1/3))^4*cos(a+b*(d*x+c)^(1/3))-20*(a+
b*(d*x+c)^(1/3))^3*sin(a+b*(d*x+c)^(1/3))-60*(a+b*(d*x+c)^(1/3))^2*cos(a+b*
(d*x+c)^(1/3))+120*cos(a+b*(d*x+c)^(1/3))+120*(a+b*(d*x+c)^(1/3))*sin(a+b*(
d*x+c)^(1/3))))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.42

$$\int x\cos\left(a+b\sqrt[3]{c+dx}\right)\,dx = -\frac{3\left(\left(60\,(dx+c)^{\frac{2}{3}}b^2-(5\,b^4dx+3\,b^4c)(dx+c)^{\frac{1}{3}}-120\right)\cos\left((dx+c)^{\frac{1}{3}}b+a\right)-\left((dx+c)^{\frac{2}{3}}b^5dx-20\,b^3d\right.\right.}{b^6d^2}$$

```
[In] integrate(x*cos(a+b*(d*x+c)^(1/3)),x, algorithm="fricas")
```

```
[Out] -3*((60*(d*x + c)^(2/3)*b^2 - (5*b^4*d*x + 3*b^4*c)*(d*x + c)^(1/3) - 120)*
cos((d*x + c)^(1/3)*b + a) - ((d*x + c)^(2/3)*b^5*d*x - 20*b^3*d*x - 18*b^3
*c + 120*(d*x + c)^(1/3)*b)*sin((d*x + c)^(1/3)*b + a))/(b^6*d^2)
```

Sympy [F]

$$\int x\cos\left(a+b\sqrt[3]{c+dx}\right)\,dx = \int x\cos\left(a+b\sqrt[3]{c+dx}\right)dx$$

```
[In] integrate(x*cos(a+b*(d*x+c)**(1/3)),x)
```

```
[Out] Integral(x*cos(a + b*(c + d*x)**(1/3)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 523 vs. $2(231) = 462$.

Time = 0.24 (sec) , antiderivative size = 523, normalized size of antiderivative = 2.00

$$\int x\cos\left(a+b\sqrt[3]{c+dx}\right)\,dx =$$

$$-\frac{3\left(a^2c\sin\left((dx+c)^{\frac{1}{3}}b+a\right)-2\left(\left(\left((dx+c)^{\frac{1}{3}}b+a\right)\sin\left((dx+c)^{\frac{1}{3}}b+a\right)+\cos\left((dx+c)^{\frac{1}{3}}b+a\right)\right)ac+\cdots\right.}{\cdots}$$

```
[In] integrate(x*cos(a+b*(d*x+c)^(1/3)),x, algorithm="maxima")
```

```
[Out] -3*(a^2*c*sin((d*x + c)^(1/3)*b + a) - 2*(((d*x + c)^(1/3)*b + a)*sin((d*x
+ c)^(1/3)*b + a) + cos((d*x + c)^(1/3)*b + a))*a*c + a^5*sin((d*x + c)^(1/
3)*b + a)/b^3 - 5*(((d*x + c)^(1/3)*b + a)*sin((d*x + c)^(1/3)*b + a) + cos
((d*x + c)^(1/3)*b + a))*a^4/b^3 + (2*((d*x + c)^(1/3)*b + a)*cos((d*x + c)
^(1/3)*b + a) + (((d*x + c)^(1/3)*b + a)^2 - 2)*sin((d*x + c)^(1/3)*b + a))
*c + 10*(2*((d*x + c)^(1/3)*b + a)*cos((d*x + c)^(1/3)*b + a) + (((d*x + c)
^(1/3)*b + a)^2 - 2)*sin((d*x + c)^(1/3)*b + a))*a^3/b^3 - 10*(3*(((d*x + c
)^(1/3)*b + a)^2 - 2)*cos((d*x + c)^(1/3)*b + a) + (((d*x + c)^(1/3)*b + a)
^3 - 6*(d*x + c)^(1/3)*b - 6*a)*sin((d*x + c)^(1/3)*b + a))*a^2/b^3 + 5*(4*
(((d*x + c)^(1/3)*b + a)^3 - 6*(d*x + c)^(1/3)*b - 6*a)*cos((d*x + c)^(1/3)
*b + a) + (((d*x + c)^(1/3)*b + a)^4 - 12*((d*x + c)^(1/3)*b + a)^2 + 24)*s
in((d*x + c)^(1/3)*b + a))*a/b^3 - (5*(((d*x + c)^(1/3)*b + a)^4 - 12*((d*x
 + c)^(1/3)*b + a)^2 + 24)*cos((d*x + c)^(1/3)*b + a) + (((d*x + c)^(1/3)*b
 + a)^5 - 20*((d*x + c)^(1/3)*b + a)^3 + 120*(d*x + c)^(1/3)*b + 120*a)*sin
((d*x + c)^(1/3)*b + a))/b^3)/(b^3*d^2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 370, normalized size of antiderivative = 1.42

$$\int x\cos\left(a+b\sqrt[3]{c+dx}\right)\,dx =$$

$$-\frac{3\left(\frac{\left(2\left((dx+c)^{\frac{1}{3}}b+a\right)b^3c-2ab^3c-5\left((dx+c)^{\frac{1}{3}}b+a\right)^4+20\left((dx+c)^{\frac{1}{3}}b+a\right)^3a-30\left((dx+c)^{\frac{1}{3}}b+a\right)^2a^2+20\left((dx+c)^{\frac{1}{3}}b+a\right)a^3-5a^4+60\left((dx\cdots\right.}{b^5}\cdots\right.}{\cdots}$$

```
[In] integrate(x*cos(a+b*(d*x+c)^(1/3)),x, algorithm="giac")
```

```
[Out] -3*((2*((d*x + c)^(1/3)*b + a)*b^3*c - 2*a*b^3*c - 5*((d*x + c)^(1/3)*b + a
)^4 + 20*((d*x + c)^(1/3)*b + a)^3*a - 30*((d*x + c)^(1/3)*b + a)^2*a^2 + 2
```

```
0*((d*x + c)^(1/3)*b + a)*a^3 - 5*a^4 + 60*((d*x + c)^(1/3)*b + a)^2 - 120*
((d*x + c)^(1/3)*b + a)*a + 60*a^2 - 120)*cos((d*x + c)^(1/3)*b + a)/b^5 +
(((d*x + c)^(1/3)*b + a)^2*b^3*c - 2*((d*x + c)^(1/3)*b + a)*a*b^3*c + a^2*
b^3*c - ((d*x + c)^(1/3)*b + a)^5 + 5*((d*x + c)^(1/3)*b + a)^4*a - 10*((d*
x + c)^(1/3)*b + a)^3*a^2 + 10*((d*x + c)^(1/3)*b + a)^2*a^3 - 5*((d*x + c)
^(1/3)*b + a)*a^4 + a^5 - 2*b^3*c + 20*((d*x + c)^(1/3)*b + a)^3 - 60*((d*x
 + c)^(1/3)*b + a)^2*a + 60*((d*x + c)^(1/3)*b + a)*a^2 - 20*a^3 - 120*(d*x
 + c)^(1/3)*b)*sin((d*x + c)^(1/3)*b + a)/b^5)/(b*d^2)
```

Mupad [F(-1)]

Timed out.

$$\int x\cos\left(a+b\sqrt[3]{c+dx}\right)\,dx=\int x\,\cos\left(a+b\,(c+d\,x)^{1/3}\right)\,dx$$

```
[In] int(x*cos(a + b*(c + d*x)^(1/3)),x)
[Out] int(x*cos(a + b*(c + d*x)^(1/3)), x)
```

3.97 $\int \cos\left(a+b\sqrt[3]{c+dx}\right)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 85

$$\int \cos\left(a+b\sqrt[3]{c+dx}\right)\,dx=\frac{6\sqrt[3]{c+dx}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^2d}-\frac{6\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^3d}+\frac{3(c+dx)^{2/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{bd}$$

```
[Out] 6*(d*x+c)^(1/3)*cos(a+b*(d*x+c)^(1/3))/b^2/d-6*sin(a+b*(d*x+c)^(1/3))/b^3/d
+3*(d*x+c)^(2/3)*sin(a+b*(d*x+c)^(1/3))/b/d
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3443, 3377, 2717}

$$\int \cos\left(a+b\sqrt[3]{c+dx}\right)\,dx=-\frac{6\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^3d}+\frac{6\sqrt[3]{c+dx}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^2d}+\frac{3(c+dx)^{2/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{bd}$$

```
[In] Int[Cos[a + b*(c + d*x)^(1/3)],x]
```

```
[Out] (6*(c + d*x)^(1/3)*Cos[a + b*(c + d*x)^(1/3)])/(b^2*d) - (6*Sin[a + b*(c +
d*x)^(1/3)])/(b^3*d) + (3*(c + d*x)^(2/3)*Sin[a + b*(c + d*x)^(1/3)])/(b*d)
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3443

```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_.), x_S
ymbol] :> Dist[1/(n*f), Subst[Int[x^(1/n - 1)*(a + b*Cos[c + d*x])^p, x], x
, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && Integer
Q[1/n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{3\text{Subst}\left(\int x^2\cos(a+bx)\,dx, x, \sqrt[3]{c+dx}\right)}{d}\\
&= \frac{3(c+dx)^{2/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{bd} - \frac{6\text{Subst}\left(\int x\sin(a+bx)\,dx, x, \sqrt[3]{c+dx}\right)}{bd}\\
&= \frac{6\sqrt[3]{c+dx}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^2d} + \frac{3(c+dx)^{2/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{bd}\\
&\quad - \frac{6\text{Subst}\left(\int \cos(a+bx)\,dx, x, \sqrt[3]{c+dx}\right)}{b^2d}\\
&= \frac{6\sqrt[3]{c+dx}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^2d} - \frac{6\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^3d} + \frac{3(c+dx)^{2/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{bd}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.76

$$
\begin{aligned}
&\int \cos\left(a+b\sqrt[3]{c+dx}\right)\,dx\\
&= \frac{6b\sqrt[3]{c+dx}\cos\left(a+b\sqrt[3]{c+dx}\right) + 3\left(-2+b^2(c+dx)^{2/3}\right)\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^3d}
\end{aligned}
$$

```
[In] Integrate[Cos[a + b*(c + d*x)^(1/3)],x]
[Out] (6*b*(c + d*x)^(1/3)*Cos[a + b*(c + d*x)^(1/3)] + 3*(-2 + b^2*(c + d*x)^(2/
3))*Sin[a + b*(c + d*x)^(1/3)])/(b^3*d)
```

Maple [A] (verified)

Time = 0.91 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.54

method	result
derivativedivides	$\frac{3a^2 \sin\left(a+b(dx+c)^{\frac{1}{3}}\right)-6a\left(\cos\left(a+b(dx+c)^{\frac{1}{3}}\right)+\left(a+b(dx+c)^{\frac{1}{3}}\right)\sin\left(a+b(dx+c)^{\frac{1}{3}}\right)\right)+3\left(a+b(dx+c)^{\frac{1}{3}}\right)^2 \sin\left(a+b(dx-\right.}{b^3d}$
default	$\frac{3a^2 \sin\left(a+b(dx+c)^{\frac{1}{3}}\right)-6a\left(\cos\left(a+b(dx+c)^{\frac{1}{3}}\right)+\left(a+b(dx+c)^{\frac{1}{3}}\right)\sin\left(a+b(dx+c)^{\frac{1}{3}}\right)\right)+3\left(a+b(dx+c)^{\frac{1}{3}}\right)^2 \sin\left(a+b(dx-\right.}{b^3d}$

```
[In] int(cos(a+b*(d*x+c)^(1/3)),x,method=_RETURNVERBOSE)
```

```
[Out] 3/d/b^3*(a^2*sin(a+b*(d*x+c)^(1/3))-2*a*(cos(a+b*(d*x+c)^(1/3))+(a+b*(d*x+c
)^(1/3))*sin(a+b*(d*x+c)^(1/3)))+(a+b*(d*x+c)^(1/3))^2*sin(a+b*(d*x+c)^(1/3
))-2*sin(a+b*(d*x+c)^(1/3))+2*(a+b*(d*x+c)^(1/3))*cos(a+b*(d*x+c)^(1/3)))
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.67

$$\int \cos\left(a+b\sqrt[3]{c+dx}\right) dx = \frac{3\left(2(dx+c)^{\frac{1}{3}}b\cos\left((dx+c)^{\frac{1}{3}}b+a\right)+\left((dx+c)^{\frac{2}{3}}b^2-2\right)\sin\left((dx+c)^{\frac{1}{3}}b+a\right)\right)}{b^3d}$$

```
[In] integrate(cos(a+b*(d*x+c)^(1/3)),x, algorithm="fricas")
```

```
[Out] 3*(2*(d*x + c)^(1/3)*b*cos((d*x + c)^(1/3)*b + a) + ((d*x + c)^(2/3)*b^2 -
2)*sin((d*x + c)^(1/3)*b + a))/(b^3*d)
```

Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.11

$$\int \cos\left(a+b\sqrt[3]{c+dx}\right) dx = \begin{cases} x\cos(a) & \text{for } b=0 \wedge (b=0 \vee d= \\ x\cos\left(a+b\sqrt[3]{c}\right) & \text{for } d=0 \\ \frac{3(c+dx)^{\frac{2}{3}}\sin\left(a+b\sqrt[3]{c+dx}\right)}{bd}+\frac{6\sqrt[3]{c+dx}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^2d}-\frac{6\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^3d} & \text{otherwise} \end{cases}$$

```
[In] integrate(cos(a+b*(d*x+c)**(1/3)),x)
```

```
[Out] Piecewise((x*cos(a), Eq(b, 0) & (Eq(b, 0) | Eq(d, 0))), (x*cos(a + b*c**(1/
3)), Eq(d, 0)), (3*(c + d*x)**(2/3)*sin(a + b*(c + d*x)**(1/3))/(b*d) + 6*(
c + d*x)**(1/3)*cos(a + b*(c + d*x)**(1/3))/(b**2*d) - 6*sin(a + b*(c + d*x
)**(1/3))/(b**3*d), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.39

$$\int \cos\left(a+b\sqrt[3]{c+dx}\right)\,dx = \frac{3\left(a^2\sin\left((dx+c)^{\frac{1}{3}}b+a\right)-2\left(\left((dx+c)^{\frac{1}{3}}b+a\right)\sin\left((dx+c)^{\frac{1}{3}}b+a\right)+\cos\left((dx+c)^{\frac{1}{3}}b+a\right)\right)a+2\left((a\right.\right.}{b^3d}$$

```
[In] integrate(cos(a+b*(d*x+c)^(1/3)),x, algorithm="maxima")
```

```
[Out] 3*(a^2*sin((d*x + c)^(1/3)*b + a) - 2*(((d*x + c)^(1/3)*b + a)*sin((d*x + c
)^(1/3)*b + a) + cos((d*x + c)^(1/3)*b + a))*a + 2*((d*x + c)^(1/3)*b + a)*
cos((d*x + c)^(1/3)*b + a) + (((d*x + c)^(1/3)*b + a)^2 - 2)*sin((d*x + c)^
(1/3)*b + a))/(b^3*d)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.95

$$\int \cos\left(a+b\sqrt[3]{c+dx}\right)\,dx = \frac{3\left(\frac{2(dx+c)^{\frac{1}{3}}\cos\left((dx+c)^{\frac{1}{3}}b+a\right)}{b}+\frac{\left(\left((dx+c)^{\frac{1}{3}}b+a\right)^2-2\left((dx+c)^{\frac{1}{3}}b+a\right)a+a^2-2\right)\sin\left((dx+c)^{\frac{1}{3}}b+a\right)}{b^2}\right)}{bd}$$

```
[In] integrate(cos(a+b*(d*x+c)^(1/3)),x, algorithm="giac")
```

```
[Out] 3*(2*(d*x + c)^(1/3)*cos((d*x + c)^(1/3)*b + a)/b + (((d*x + c)^(1/3)*b + a
)^2 - 2*((d*x + c)^(1/3)*b + a)*a + a^2 - 2)*sin((d*x + c)^(1/3)*b + a)/b^2
)/(b*d)
```

Mupad [B] (verification not implemented)

Time = 14.02 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.80

$$\int \cos \left(a+b\sqrt[3]{c+dx}\right)\,dx$$
$$=\frac{6\,b\,\cos\left(a+b\,{\left(c+d\,x\right)}^{1/3}\right)\,{\left(c+d\,x\right)}^{1/3}-6\,\sin\left(a+b\,{\left(c+d\,x\right)}^{1/3}\right)+3\,b^2\,\sin\left(a+b\,{\left(c+d\,x\right)}^{1/3}\right)\,{\left(c+d\,x\right)}^{2/3}}{b^3\,d}$$

```
[In] int(cos(a + b*(c + d*x)^(1/3)),x)
[Out] (6*b*cos(a + b*(c + d*x)^(1/3))*(c + d*x)^(1/3) - 6*sin(a + b*(c + d*x)^(1/
3)) + 3*b^2*sin(a + b*(c + d*x)^(1/3))*(c + d*x)^(2/3))/(b^3*d)
```

3.98 $\int \frac{\cos\left(a+b\sqrt[3]{c+dx}\right)}{x}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 234

$$\begin{aligned}\int \frac{\cos\left(a+b\sqrt[3]{c+dx}\right)}{x}\,dx &= \cos\left(a+b\sqrt[3]{c}\right)\operatorname{CosIntegral}\left(b\sqrt[3]{c}-b\sqrt[3]{c+dx}\right)\\ &\quad+\cos\left(a+(-1)^{2/3}b\sqrt[3]{c}\right)\operatorname{CosIntegral}\left((-1)^{2/3}b\sqrt[3]{c}-b\sqrt[3]{c+dx}\right)\\ &\quad+\cos\left(a-\sqrt[3]{-1}b\sqrt[3]{c}\right)\operatorname{CosIntegral}\left(\sqrt[3]{-1}b\sqrt[3]{c}+b\sqrt[3]{c+dx}\right)\\ &\quad+\sin\left(a+b\sqrt[3]{c}\right)\operatorname{Si}\left(b\sqrt[3]{c}-b\sqrt[3]{c+dx}\right)\\ &\quad+\sin\left(a+(-1)^{2/3}b\sqrt[3]{c}\right)\operatorname{Si}\left((-1)^{2/3}b\sqrt[3]{c}-b\sqrt[3]{c+dx}\right)-\sin\left(a-\sqrt[3]{-1}b\sqrt[3]{c}\right)\operatorname{Si}\left(\sqrt[3]{-1}b\sqrt[3]{c}+b\sqrt[3]{c+dx}\right)\end{aligned}$$

```
[Out] Ci(b*c^(1/3)-b*(d*x+c)^(1/3))*cos(a+b*c^(1/3))+Ci((-1)^(1/3)*b*c^(1/3)+b*(d
*x+c)^(1/3))*cos(a-(-1)^(1/3)*b*c^(1/3))+Ci((-1)^(2/3)*b*c^(1/3)-b*(d*x+c)^
(1/3))*cos(a+(-1)^(2/3)*b*c^(1/3))+Si(b*c^(1/3)-b*(d*x+c)^(1/3))*sin(a+b*c^
(1/3))-Si((-1)^(1/3)*b*c^(1/3)+b*(d*x+c)^(1/3))*sin(a-(-1)^(1/3)*b*c^(1/3))
+Si((-1)^(2/3)*b*c^(1/3)-b*(d*x+c)^(1/3))*sin(a+(-1)^(2/3)*b*c^(1/3))
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used

= {3513, 3384, 3380, 3383}

$$\int \frac{\cos\left(a+b\sqrt[3]{c+dx}\right)}{x}\,dx = \cos\left(a+b\sqrt[3]{c}\right)\text{CosIntegral}\left(b\sqrt[3]{c}-b\sqrt[3]{c+dx}\right)$$
$$+\cos\left(a+(-1)^{2/3}b\sqrt[3]{c}\right)\text{CosIntegral}\left((-1)^{2/3}b\sqrt[3]{c}-b\sqrt[3]{c+dx}\right)$$
$$+\cos\left(a-\sqrt[3]{-1}b\sqrt[3]{c}\right)\text{CosIntegral}\left(\sqrt[3]{-1}\sqrt[3]{c}b+\sqrt[3]{c+dx}b\right)$$
$$+\sin\left(a+b\sqrt[3]{c}\right)\text{Si}\left(b\sqrt[3]{c}-b\sqrt[3]{c+dx}\right)$$
$$+\sin\left(a+(-1)^{2/3}b\sqrt[3]{c}\right)\text{Si}\left((-1)^{2/3}b\sqrt[3]{c}-b\sqrt[3]{c+dx}\right)-\sin\left(a-\sqrt[3]{-1}b\sqrt[3]{c}\right)\text{Si}\left(\sqrt[3]{-1}\sqrt[3]{c}b+\sqrt[3]{c+dx}b\right)$$

```
[In] Int[Cos[a + b*(c + d*x)^(1/3)]/x,x]
```

```
[Out] Cos[a + b*c^(1/3)]*CosIntegral[b*c^(1/3) - b*(c + d*x)^(1/3)] + Cos[a + (-1
)^(2/3)*b*c^(1/3)]*CosIntegral[(-1)^(2/3)*b*c^(1/3) - b*(c + d*x)^(1/3)] +
Cos[a - (-1)^(1/3)*b*c^(1/3)]*CosIntegral[(-1)^(1/3)*b*c^(1/3) + b*(c + d*x
)^(1/3)] + Sin[a + b*c^(1/3)]*SinIntegral[b*c^(1/3) - b*(c + d*x)^(1/3)] +
Sin[a + (-1)^(2/3)*b*c^(1/3)]*SinIntegral[(-1)^(2/3)*b*c^(1/3) - b*(c + d*x
)^(1/3)] - Sin[a - (-1)^(1/3)*b*c^(1/3)]*SinIntegral[(-1)^(1/3)*b*c^(1/3) +
 b*(c + d*x)^(1/3)]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3513

```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_.)*((g_
.) + (h_.)*(x_))^(m_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Cos[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
```

```
0] && IntegerQ[1/n]
```

Rubi steps

$$\text{integral} = \frac{3\text{Subst}\left(\int \left(-\frac{d\cos(a+bx)}{3\left(\sqrt[3]{c}-x\right)}-\frac{d\cos(a+bx)}{3\left(-\sqrt[3]{-1}\sqrt[3]{c}-x\right)}-\frac{d\cos(a+bx)}{3\left((-1)^{2/3}\sqrt[3]{c}-x\right)}\right)dx,x,\sqrt[3]{c+dx}\right)}{d}$$

$$= -\text{Subst}\left(\int \frac{\cos(a+bx)}{\sqrt[3]{c}-x}\,dx,x,\sqrt[3]{c+dx}\right) - \text{Subst}\left(\int \frac{\cos(a+bx)}{-\sqrt[3]{-1}\sqrt[3]{c}-x}\,dx,x,\sqrt[3]{c+dx}\right) - \text{Subst}\left(\int \frac{\cos(a+bx)}{(-1)^{2/3}\sqrt[3]{c}-x}\,dx,x,\sqrt[3]{c+dx}\right)$$

$$= -\left(\cos\left(a+b\sqrt[3]{c}\right)\text{Subst}\left(\int \frac{\cos\left(b\sqrt[3]{c}-bx\right)}{\sqrt[3]{c}-x}\,dx,x,\sqrt[3]{c+dx}\right)\right) - \cos\left(a-\sqrt[3]{-1}b\sqrt[3]{c}\right)\text{Subst}\left(\int \frac{\cos\left(\sqrt[3]{-1}b\sqrt[3]{c}+bx\right)}{-\sqrt[3]{-1}\sqrt[3]{c}-x}\,dx,x,\sqrt[3]{c+dx}\right) - \cos\left(a+(-1)^{2/3}b\sqrt[3]{c}\right)\text{Subst}\left(\int \frac{\cos\left((-1)^{2/3}b\sqrt[3]{c}-bx\right)}{(-1)^{2/3}\sqrt[3]{c}-x}\,dx,x,\sqrt[3]{c+dx}\right) - \sin\left(a+b\sqrt[3]{c}\right)\text{Subst}\left(\int \frac{\sin\left(b\sqrt[3]{}\right.}{\sqrt[3]{}}\right.$$

$$= \cos\left(a+b\sqrt[3]{c}\right)\text{CosIntegral}\left(b\sqrt[3]{c}-b\sqrt[3]{c+dx}\right) + \cos\left(a+(-1)^{2/3}b\sqrt[3]{c}\right)\text{CosIntegral}\left((-1)^{2/3}b\sqrt[3]{c}-b\sqrt[3]{c+dx}\right) + \cos\left(a-\sqrt[3]{-1}b\sqrt[3]{c}\right)\text{CosIntegral}\left(\sqrt[3]{-1}b\sqrt[3]{c}+b\sqrt[3]{c+dx}\right) + \sin\left(a+b\sqrt[3]{c}\right)\text{Si}\left(b\sqrt[3]{c}-b\sqrt[3]{c+dx}\right) + \sin\left(a+(-1)^{2/3}b\sqrt[3]{c}\right)\text{Si}\left((-1)^{2/3}b\sqrt[3]{c}-b\sqrt[3]{c+dx}\right) - \sin\left(a-\sqrt[3]{-1}b\sqrt[3]{c}\right)\text{Si}\left(\sqrt[3]{-1}b\sqrt[3]{c}+b\sqrt[3]{c+dx}\right)$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 11.08 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.04

$$\int \frac{\cos\left(a+b\sqrt[3]{c+dx}\right)}{x}\,dx = \frac{1}{2}\Big(\text{RootSum}\Big[c - \#1^3\&, \cos(a+b\#1)\,\text{CosIntegral}\left(b\left(\sqrt[3]{c+dx}-\#1\right)\right) - i\,\text{CosIntegral}\left(b\left(\sqrt[3]{c+dx}-\#1\right)\right)\sin(a+b\#1) - i\cos(a+b\#1)\text{Si}\left(b\left(\sqrt[3]{c+dx}-\#1\right)\right) - \sin(a+b\#1)\text{Si}\left(b\left(\sqrt[3]{c+dx}-\#1\right)\right)\&\Big] + \text{RootSum}\Big[c - \#1^3\&, \cos(a+b\#1)\,\text{CosIntegral}\left(b\left(\sqrt[3]{c+dx}-\#1\right)\right) + i\,\text{CosIntegral}\left(b\left(\sqrt[3]{c+dx}-\#1\right)\right)\sin(a+b\#1) + i\cos(a+b\#1)\text{Si}\left(b\left(\sqrt[3]{c+dx}-\#1\right)\right) - \sin(a+b\#1)\text{Si}\left(b\left(\sqrt[3]{c+dx}-\#1\right)\right)\&\Big]\Big)$$

```
[In] Integrate[Cos[a + b*(c + d*x)^(1/3)]/x,x]
```

```
[Out] (RootSum[c - #1^3 & , Cos[a + b*#1]*CosIntegral[b*((c + d*x)^(1/3) - #1)] -
 I*CosIntegral[b*((c + d*x)^(1/3) - #1)]*Sin[a + b*#1] - I*Cos[a + b*#1]*Si
nIntegral[b*((c + d*x)^(1/3) - #1)] - Sin[a + b*#1]*SinIntegral[b*((c + d*x
)^(1/3) - #1)] & ] + RootSum[c - #1^3 & , Cos[a + b*#1]*CosIntegral[b*((c +
 d*x)^(1/3) - #1)] + I*CosIntegral[b*((c + d*x)^(1/3) - #1)]*Sin[a + b*#1]
+ I*Cos[a + b*#1]*SinIntegral[b*((c + d*x)^(1/3) - #1)] - Sin[a + b*#1]*Sin
Integral[b*((c + d*x)^(1/3) - #1)] & ])/2
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.10 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.19

method	result
derivativedivides	$\frac{a^2 b^3 \left(\sum_{\_R1=\text{RootOf}\left(-b^3 c+\_Z^3-3a\_Z^2+3a^2\_Z-a^3\right)} \frac{\text{Si}\left(-b(dx+c)^{\frac{1}{3}}+\_R1-a\right)\sin\left(\_R1\right)+\text{Ci}\left(b(dx+c)^{\frac{1}{3}}-\_R1+a\right)}{\_R1^2-2\_R1a+a^2}\right.}{}$
default	$\frac{a^2 b^3 \left(\sum_{\_R1=\text{RootOf}\left(-b^3 c+\_Z^3-3a\_Z^2+3a^2\_Z-a^3\right)} \frac{\text{Si}\left(-b(dx+c)^{\frac{1}{3}}+\_R1-a\right)\sin\left(\_R1\right)+\text{Ci}\left(b(dx+c)^{\frac{1}{3}}-\_R1+a\right)}{\_R1^2-2\_R1a+a^2}\right.}{}$

```
[In] int(cos(a+b*(d*x+c)^(1/3))/x,x,method=_RETURNVERBOSE)
```

```
[Out] 3/b^3*(1/3*a^2*b^3*sum(1/(_R1^2-2*_R1*a+a^2)*(Si(-b*(d*x+c)^(1/3)+_R1-a)*si
n(_R1)+Ci(b*(d*x+c)^(1/3)-_R1+a)*cos(_R1)),_R1=RootOf(-b^3*c+_Z^3-3*_Z^2*a+
3*_Z*a^2-a^3))-2/3*a*b^3*sum(_R1/(_R1^2-2*_R1*a+a^2)*(Si(-b*(d*x+c)^(1/3)+_
R1-a)*sin(_R1)+Ci(b*(d*x+c)^(1/3)-_R1+a)*cos(_R1)),_R1=RootOf(-b^3*c+_Z^3-3
*_Z^2*a+3*_Z*a^2-a^3))+1/3*b^3*sum(_R1^2/(_R1^2-2*_R1*a+a^2)*(Si(-b*(d*x+c)
^(1/3)+_R1-a)*sin(_R1)+Ci(b*(d*x+c)^(1/3)-_R1+a)*cos(_R1)),_R1=RootOf(-b^3*
c+_Z^3-3*_Z^2*a+3*_Z*a^2-a^3)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.23

$$\begin{aligned}\int \frac{\cos\left(a+b\sqrt[3]{c+dx}\right)}{x}\,dx = {} & \frac{1}{2}\operatorname{Ei}\left(i\,(dx+c)^{\frac{1}{3}}b + \frac{1}{2}\left(i\,b^3c\right)^{\frac{1}{3}}\left(-i\sqrt{3}-1\right)\right)e^{\left(\frac{1}{2}\left(i\,b^3c\right)^{\frac{1}{3}}\left(i\sqrt{3}+1\right)+i\,a\right)} \\ & + \frac{1}{2}\operatorname{Ei}\left(-i\,(dx+c)^{\frac{1}{3}}b + \frac{1}{2}\left(-i\,b^3c\right)^{\frac{1}{3}}\left(-i\sqrt{3}-1\right)\right)e^{\left(\frac{1}{2}\left(-i\,b^3c\right)^{\frac{1}{3}}\left(i\sqrt{3}+1\right)-i\,a\right)} \\ & + \frac{1}{2}\operatorname{Ei}\left(i\,(dx+c)^{\frac{1}{3}}b + \frac{1}{2}\left(i\,b^3c\right)^{\frac{1}{3}}\left(i\sqrt{3}-1\right)\right)e^{\left(\frac{1}{2}\left(i\,b^3c\right)^{\frac{1}{3}}\left(-i\sqrt{3}+1\right)+i\,a\right)} \\ & + \frac{1}{2}\operatorname{Ei}\left(-i\,(dx+c)^{\frac{1}{3}}b + \frac{1}{2}\left(-i\,b^3c\right)^{\frac{1}{3}}\left(i\sqrt{3}-1\right)\right)e^{\left(\frac{1}{2}\left(-i\,b^3c\right)^{\frac{1}{3}}\left(-i\sqrt{3}+1\right)-i\,a\right)} \\ & + \frac{1}{2}\operatorname{Ei}\left(i\,(dx+c)^{\frac{1}{3}}b + \left(i\,b^3c\right)^{\frac{1}{3}}\right)e^{\left(i\,a-\left(i\,b^3c\right)^{\frac{1}{3}}\right)} \\ & + \frac{1}{2}\operatorname{Ei}\left(-i\,(dx+c)^{\frac{1}{3}}b + \left(-i\,b^3c\right)^{\frac{1}{3}}\right)e^{\left(-i\,a-\left(-i\,b^3c\right)^{\frac{1}{3}}\right)}\end{aligned}$$

```
[In] integrate(cos(a+b*(d*x+c)^(1/3))/x,x, algorithm="fricas")
```

```
[Out] 1/2*Ei(I*(d*x + c)^(1/3)*b + 1/2*(I*b^3*c)^(1/3)*(-I*sqrt(3) - 1))*e^(1/2*(
I*b^3*c)^(1/3)*(I*sqrt(3) + 1) + I*a) + 1/2*Ei(-I*(d*x + c)^(1/3)*b + 1/2*(
-I*b^3*c)^(1/3)*(-I*sqrt(3) - 1))*e^(1/2*(-I*b^3*c)^(1/3)*(I*sqrt(3) + 1) -
 I*a) + 1/2*Ei(I*(d*x + c)^(1/3)*b + 1/2*(I*b^3*c)^(1/3)*(I*sqrt(3) - 1))*e
^(1/2*(I*b^3*c)^(1/3)*(-I*sqrt(3) + 1) + I*a) + 1/2*Ei(-I*(d*x + c)^(1/3)*b
```

```
 + 1/2*(-I*b^3*c)^(1/3)*(I*sqrt(3) - 1))*e^(1/2*(-I*b^3*c)^(1/3)*(-I*sqrt(3
) + 1) - I*a) + 1/2*Ei(I*(d*x + c)^(1/3)*b + (I*b^3*c)^(1/3))*e^(I*a - (I*b
^3*c)^(1/3)) + 1/2*Ei(-I*(d*x + c)^(1/3)*b + (-I*b^3*c)^(1/3))*e^(-I*a - (-
I*b^3*c)^(1/3))
```

Sympy [F]

$$\int \frac{\cos \left(a+b\sqrt[3]{c+dx}\right)}{x}\,dx=\int \frac{\cos \left(a+b\sqrt[3]{c+dx}\right)}{x}\,dx$$

```
[In] integrate(cos(a+b*(d*x+c)**(1/3))/x,x)
[Out] Integral(cos(a + b*(c + d*x)**(1/3))/x, x)
```

Maxima [F]

$$\int \frac{\cos \left(a+b\sqrt[3]{c+dx}\right)}{x}\,dx=\int \frac{\cos \left((dx+c)^{\frac{1}{3}}b+a\right)}{x}\,dx$$

```
[In] integrate(cos(a+b*(d*x+c)^(1/3))/x,x, algorithm="maxima")
[Out] integrate(cos((d*x + c)^(1/3)*b + a)/x, x)
```

Giac [F]

$$\int \frac{\cos \left(a+b\sqrt[3]{c+dx}\right)}{x}\,dx=\int \frac{\cos \left((dx+c)^{\frac{1}{3}}b+a\right)}{x}\,dx$$

```
[In] integrate(cos(a+b*(d*x+c)^(1/3))/x,x, algorithm="giac")
[Out] integrate(cos((d*x + c)^(1/3)*b + a)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos \left(a+b\sqrt[3]{c+dx}\right)}{x}\,dx = \int \frac{\cos \left(a+b\,{\left(c+d\,x\right)}^{1/3}\right)}{x}\,dx$$

```
[In] int(cos(a + b*(c + d*x)^(1/3))/x,x)
[Out] int(cos(a + b*(c + d*x)^(1/3))/x, x)
```

3.99 $\int \frac{\cos\left(a+b\sqrt[3]{c+dx}\right)}{x^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 332

$$\begin{aligned}\int \frac{\cos\left(a+b\sqrt[3]{c+dx}\right)}{x^2}\,dx &= -\frac{\cos\left(a+b\sqrt[3]{c+dx}\right)}{x}-\frac{bd\,\mathrm{CosIntegral}\left(b\sqrt[3]{c}-b\sqrt[3]{c+dx}\right)\sin\left(a+b\sqrt[3]{c}\right)}{3c^{2/3}}\\
&\quad+\frac{\sqrt[3]{-1}bd\,\mathrm{CosIntegral}\left(\sqrt[3]{-1}b\sqrt[3]{c}+b\sqrt[3]{c+dx}\right)\sin\left(a-\sqrt[3]{-1}b\sqrt[3]{c}\right)}{3c^{2/3}}\\
&\quad-\frac{(-1)^{2/3}bd\,\mathrm{CosIntegral}\left((-1)^{2/3}b\sqrt[3]{c}-b\sqrt[3]{c+dx}\right)\sin\left(a+(-1)^{2/3}b\sqrt[3]{c}\right)}{3c^{2/3}}\\
&\quad+\frac{bd\cos\left(a+b\sqrt[3]{c}\right)\mathrm{Si}\left(b\sqrt[3]{c}-b\sqrt[3]{c+dx}\right)}{3c^{2/3}}\\
&\quad+\frac{(-1)^{2/3}bd\cos\left(a+(-1)^{2/3}b\sqrt[3]{c}\right)\mathrm{Si}\left((-1)^{2/3}b\sqrt[3]{c}-b\sqrt[3]{c+dx}\right)}{3c^{2/3}}\\
&\quad+\frac{\sqrt[3]{-1}bd\cos\left(a-\sqrt[3]{-1}b\sqrt[3]{c}\right)\mathrm{Si}\left(\sqrt[3]{-1}b\sqrt[3]{c}+b\sqrt[3]{c+dx}\right)}{3c^{2/3}}\end{aligned}$$

```
[Out] -cos(a+b*(d*x+c)^(1/3))/x+1/3*b*d*cos(a+b*c^(1/3))*Si(b*c^(1/3)-b*(d*x+c)^(
1/3))/c^(2/3)+1/3*(-1)^(2/3)*b*d*cos(a+(-1)^(2/3)*b*c^(1/3))*Si((-1)^(2/3)*
b*c^(1/3)-b*(d*x+c)^(1/3))/c^(2/3)+1/3*(-1)^(1/3)*b*d*cos(a-(-1)^(1/3)*b*c^
(1/3))*Si((-1)^(1/3)*b*c^(1/3)+b*(d*x+c)^(1/3))/c^(2/3)-1/3*b*d*Ci(b*c^(1/3
)-b*(d*x+c)^(1/3))*sin(a+b*c^(1/3))/c^(2/3)+1/3*(-1)^(1/3)*b*d*Ci((-1)^(1/3
)*b*c^(1/3)+b*(d*x+c)^(1/3))*sin(a-(-1)^(1/3)*b*c^(1/3))/c^(2/3)-1/3*(-1)^(
```

```
2/3)*b*d*Ci((-1)^(2/3)*b*c^(1/3)-b*(d*x+c)^(1/3))*sin(a+(-1)^(2/3)*b*c^(1/3
))/c^(2/3)
```

Rubi [A] (verified)

Time = 1.09 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3513, 3423, 3414, 3384, 3380, 3383}

$$\begin{aligned}&\int \frac{\cos\left(a+b\sqrt[3]{c+dx}\right)}{x^2}\,dx\\&=-\frac{bd\sin\left(a+b\sqrt[3]{c}\right)\text{CosIntegral}\left(b\sqrt[3]{c}-b\sqrt[3]{c+dx}\right)}{3c^{2/3}}\\&+\frac{\sqrt[3]{-1}bd\sin\left(a-\sqrt[3]{-1}b\sqrt[3]{c}\right)\text{CosIntegral}\left(\sqrt[3]{-1}\sqrt[3]{c}b+\sqrt[3]{c+dx}b\right)}{3c^{2/3}}\\&-\frac{(-1)^{2/3}bd\sin\left(a+(-1)^{2/3}b\sqrt[3]{c}\right)\text{CosIntegral}\left((-1)^{2/3}b\sqrt[3]{c}-b\sqrt[3]{c+dx}\right)}{3c^{2/3}}\\&+\frac{bd\cos\left(a+b\sqrt[3]{c}\right)\text{Si}\left(b\sqrt[3]{c}-b\sqrt[3]{c+dx}\right)}{3c^{2/3}}\\&+\frac{(-1)^{2/3}bd\cos\left(a+(-1)^{2/3}b\sqrt[3]{c}\right)\text{Si}\left((-1)^{2/3}b\sqrt[3]{c}-b\sqrt[3]{c+dx}\right)}{3c^{2/3}}\\&+\frac{\sqrt[3]{-1}bd\cos\left(a-\sqrt[3]{-1}b\sqrt[3]{c}\right)\text{Si}\left(\sqrt[3]{-1}\sqrt[3]{c}b+\sqrt[3]{c+dx}b\right)}{3c^{2/3}}-\frac{\cos\left(a+b\sqrt[3]{c+dx}\right)}{x}\end{aligned}$$

```
[In] Int[Cos[a + b*(c + d*x)^(1/3)]/x^2,x]
```

```
[Out] -(Cos[a + b*(c + d*x)^(1/3)]/x) - (b*d*CosIntegral[b*c^(1/3) - b*(c + d*x)^
(1/3)]*Sin[a + b*c^(1/3)])/(3*c^(2/3)) + ((-1)^(1/3)*b*d*CosIntegral[(-1)^(
1/3)*b*c^(1/3) + b*(c + d*x)^(1/3)]*Sin[a - (-1)^(1/3)*b*c^(1/3)])/(3*c^(2/
3)) - ((-1)^(2/3)*b*d*CosIntegral[(-1)^(2/3)*b*c^(1/3) - b*(c + d*x)^(1/3)]
*Sin[a + (-1)^(2/3)*b*c^(1/3)])/(3*c^(2/3)) + (b*d*Cos[a + b*c^(1/3)]*SinIn
tegral[b*c^(1/3) - b*(c + d*x)^(1/3)])/(3*c^(2/3)) + ((-1)^(2/3)*b*d*Cos[a
+ (-1)^(2/3)*b*c^(1/3)]*SinIntegral[(-1)^(2/3)*b*c^(1/3) - b*(c + d*x)^(1/3
)])/(3*c^(2/3)) + ((-1)^(1/3)*b*d*Cos[a - (-1)^(1/3)*b*c^(1/3)]*SinIntegral
[(-1)^(1/3)*b*c^(1/3) + b*(c + d*x)^(1/3)])/(3*c^(2/3))
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Int
[ExpandIntegrand[Sin[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 3423

```
Int[Cos[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_
), x_Symbol] :> Simp[e^m*(a + b*x^n)^(p + 1)*(Cos[c + d*x]/(b*n*(p + 1))),
x] + Dist[d*(e^m/(b*n*(p + 1))), Int[(a + b*x^n)^(p + 1)*Sin[c + d*x], x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && ILtQ[p, -1] && EqQ[m, n - 1] && (I
ntegerQ[n] || GtQ[e, 0])
```

Rule 3513

```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_.)*((g_
.) + (h_.)*(x_))^(m_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Cos[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rubi steps

$$\text{integral} = \frac{3\text{Subst}\left(\int \frac{x^2\cos(a+bx)}{\left(-\frac{c}{d}+\frac{x^3}{d}\right)^2}\,dx, x, \sqrt[3]{c+dx}\right)}{d}$$

$$= -\frac{\cos\left(a+b\sqrt[3]{c+dx}\right)}{x} - b\text{Subst}\left(\int \frac{\sin(a+bx)}{-\frac{c}{d}+\frac{x^3}{d}}\,dx, x, \sqrt[3]{c+dx}\right)$$

$$= -\frac{\cos\left(a+b\sqrt[3]{c+dx}\right)}{x} - b\text{Subst}\left(\int\left(-\frac{d\sin(a+bx)}{3c^{2/3}\left(\sqrt[3]{c}-x\right)} - \frac{d\sin(a+bx)}{3c^{2/3}\left(\sqrt[3]{c}+\sqrt[3]{-1}x\right)} - \frac{d\sin(a+bx)}{3c^{2/3}\left(\sqrt[3]{c}-(-1)^{2/3}x\right)}\right)dx, x, \sqrt[3]{c+dx}\right)$$

$$= -\frac{\cos\left(a+b\sqrt[3]{c+dx}\right)}{x} + \frac{(bd)\text{Subst}\left(\int\frac{\sin(a+bx)}{\sqrt[3]{c}-x}dx, x, \sqrt[3]{c+dx}\right)}{3c^{2/3}} + \frac{(bd)\text{Subst}\left(\int\frac{\sin(a+bx)}{\sqrt[3]{c}+\sqrt[3]{-1}x}dx, x, \sqrt[3]{c+dx}\right)}{3c^{2/3}} + \frac{(bd)\text{Subst}\left(\int\frac{\sin(a+bx)}{\sqrt[3]{c}-(-1)^{2/3}x}dx, x, \sqrt[3]{c+dx}\right)}{3c^{2/3}}$$

$$= -\frac{\cos\left(a+b\sqrt[3]{c+dx}\right)}{x} - \frac{\left(bd\cos\left(a+b\sqrt[3]{c}\right)\right)\text{Subst}\left(\int\frac{\sin\left(b\sqrt[3]{c}-bx\right)}{\sqrt[3]{c}-x}dx, x, \sqrt[3]{c+dx}\right)}{3c^{2/3}} + \frac{\left(bd\cos\left(a-\sqrt[3]{-1}b\sqrt[3]{c}\right)\right)\text{Subst}\left(\int\frac{\sin\left(\sqrt[3]{-1}b\sqrt[3]{c}+bx\right)}{\sqrt[3]{c}-(-1)^{2/3}x}dx, x, \sqrt[3]{c+dx}\right)}{3c^{2/3}} - \frac{\left(bd\cos\left(a+(-1)^{2/3}b\sqrt[3]{c}\right)\right)\text{Subst}\left(\int\frac{\sin\left((-1)^{2/3}b\sqrt[3]{c}-bx\right)}{\sqrt[3]{c}+\sqrt[3]{-1}x}dx, x, \sqrt[3]{c+dx}\right)}{3c^{2/3}} + \frac{\left(bd\sin\left(a+b\sqrt[3]{c}\right)\right)\text{Subst}\left(\int\frac{\cos\left(b\sqrt[3]{c}-bx\right)}{\sqrt[3]{c}-x}dx, x, \sqrt[3]{c+dx}\right)}{3c^{2/3}} + \frac{\left(bd\sin\left(a-\sqrt[3]{-1}b\sqrt[3]{c}\right)\right)\text{Subst}\left(\int\frac{\cos\left(\sqrt[3]{-1}b\sqrt[3]{c}+bx\right)}{\sqrt[3]{c}-(-1)^{2/3}x}dx, x, \sqrt[3]{c+dx}\right)}{3c^{2/3}} + \frac{\left(bd\sin\left(a+(-1)^{2/3}b\sqrt[3]{c}\right)\right)\text{Subst}\left(\int\frac{\cos\left((-1)^{2/3}b\sqrt[3]{c}-bx\right)}{\sqrt[3]{c}+\sqrt[3]{-1}x}dx, x, \sqrt[3]{c+dx}\right)}{3c^{2/3}}$$

$$= -\frac{\cos\left(a+b\sqrt[3]{c+dx}\right)}{x} - \frac{bd\,\mathrm{CosIntegral}\left(b\sqrt[3]{c}-b\sqrt[3]{c+dx}\right)\sin\left(a+b\sqrt[3]{c}\right)}{3c^{2/3}} + \frac{\sqrt[3]{-1}bd\,\mathrm{CosIntegral}\left(\sqrt[3]{-1}b\sqrt[3]{c}+b\sqrt[3]{c+dx}\right)\sin\left(a-\sqrt[3]{-1}b\sqrt[3]{c}\right)}{3c^{2/3}} - \frac{(-1)^{2/3}bd\,\mathrm{CosIntegral}\left((-1)^{2/3}b\sqrt[3]{c}-b\sqrt[3]{c+dx}\right)\sin\left(a+(-1)^{2/3}b\sqrt[3]{c}\right)}{3c^{2/3}} + \frac{bd\cos\left(a+b\sqrt[3]{c}\right)\mathrm{Si}\left(b\sqrt[3]{c}-b\sqrt[3]{c+dx}\right)}{3c^{2/3}} + \frac{(-1)^{2/3}bd\cos\left(a+(-1)^{2/3}b\sqrt[3]{c}\right)\mathrm{Si}\left((-1)^{2/3}b\sqrt[3]{c}-b\sqrt[3]{c+dx}\right)}{3c^{2/3}} + \frac{\sqrt[3]{-1}bd\cos\left(a-\sqrt[3]{-1}b\sqrt[3]{c}\right)\mathrm{Si}\left(\sqrt[3]{-1}b\sqrt[3]{c}+b\sqrt[3]{c+dx}\right)}{3c^{2/3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 0.77 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.42

$$\int \frac{\cos\left(a+b\sqrt[3]{c+dx}\right)}{x^2}\,dx = -\frac{\cos\left(a+b\sqrt[3]{c+dx}\right)}{x} - \frac{1}{6}ibd\mathrm{RootSum}\left[c-\#1^3\&,\frac{e^{-ia-ib\#1}\,\mathrm{ExpIntegralEi}\left(-ib\left(\sqrt[3]{c+dx}-\#1\right)\right)}{\#1^2}\&\right] + \frac{1}{6}ibd\mathrm{RootSum}\left[c-\#1^3\&,\frac{e^{ia+ib\#1}\,\mathrm{ExpIntegralEi}\left(ib\left(\sqrt[3]{c+dx}-\#1\right)\right)}{\#1^2}\&\right]$$

```
[In] Integrate[Cos[a + b*(c + d*x)^(1/3)]/x^2,x]
```

```
[Out] -(Cos[a + b*(c + d*x)^(1/3)]/x) - (I/6)*b*d*RootSum[c - #1^3 & , (E^((-I)*a
 - I*b*#1)*ExpIntegralEi[(-I)*b*((c + d*x)^(1/3) - #1)])/#1^2 & ] + (I/6)*b
*d*RootSum[c - #1^3 & , (E^(I*a + I*b*#1)*ExpIntegralEi[I*b*((c + d*x)^(1/3
) - #1)])/#1^2 & ]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.33 (sec) , antiderivative size = 931, normalized size of antiderivative = 2.80

method	result	size
derivativedivides	Expression too large to display	931
default	Expression too large to display	931

```
[In] int(cos(a+b*(d*x+c)^(1/3))/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 3*d/b^3*(b^6*a^2*(cos(a+b*(d*x+c)^(1/3))*(1/3/c/b^3*(a+b*(d*x+c)^(1/3))-1/3
*a/b^3/c)/(b^3*c+a^3-3*a^2*(a+b*(d*x+c)^(1/3))+3*a*(a+b*(d*x+c)^(1/3))^2-(a
+b*(d*x+c)^(1/3))^3)-2/9/c/b^3*sum(1/(_R1^2-2*_R1*a+a^2)*(Si(-b*(d*x+c)^(1/
3)+_R1-a)*sin(_R1)+Ci(b*(d*x+c)^(1/3)-_R1+a)*cos(_R1)),_R1=RootOf(-b^3*c+_Z
^3-3*_Z^2*a+3*_Z*a^2-a^3))+1/9/c/b^3*sum(1/(-_RR1+a)*(-Si(-b*(d*x+c)^(1/3)+
_RR1-a)*cos(_RR1)+Ci(b*(d*x+c)^(1/3)-_RR1+a)*sin(_RR1)),_RR1=RootOf(-b^3*c+
_Z^3-3*_Z^2*a+3*_Z*a^2-a^3)))+cos(a+b*(d*x+c)^(1/3))*(-2/3*a*b^3/c*(a+b*(d*
x+c)^(1/3))^2+2/3*a^2*b^3/c*(a+b*(d*x+c)^(1/3)))/(b^3*c+a^3-3*a^2*(a+b*(d*x
+c)^(1/3))+3*a*(a+b*(d*x+c)^(1/3))^2-(a+b*(d*x+c)^(1/3))^3)+2/9*a*b^3/c*sum
((_R1+a)/(_R1^2-2*_R1*a+a^2)*(Si(-b*(d*x+c)^(1/3)+_R1-a)*sin(_R1)+Ci(b*(d*x
+c)^(1/3)-_R1+a)*cos(_R1)),_R1=RootOf(-b^3*c+_Z^3-3*_Z^2*a+3*_Z*a^2-a^3))-2
/9*a*b^3/c*sum(_RR1/(-_RR1+a)*(-Si(-b*(d*x+c)^(1/3)+_RR1-a)*cos(_RR1)+Ci(b*
(d*x+c)^(1/3)-_RR1+a)*sin(_RR1)),_RR1=RootOf(-b^3*c+_Z^3-3*_Z^2*a+3*_Z*a^2-
a^3))+cos(a+b*(d*x+c)^(1/3))*(2/3*a*b^3/c*(a+b*(d*x+c)^(1/3))^2-a^2*b^3/c*(
a+b*(d*x+c)^(1/3))+1/3*b^3*(b^3*c+a^3)/c)/(b^3*c+a^3-3*a^2*(a+b*(d*x+c)^(1/
3))+3*a*(a+b*(d*x+c)^(1/3))^2-(a+b*(d*x+c)^(1/3))^3)-2/9*a*b^3/c*sum(_R1/(_
R1^2-2*_R1*a+a^2)*(Si(-b*(d*x+c)^(1/3)+_R1-a)*sin(_R1)+Ci(b*(d*x+c)^(1/3)-_
R1+a)*cos(_R1)),_R1=RootOf(-b^3*c+_Z^3-3*_Z^2*a+3*_Z*a^2-a^3))-1/9*b^3/c*su
m((b^3*c+2*_RR1^2*a-3*_RR1*a^2+a^3)/(_RR1^2-2*_RR1*a+a^2)*(-Si(-b*(d*x+c)^(
1/3)+_RR1-a)*cos(_RR1)+Ci(b*(d*x+c)^(1/3)-_RR1+a)*sin(_RR1)),_RR1=RootOf(-b
^3*c+_Z^3-3*_Z^2*a+3*_Z*a^2-a^3)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 406, normalized size of antiderivative = 1.22

$$\int \frac{\cos\left(a+b\sqrt[3]{c+dx}\right)}{x^2}\,dx =$$

$$-\frac{2\left(i\,b^3c\right)^{\frac{1}{3}}dx\mathrm{Ei}\left(i\,(dx+c)^{\frac{1}{3}}b+\left(i\,b^3c\right)^{\frac{1}{3}}\right)e^{\left(i\,a-\left(i\,b^3c\right)^{\frac{1}{3}}\right)}+2\left(-i\,b^3c\right)^{\frac{1}{3}}dx\mathrm{Ei}\left(-i\,(dx+c)^{\frac{1}{3}}b+\left(-i\,b^3c\right)^{\frac{1}{3}}\right)e^{\left(-\cdots\right.}}{\cdots}$$

```
[In] integrate(cos(a+b*(d*x+c)^(1/3))/x^2,x, algorithm="fricas")
```

```
[Out] -1/12*(2*(I*b^3*c)^(1/3)*d*x*Ei(I*(d*x + c)^(1/3)*b + (I*b^3*c)^(1/3))*e^(I
*a - (I*b^3*c)^(1/3)) + 2*(-I*b^3*c)^(1/3)*d*x*Ei(-I*(d*x + c)^(1/3)*b + (-
I*b^3*c)^(1/3))*e^(-I*a - (-I*b^3*c)^(1/3)) - (I*b^3*c)^(1/3)*(I*sqrt(3)*d*
x + d*x)*Ei(I*(d*x + c)^(1/3)*b + 1/2*(I*b^3*c)^(1/3)*(-I*sqrt(3) - 1))*e^(
1/2*(I*b^3*c)^(1/3)*(I*sqrt(3) + 1) + I*a) - (-I*b^3*c)^(1/3)*(I*sqrt(3)*d*
x + d*x)*Ei(-I*(d*x + c)^(1/3)*b + 1/2*(-I*b^3*c)^(1/3)*(-I*sqrt(3) - 1))*e
^(1/2*(-I*b^3*c)^(1/3)*(I*sqrt(3) + 1) - I*a) - (I*b^3*c)^(1/3)*(-I*sqrt(3)
*d*x + d*x)*Ei(I*(d*x + c)^(1/3)*b + 1/2*(I*b^3*c)^(1/3)*(I*sqrt(3) - 1))*e
^(1/2*(I*b^3*c)^(1/3)*(-I*sqrt(3) + 1) + I*a) - (-I*b^3*c)^(1/3)*(-I*sqrt(3
)*d*x + d*x)*Ei(-I*(d*x + c)^(1/3)*b + 1/2*(-I*b^3*c)^(1/3)*(I*sqrt(3) - 1)
)*e^(1/2*(-I*b^3*c)^(1/3)*(-I*sqrt(3) + 1) - I*a) + 12*c*cos((d*x + c)^(1/3
)*b + a))/(c*x)
```

Sympy [F]

$$\int \frac{\cos\left(a+b\sqrt[3]{c+dx}\right)}{x^2}\,dx=\int \frac{\cos\left(a+b\sqrt[3]{c+dx}\right)}{x^2}\,dx$$

```
[In] integrate(cos(a+b*(d*x+c)**(1/3))/x**2,x)
```

```
[Out] Integral(cos(a + b*(c + d*x)**(1/3))/x**2, x)
```

Maxima [F]

$$\int \frac{\cos\left(a+b\sqrt[3]{c+dx}\right)}{x^2}\,dx=\int \frac{\cos\left((dx+c)^{\frac{1}{3}}b+a\right)}{x^2}\,dx$$

```
[In] integrate(cos(a+b*(d*x+c)^(1/3))/x^2,x, algorithm="maxima")
```

```
[Out] integrate(cos((d*x + c)^(1/3)*b + a)/x^2, x)
```

Giac [F]

$$\int \frac{\cos\left(a+b\sqrt[3]{c+dx}\right)}{x^2}\,dx=\int \frac{\cos\left((dx+c)^{\frac{1}{3}}b+a\right)}{x^2}\,dx$$

```
[In] integrate(cos(a+b*(d*x+c)^(1/3))/x^2,x, algorithm="giac")
```

```
[Out] integrate(cos((d*x + c)^(1/3)*b + a)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos\left(a+b\sqrt[3]{c+dx}\right)}{x^2}\,dx=\int \frac{\cos\left(a+b\,(c+d\,x)^{1/3}\right)}{x^2}\,dx$$

```
[In] int(cos(a + b*(c + d*x)^(1/3))/x^2,x)
[Out] int(cos(a + b*(c + d*x)^(1/3))/x^2, x)
```

CHAPTER 4

APPENDIX

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
```

```
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafCou
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count is
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $"<
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<>T
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
```

```
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
```

```
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
```

```
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                convert(leaf_count_result,string)," vs. $2 (",
```

```
                                    convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_c
                    end if
                else #result contains complex but optimal is not
                    if debug then
                            print("result contains complex but optimal is not");
                    fi;
                    return "C","Result contains complex when optimal does not.";
                fi;
            else # result do not contain complex
                 # this assumes optimal do not as well. No check is needed here.
                if debug then
                       print("result do not contain complex, this assumes optimal do not as well")
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    if debug then
                        print("leaf_count_result<=2*leaf_count_optimal");
                    fi;
                    return "A"," ";
                else
                    if debug then
                        print("leaf_count_result>2*leaf_count_optimal");
                    fi;
                    return "B",cat("Leaf count of result is larger than twice the leaf count of opt
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_count
                fi;
            fi;
        else #ExpnType(result) > ExpnType(optimal)
            if debug then
                print("ExpnType(result) > ExpnType(optimal)");
            fi;
            return "C",cat("Result contains higher order function than in optimal. Order ",
                           convert(ExpnType_result,string)," vs. order ",
                           convert(ExpnType_optimal,string),".");
        fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:
```

```
# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
```

```
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnTy
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is large
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
         'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
         'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
         'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
         'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-t
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
      return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])    #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinsta
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)



    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger th
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_c
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_result


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```